1. Domains of quantification in natural language

We design new languages, by and large, in order to bypass complexities and limitations within the languages we already have. But when we are concerned with language itself we should guard against projecting the simple and powerful syntax and semantics we have concocted back into the sentences we encounter. For some of the features of English, French, or Ancient Greek we routinely abstract away from in the process of formalization might be linguistic universals – the very features that set human languages apart from all the other conceivable ones.¹ How similar natural languages really are to formal ones is an empirical question for linguistics. ² The answer is of philosophical concern: if the difference turns out to be great, we should worry whether we can even express in the vernacular the results of logic, mathematics, or physics.

When it comes to quantification, natural languages and the standard formal ones differ in at least three important ways: in terms of binding, scope, and domain. The first two have been much discussed in the last decades. The syntactic devices handling binding and scope in familiar formal languages – variables and parentheses – are entirely missing from the surface of the languages we speak. We are so used to reintroducing them at the level of logical form that it is useful to remind ourselves every now and then that there is no pre-theoretical evidence that they are actually there. Nor is it true to say that they must be, for otherwise quantification would be a mystery – we have empirically successful theories of quantification that do away with variables,

¹ I take it that a linguistic universal is not simply a feature that all actual human languages happen to share. Rather, it is something human languages must share – a feature required for the primary acquisition of the language by human beings under ordinary conditions.

² Richard Montague famously said that there are no important theoretical differences between natural and formal languages; cf. Montague (1970): 222. He illustrated his contention by providing a translation of a fragment of English into the language of type-theoretic intensional logic. The illustration fails to prove the point for a number of reasons. The fragment of English is small and its grammar artificially simple. The translation was supposed to preserve syntactic constituency, but not atomicity. (A number of lexical items, most notably the quantifiers, were translated as complex λ-terms.) And while the translation may have preserved truth-conditions, it was never meant to preserve the intuitive semantic values of words. (For example, the referents of proper names turned out to be functions from indices to sets of sets of individuals.)
and others that allow quantifiers to bind variables outside their scope. Our usual way of handling quantification certainly feels more natural than the way this is done in variable-free or dynamic theories. But it is not easy to tell whether this feeling tracks anything beyond sheer familiarity.

The third difference between natural and artificial languages in expressing quantification – the domain of quantifiers – is the topic of the present paper. Quantifiers in familiar formal languages take a formula as input and yield another as output, while typical natural language quantifiers combine with a nominal expression to form a complex quantificational phrase. This syntactic difference is correlated with a semantic one. The function of the nominal expression with which the quantifier combines is to restrict what the quantifier ranges over. By contrast, in formal languages there tends to be nothing in a quantified formula that restricts the domain. The semantics usually specifies a single domain of quantification for the entire language.

These syntactic and semantic differences are illustrated by (1) and (2), a simple quantified sentence in English and its usual translation into the language of first-order logic:

(1) Every dog is asleep.
(2) ∀x (dog(x) → asleep(x))

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3 For a discussion of syntax and semantics for natural language without variables, see Jacobson (1999) and (2002). (Note that the usual formulations of variable-free semantics employ variables in the meta-language. However, all those variables are bound: the expressions of the object-language are associated with λ-expressions in the meta-language. This makes the use of variables a mere convenience; cf. Jacobson (1999): 121.) Many dynamic theories allow for a quantifier to bind a variable beyond its scope; see for example Groenendijk and Stokhof (1990) and (1991). (Note that one could introduce a non-syntactic notion of scope within these languages. Given that notion, quantifiers cannot bind variables outside their scope.)

4 Even if we take variables and parentheses for granted we need extra machinery to capture the behavior of natural language quantifiers. We have different types of expressions that can be bound – ordinary pronouns, reflexives, and perhaps various sorts of empty elements. These expressions can occur in different linguistic environments and have uses that are unlike the uses of variables in formal languages. Despite its many empirical problems, the structural relation of c-command may be a good first approximation of syntactic scope. But it can only be plausible as a full characterization if we allow type-shifting or quantifier raising. Unlike their formal counterparts, natural language quantifiers seem to differ from one another in the scope possibilities they permit – so much so, that some have suggested different syntactic positions for different types of quantifiers. Philosophers like to dismiss these as idiosyncratic features of particular natural languages, but this is not a plausible stance. Many of them are likely linguistic universals.
Standard syntactic tests establish that ‘every dog’ is a constituent of (1) while ‘every … is asleep’ or ‘every … asleep’ are not. Thus the syntactic relationship between ‘every’ and ‘dog’ is more intimate than the syntactic relationship between ‘every’ and ‘asleep’. By contrast, the quantifier ‘∀x’ bears the exact same syntactic relationship to the two predicates ‘dog(x)’ and ‘asleep(x)’ within (2). On the semantic side, in (1) ‘dog’ constrains what ‘every’ quantifies over and ‘asleep’ does not: the sentence is about dogs, not about things asleep. By contrast, the predicate ‘dog(x)’ in (2) plays no role in domain selection.

The suggestion that unlike (2), (1) is about dogs is likely to meet with some skepticism. Intuitions of aboutness are notoriously shaky, presumably because there is more than one sense in which a sentence can be about something or other. My hope is that asymmetry in topicalization can help to single out the sense relevant here: ‘Dogs, every one of them is asleep’ is fairly natural, ‘Things asleep, every dog is one of them’ not so much. A reasonable conjecture is that the possibility of topicalizing ‘dog’ in a paraphrase is a consequence of the fact that the truth-value of (1) does not depend on how things stand with non-dogs: had there been more cats asleep or fewer mice awake it would have made no difference. In this sense (1) makes a claim about dogs and says nothing whatsoever about non-dogs. By contrast, the truth-value of (1) does depend on how things stand with things not asleep: it matters whether there are dogs among them. Accordingly, (1) is no more about things asleep than about things not asleep.

There is a general pattern here. While the truth-value of none of the sentences under (3) depends on how things are with things that are not Fs, the truth-value of some (to wit, the last three) does depend on how things are with things that are not G:

(3) Some/no/five/more than six but fewer than nine/exactly 60% of/almost all Fs are G

This asymmetry appears to be a genuine linguistic universal; it would be highly desirable to capture it in semantic theories for natural languages. A straightforward way to do so is to

5 The putative universal is a consequence of two others: that binary natural language quantifiers satisfy the conditions extension and conservativity. For definitions and discussion see chapter 4.5 in Peters and Westerståhl (2006). Extension is an uncontroversial feature of natural language quantifiers. While there are putative
endorse the idea that at the level of logical form all sentences under (3) have a *tripartite structure*:\(^6\) the leftmost (quantifier) position occupied by a determiner, the middle (restrictor) position occupied by ‘F’s and the rightmost (nuclear scope) position occupied by ‘Gs’. Let’s also assume that the expressions in the latter two positions contribute to the interpretation of the quantified sentence as their names suggest – the first by restricting the domain, the second by providing the scope.\(^7\) Then the sentences under (3) express some relation of quantity between Gs and non-Gs within a domain restricted to Fs. This makes the desired prediction: whether there are things that are not F is irrelevant to the truth or falsity of these sentences, no matter what the quantifier might be.\(^8\)

It is an interesting hypothesis that all quantification in natural language works this way – that every quantified sentence in every natural language contains some expression or other whose interpretation constrains the domain of quantification. If the hypothesis is correct, quantificational domains are fixed in a fundamentally different way in natural languages and in standard artificial languages. This would be a difference potentially far more significant than the ones concerning binding and scope. For those of us who think sentences with different logical forms express different propositions it would mean that *no proposition expressed in a typical formal language is expressible in any natural language.*\(^9\)

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\(^6\) The idea of tripartite structure goes back to Lewis (1975), Kamp (1981), and Heim (1982). For a historical introduction of the idea, see chapter 2.1. of Hajičová, Partee and Sgall (1998).

\(^7\) The labels *restrictor* and *nuclear scope* are from Heim (1982). Many who employ the terms do not take restrictors to be literally restricting quantificational domains, as I suggest. But a mere mnemonic yields no explanation for the semantic asymmetry between these two positions.

\(^8\) The point that the conservativity of natural language quantification is explained by such a hypothesis (and remains unexplained within standard generalized quantifier theory) is emphasized in Ben-Yami (2009).

\(^9\) Some would dismiss such a hypothesis on the grounds that expressions of artificial languages are understood by translating them to expressions of a natural language. As far as I know, this claim has never been supported by empirical evidence. *Prime facie,* it is rather implausible – I know mathematicians who are brilliant with complex formulae but horrible at explaining their proofs in plain English. Given my experience of teaching logic, I am inclined to think that after a brief (and often misleading) elucidation, people tend to acquire formal languages by immersion, not by translation.
I begin by clarifying the hypothesis (section 2) and by arguing that there is no easy way to defeat it (section 3). Then I will present a counterexample (section 4) and argue that it generalizes broadly (section 5). The counterexample is a hitherto largely neglected reading of quantified attitude-reports, which is of independent interest. The upshot of the paper is that although there is a real threat to our comfortable assumption that the sort of bare quantification we routinely employ in formal languages can be fully captured in English, there is also reason to think that the threat can be averted.

2. Clarification

Let’s say that a quantifier occurs bare within a sentence just in case none of the domains of that occurrence are restricted by the extension of any expression in the sentence. If we ignore multiple occurrences of a quantifier in a single sentence as well as multiple domains associated with an occurrence of a single quantifier (which I will henceforth do) we can simplify the definition: a quantifier is bare in a sentence iff its domain is not restricted by the extension of any expression in the sentence. A quantifier is bare simpliciter just in case it is bare in some sentence. The hypothesis under consideration is that natural languages contain no bare quantifiers.

This hypothesis cannot be proved – in fact, I don’t even know a knock-down argument against the thesis that quantification in natural languages is always bare. The view that run-of-the-mill occurrences of quantificational determiners are not bare is supported solely by the explanatory considerations presented in the previous section. These are probative but certainly not conclusive. The interesting question is whether the hypothesis can be refuted – whether there are quantifiers in natural languages that adequate semantic theories must construe as bare.

The definition of a bare quantifier is in need of elucidation. Let’s start with the word ‘quantifier’. Quantifiers are expressions of generality which occur in a wide variety of syntactic categories –

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10 ‘More dogs than cats are asleep’ plausibly quantifies over two domains – one that consists of dogs and another that consists of cats. The sentence says of these domains that more things are asleep in the former than in the latter.
they can be articles (‘the’), numerals (‘five’), adjectives (‘countless’), adverbs (‘usually’), nouns (‘everyone’), and of course determiners (‘many’). Complex phrases can also be quantifiers (‘at least five’, ‘more … than’, ‘exactly five times as many … as’, etc.). Whether an expression is a quantifier is sometimes a matter of controversy (cf. the definite and indefinite articles, tense markers, modal auxiliaries, and even propositional attitude verbs) and sometimes a matter of indifference (cf. the adverbial ‘most’ in ‘Cats fear dogs most’). I will not rest my case on these.

By ‘domain’ I mean the things a quantifier ranges over. I do not mean the set of those things; some domains – like the domains of quantifiers in the axioms of set-theory – cannot be sets. When I say that the extension of an expression ‘restricts’ the domain of a quantifier I mean that everything in the domain is included in the extension. By ‘extension’ I mean the things the expression can be truthfully predicated of. Again, I do not assume that these things always form a set; the things of which ‘set’ can be truthfully predicated don’t. We do best talking plurally when discussing what belongs to a domain or an extension.

Expressions that have extensions in the sense I am using this term are expressions that can be *predicated*. But this does not mean that they all must be *predicates*. For example, I maintain that ‘dog’ is a referring expression. I am uncertain whether it refers singularly to the species *canis lupus familiaris* or plurally to each individual dog. Either way, it is not a predicate. But whether I am right about this is irrelevant for the purposes of this paper. Everyone agrees that ‘dog’ can be used to predicate when it occurs within the phrase ‘is a dog.’ The things it can be truthfully predicated of constitute the extension of ‘dog’.

My final comment concerns the seemingly straightforward issue whether an expression occurs ‘in’ a sentence. In assessing this, I will always consider the logical form of the sentence. By ‘logical form’ I mean a level of representation which encompasses all and only information

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11 Adjectives are rarely mentioned among the categories that can contain quantifiers. But since adverbs of quantification exist and many of them have adjectival correlates (‘usually’/‘usual’, ‘frequently’/‘frequent’, ‘rarely’/‘rare’), this is probably just an oversight.

12 There is solid evidence that predicative occurrences of nouns and adjectives have extra syntactic structure; cf. Bowers (2001). Following Chierchia and Turner (1988) Bowers assumes that the extra structure is interpreted as a type-shifter mapping properties (which he considers as individuals) to propositional functions.
required for semantic interpretation. Whether such a level of representation has anything to do with logic is a question I want to leave open in this paper. As usual in the semantic literature, I assume that logical form is systematically derived in syntax and thus respects constituency. Some authors take logical form to be by necessity distinct form surface form. While I am sympathetic towards movement and empty elements in syntax I won’t take stand on the precise nature of the relationship between surface form and logical form. Nonetheless, as a methodological principle, I will assume that unless there is some reason to think otherwise, only words we hear are present at logical form.

Let’s see the definition in action. Consider the sentence ‘Few dogs are asleep’. The logical form of this sentence contains an expression (to wit, the noun ‘dog’) whose extension (i.e. the things of which ‘dog’ can be truthfully predicated) arguably includes everything the quantifier (i.e. the determiner ‘few’) ranges over. If so, ‘few’ is not bare in this sentence. Nonetheless, ‘few’ might still be bare simpliciter; whether it is depends on whether it is bare in some other sentence.

Before I try to address the question whether natural languages have bare quantifiers, I need to distinguish it from three others in the neighborhood. The first such question is whether natural languages can express unrestricted quantification. An occurrence of a quantifier is unrestricted just in case nothing restricts its domain; it is bare if no extension of an expression in the sentence restricts it. Thus unrestricted occurrences of quantifiers must be bare but the converse does not hold. The domains of quantifiers in standard formal languages are restricted (to the universe of a model) but bare.

Quantificational domains in natural languages are often restricted by context. One might conjecture that contextual restrictions on quantificational domains are due to contextual restrictions on the extension of some expression within the sentence where the quantifier occurs.¹³ Suppose we are visiting a friend who owns seven dogs. They are all in the back yard – four of them running around, three sound asleep. In the case one can certainly say something true

¹³ See Stanley and Szabó (2000) and Stanley (2002). Both papers discuss quantificational determiners only; they identify the extension of the nominal expression in construction with the quantifier as the source of contextual restriction.
by uttering ‘Exactly three dogs are asleep’ despite the fact that the number of sleeping dogs in the universe far exceeds three. The dogs not in the yard don’t count. Proponents of the conjecture would insist that this is because in this context predicating ‘dog’ of one of those remote dogs would not be true. The idea that contextual domain restriction operates on an over restrictor has plausibility in this particular case but it is doubtful whether it is defensible for all natural language quantifiers.

The question whether natural languages have bare quantifiers must also be distinguished from the question whether natural languages have the resources to express absolute quantification, i.e. quantification over the all-inclusive domain. It is a matter of considerable philosophical controversy whether the idea of absolute quantification is coherent. One prominent reason for doubt is a certain take on the set-theoretic paradoxes. Following Michael Dummett, some have argued that the right moral to draw from these is that certain concepts – such as the concept of set, the concept of ordinal, and of course the concept of entity – are indefinitely extensible. For any particular way to try to specify what belongs to their extensions we can specify a more inclusive extension by identifying something that belongs to the latter but not to the former. If there really are indefinitely extensible concepts, absolute quantification seems unattainable. And even if it were attainable, one might seriously doubt whether our quantifiers can pull off the trick.

This is not a question I plan to settle: whether natural language quantifiers are absolute and whether they are bare are orthogonal problems. Bare quantifiers can certainly have less than all-inclusive domains – the quantifiers of first-order logic are a case in point. Given standard model-theoretic semantics, these quantifiers range over the universe of any model relative to which the

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14 This much is uncontroversial. But there is considerable disagreement whether the fact that the sentence can be used to say something true in this context means that the sentence itself is true in this context. Some will say the sentence fails to express anything truth-evaluable and some that it expresses something false. Those who are attracted to these views must handle domain restriction at the level of speech acts.

15 Later I will argue that natural languages have bare quantifiers. I certainly don’t think that quantificational domains in ordinary utterances involving my examples are contextually unrestricted.

16 A recent collection of papers on the issue is Rayo and Uzquiano (2006).
language is interpreted. These are sets and, as we know from Cantor’s theorem, no set is all-inclusive. Conversely, absolute quantification needn’t be bare. Suppose, for example, that the extension of ‘entity’ is all-inclusive – then ‘At least six entities are asleep’ quantifies over the all-inclusive domain. But the occurrence of the quantifier ‘at least six’ in this sentence is not bare: its domain is restricted by the extension of ‘entity.’ This is not restricting in the sense of excluding something from the domain; the domain is restricted because the interpretation of an expression within the sentence makes a demand on what is included in the domain. That the demand is vacuous changes nothing.\textsuperscript{17,18}

Finally, the question whether a quantifier is bare is distinct from the question whether it is \textit{unary}. Unary quantifiers combine with a single expression; binary quantifiers combine simultaneously with two. The quantifiers of first-order logic are both unary and bare; quantificational determiners in natural languages are often construed as a binary and not bare. But the association between being unary and being bare and between being binary and not being bare is far from necessary.

One could construe ordinary quantificational determiners as binary but bare. It is perfectly coherent to think that ‘Most dogs are asleep’ quantifies over a domain including both dogs and non-dogs – it expresses a proposition that is true just in case a certain quantitative relation holds between the dogs within the domain and the things asleep within the domain.\textsuperscript{19} Conversely, it is easy to interpret the unary quantifier in ‘\(\exists x (\text{dog}(x) \land \text{asleep}(x))\)’ as non-bare. All we need is a semantic clause that “reaches into” the complex predicate in the scope of the quantifier, finds an

\textsuperscript{17} After all, I take it that one would not want to say that ‘at least five’ occurs bare in ‘At least five things that are prime numbers between 19 and 23 are famous’ just because (necessarily) there are no prime numbers between 19 and 23.

\textsuperscript{18} Note that unrestricted quantifiers need not be absolute. It might be that nothing restricts the domain but the domain still fails to be all-inclusive simply because there is no such thing as an all-inclusive domain. I already mentioned that absolute quantification needn’t be bare – \textit{a fortiori} it needn’t be unrestricted. (However, if absolute quantification is restricted, it must be vacuously restricted.) So whether there is absolute quantification and whether there is unrestricted quantification are orthogonal issues.

\textsuperscript{19} Such a semantics can get the truth-conditions right but will arguably miss a connection between the syntactic position and semantic function of the two predicates in the sentence.
expression, and uses its extension to restrict the domain.20 Let $\sigma$ be the interpretation function for the language, $f$ an assignment function, and $P$ a one-place predicate; then $f^\sigma(P)$ is a function that assigns the same values to all variables as $f$ as long as that value satisfies $\sigma(P)$, and is otherwise undefined. Then (4) would do the trick.21

\begin{equation}
\exists x \ (\text{dog}(x) \land \text{asleep}(x))
\end{equation}

is true with respect to assignment function $f$ iff there is an object $o$ such that ‘asleep($x$)’ is true with respect to $f^{\sigma(\text{dog})}_{[x:o]}$.

So, the syntactic question whether a quantifier is unary is in principle completely independent of the semantic question whether it is bare.

Having separated bare quantifiers from unrestricted, absolute, and unary quantifiers I hasten to emphasize that the existence of any of these devices in natural languages cannot be discussed in complete isolation from the others. If there are unary quantifiers in natural languages they are very likely to be bare. After all, unary quantifiers can be combined with lexical predicates and it is far from clear how such occurrences could fail to be bare. And while quantificational domains may be restricted in a variety of ways, the most straightforward device of domain restriction is the extension of an expression within the quantified sentence. If natural languages contain bare quantifiers some of their occurrences will likely be altogether unrestricted.22 And without any constraint being put on the domain of quantification we will likely end up quantifying over the all-inclusive domain – unless the existence of such a domain is impossible. If bare quantification is abundant within natural languages, as I will argue, then opponents of absolute quantification

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20 Semantic rules that appeal to semantic values of non-immediate constituents violate local compositionality and are often shunned upon by semanticists.

21 I do not assume that the task of a semantic theory is to assign truth-conditions. I do, however, assume that an adequate semantic theory for a language (perhaps in conjunction with some extra principles, like instances of a disquotational schema) must entail adequate truth-conditions for the sentences of that language. When I “read off” the domain from the right hand side of these semantic clauses I assume that in the meta-language all linguistic restriction of quantification is made manifest.

22 Note, however, the complication mentioned at the very end of this paper. There might be a way to construe the bare occurrences of quantifiers identified in this paper as restricted by the extension of a predicate constructible from expressions within the quantified sentence, as long as one is willing to live with a merely past, merely possible, and merely imagined entities.
\end{flushleft}
will have to base their opposition entirely on the contentions claim that quantification over absolutely everything is incoherent.

3. Non-triviality

There are a number of constructions in English which include *prima facie* bare occurrences of quantifiers. In this section, I will argue that none of these occurrences are obviously bare. All this is to prepare the ground for what I take to be a much stronger argument in the next section for the presence of bare quantifiers in natural languages.

Let’s begin with quantificational sentences where a determiner all by itself takes the subject position:

(5) Some are in the dining room.
(6) Many were completely forgotten.

These sentences sound fine if they are used in the right context. For example, (5) can be felicitously uttered as a response to the question ‘Are there any cookies left?’ and (6) as following the remark ‘It is good that Chekhov’s early short stories have been reprinted.’ In such contexts, (5) is quantifying over cookies and (6) over Chekhov’s early short stories. But there seems to be no words or phrases in these sentences whose interpretation is responsible for the domain restrictions. So, it looks like ‘some’ and ‘many’ are bare quantifiers.

But the appearance may well be deceptive. It is plausible that in the cases described (5) and (6) are used elliptically, which means – according to many theories of ellipsis – that the antecedent material is actually present in the sentences in a phonologically null form. Moreover, if ‘some’ and ‘every’ are not ambiguous then they are binary quantificational determiners in all their occurrences, which is to say that they always need a nominal expression to form a quantifying phrase and only then can they combine with a verb phrase to yield a full quantified sentence. When the nominal expression is not given explicitly, it must be picked up from the context. Postulating a unary ‘some’ and ‘many’ on the basis of examples like (5) and (6) seems to be an overreaction. In the right context *both* the nominal expression and the verb phrase could be dropped: (7) is fine as a response to the question ‘Are the cookies in the dining room?’ and (8)
works as a continuation of ‘It is a mistake to think that none of Chekhov’s early short stories could ever be completely forgotten.’

(7) Some are.
(8) Many were.

Should we see (7) and (8) as evidence that besides the binary and unary interpenetrations, ‘some’ and ‘many’ permit a nullary reading as well? I think it’s better to stick with the ellipsis story. But then we have no evidence for bare quantification in examples (5) – (8).

Next, let us take a look at the class of single word quantifiers, such as ‘anyone’, ‘everything’, ‘nowhere’, or ‘somehow’. These are surely unary and they typically occur non-elliptically. They might be viewed as paradigm cases of bare quantifiers.

The obvious response is that these expressions are compound quantifiers and they contain a restricting expression. This suggestion makes perfect sense, when it comes to single word quantifiers formed from ‘-one’ and ‘-body’ – morphemes which apply to people. The idea is truth-conditionally unobjectionable: ‘Someone came’ is equivalent to ‘Some person came’ and ‘Nobody cares’ to ‘No person cares’. We can say something similar about ‘-thing’ within ‘everything’, ‘something’ and ‘nothing’. One may object that ‘-thing’ is a bound morpheme and its application is less restricted than that of the noun ‘thing’, but even if correct, this suggestion makes no substantive difference to our present concern. Perhaps not everything is a thing – still, it remains plausible that ‘everything’ in all its occurrences quantifies over whatever the morpheme ‘-thing’ within that occurrence applies to.23

There is an extra wrinkle when it comes to compounds formed from ‘-where’. It is easy to imagine that this morpheme applies to places, but ‘It is cold everywhere’ cannot be unpacked as ‘It is cold every place’. We need something like ‘It is cold at every place’. One might worry that

23 There is a possible counterexample to this claim. Consider the pair of sentences ‘Everything is quiet’ and ‘Every thing is quiet.’ Imagine that you are in a room and that you hear a noise, but not a noise coming from any particular thing there. In this case the first sentence seems false but the second true. Moreover, it seems hard to point at something that could be both in the extension of ‘-thing’ and also in the extension of ‘not quiet’. While I think this is an interesting case, I don’t think is shows that ‘everything’ is a bare quantifier. This is because it is questionable is whether ‘everything’ as it occurs in ‘Everything is quiet’ is a quantifier at all. ‘Everything is quiet’ is equivalent to ‘It is quiet’; if the latter isn’t quantifying over anything (which seems plausible), the former probably isn’t either.
this shows that the morpheme must have a more complex meaning and thus its extension cannot restrict the quantificational domain to places. But the worry is misplaced: we don’t have to assume that the meaning of this word is compositionally derivable from the meaning of ‘every’ and ‘-where’. ‘Everywhere’ is an item in the lexicon whose meaning must be acquired individually. This fact does not conflict with the plausible idea that in addition to this meaning, the word also has a morphological constituent whose interpretation serves a particular semantic function. The same can be said about ‘-how’: its function is to restrict the domain of quantification to ways. The meaning of ‘somehow’ is not fully determined by the meanings of its parts: ‘They managed somehow’ is interpreted as ‘They managed in some way.’

The next candidates for bare quantification are existentials. The colloquial way to assert existence in English is to use a ‘there’-construction:

(9) There is a solution for this problem.

Philosophers occasionally say that ‘there’ (or ‘there is’) expresses the existential quantifier in (9) and if that were true this would surely be a bare occurrence of the existential quantifier. But the proposal is exceedingly implausible. If ‘there’ (or ‘there is’) expresses the existential quantifier why can it never substitute for ‘some’? And what is the semantic function of the indefinite article in (9)? Why can it be replaced by ‘no’ and (assuming the noun and the copula are appropriately pluralized) by ‘at least five’, ‘many’, ‘a lot of’ or ‘no more than a handful of’? In light of all these puzzling questions it is hardly surprising that such a proposal has no adherents among semanticists. It seems more likely that ‘there’ makes no semantic contribution to the sentence

24 This is not to say that ‘somewhere’ and ‘somehow’ are semantically alike. There is a case for saying that the position occupied by the quantifier in ‘Jill lives somewhere’ is an argument of the verb, but this is certainly not true for ‘Jill lives somehow.’ But this difference need not affect the point that both ‘-where’ and ‘-how’ can be seen as restricting the domains of quantification in these sentences.

25 Quantifying over ways may raise some eyebrows. The trouble is not so much that ways are mysterious; it is rather that they are cheap. The inference schema ‘S; therefore somehow S’ seems impeccable. But it seems hard to believe that we are committed to ways by virtue of saying anything at all! Note, however, that postulating such unavoidable ontological commitments is not unprecedented. After all, the inference schema ‘S; therefore possibly S’ is equally impeccable and this has not stopped some philosophers to construe ‘possibly’ as a quantifier. Moreover, if possible worlds are really ways the world could be then quantification over possible worlds is but a special case of quantification over ways. For those who would rather eschew ontological commitment to ways, Rayo and Yablo (2001) offer an elegant semantics. Of course, if they are right, ‘somehow’ is not a bare quantifier.
whatsoever (it is there only because the subject position must be filled somehow) and that the quantificational force of the sentence has a different source. Those of us who have sympathies for Russell’s theory of descriptions are inclined to think that the source is the indefinite article itself.

One might concede all this and remain convinced that quantification is bare in (9). Consider the sentence (10):

(10) There is a solution for this problem in a paper published last year.

The function of the prepositional phrase – also called the coda of the ‘there’-sentence – is to constrain where a solution for the problem is said to be. Thus, it might be viewed as restricting the domain of existential quantification expressed by the indefinite article. In other words, the logical form of (10) might look like (10’):

(10’) ∃x: in a paper published last year (x)(solution for this problem (x))

If this is on the right track, (9) should probably be seen as expressing bare existential quantification. In general, the idea is that ‘there’-sentences contain an optional coda whose semantic function is to restrict the domain of quantification. When the coda is missing the domain remains unrestricted.26

But codas do not work this way. We can tell because codas can contain further quantifiers, and those can take wide scope. If the coda functions as a restrictor on the existential quantifier, the only reading we get for (11) is (11’), but that reading requires the same solution in every textbook. The more natural reading permitting different solutions in different textbooks remains unaccounted for.

(11) There is a solution for this problem in every introductory textbook
(11’) ∃x: in every introductory textbook (x)(solution for this problem (x))

Here is a better idea about the semantics of (11). Say that the phrase ‘a solution for this problem’ consists of the existential quantifier ‘a’ and the nominal expression ‘solution for this problem’, where the semantic function of the latter is to restrict the domain of the former. The scope of the

26 For a proposal along these lines see Zucchi (1995).
quantifier is unspecified – the sentence says that a solution for this problem is \( F \), where the semantic value of this empty expression is fixed by the context. In extremely permissive contexts the sentence can express the proposition that a solution for this problem exists; in more usual contexts it expresses the proposition that a solution for this problem has already been found.\(^{27}\) If there is a coda, as in (10) or (11), it functions as a modifier for the rest of the sentence.\(^{28}\) I cannot defend this semantic analysis here – for my current purposes it is enough if it has some plausibility. That already shows that ‘there’-sentences provide no clear examples of bare quantification.\(^{29}\)

Plural ‘there’-sentences can be transformed without obvious change in truth-conditions into (somewhat archaic but impeccable) plural predications, as (12) and (13) illustrate:

(12) There are twenty trees in the quad.
(13) The trees in the quad are twenty.

Some might say that ‘twenty’ is ambiguous and only one of its readings – the one that occurs in the ‘there’-sentence – is a quantifier. I will not follow suit. This strategy leads to the postulation of systematic ambiguity for infinitely many expressions: ‘few’, ‘several’, ‘a lot’, ‘many’, ‘two’, three’, ‘four’, … , ‘at least two dozen but definitely not as many as forty’, etc. The systematic ambiguity could be handled through type-shifting principles but I’d rather stick to the idea of a single semantic value for quantifiers if I can. More importantly, I prefer to call expressions of generality quantifiers whatever their syntactic category or semantic type. But then, given that there appears to be no expression in (13) that combines with ‘twenty’ to form a quantificational phrase, it looks as if I have to accept that this sentence contains a bare occurrence of a quantifier.

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\(^{27}\) This proposal is due to Barwise and Cooper (1981).

\(^{28}\) For arguments that this is the best take on codas, including the one I appealed to above, see Francez (2009).

\(^{29}\) Sentences like ‘There are some’ contain no predicate that overtly contains the domain of quantification. For reasons discussed in connection with examples (5) – (8), I am inclined to think such sentences are elliptical and thus provide no convincing example of bare quantification.
Not so. Even if ‘twenty’ were a unary quantifier it would not follow that it is bare. To make a case for the latter one would need to argue that the interpretation of the expression ‘trees in the quad’ does not restrict the domain of ‘twenty’ in (13). This seems unlikely in the light of the fact that an admittedly awkward but truth-conditionally adequate topicalizing paraphrase is available for the sentence: ‘Trees in the quad, the trees in the quad are twenty of them.’ Moreover, there is reason to believe that the nominal expression ‘trees in the quad’ does combine with ‘twenty’ and the logical form of (13) looks like (14):

(14) The trees in the quad are twenty [trees in the quad].

It is exceedingly hard to put a nominal expression after ‘twenty’; the sentence ‘The trees in the quad are twenty native plants’ marginal at best. This is expected if we assume that the syntactic position into which we try to insert ‘native plants’ is already occupied by another expression, which happens to be unpronounced.

Similar things can be said about floating quantifiers. In (15) the quantifier ‘all’ appears on the surface without a restrictor.

(15) The trees in the quad have all been chopped down.

Even if ‘all’ is a unary quantifier in this sentence, that doesn’t mean it is bare: the sentence seems to be quantifying over trees in the quad. Moreover, the syntactic literature on floating quantification typically assumes that at the level of logical form ‘all’ and ‘the trees in the quad’ form a unit.30 Thus, the fact that certain quantifiers can appear on the surface detached from their restrictors is not a strong argument in favor of the presence of bare quantification in English.

More promising is the case of frequency adverbs, such as ‘usually’, ‘rarely’, ‘sometimes’, ‘always’, or ‘never’. Traditionally these have been viewed as quantifiers over times. But since they can be used in atemporal settings as well – as in ‘These problems usually have multiple

30 For a classic treatment, see Sportiche (1988).
solutions’– it’s best to assume that they quantify over cases. (16) seems reasonably well paraphrased as (16’) and (17) as (17’):

(16) Our dog usually chases your cat.
(16’) In most cases, our dog chases your cat.
(17) Our dog always runs.
(17’) In all cases, our dog runs.

Of course, context restricts the domain but none of the words or phrases in the sentences seems to. If the appearances are correct, frequency adverbs are bare quantifiers.31

What are cases? Lewis famously argued that talk of cases in (16’) or (17’) should not be taken seriously – he analyzed adverbial quantifiers as binding all variables unselectively within their scope.32 But Lewis’s is not the only viable view about the semantics of adverbial quantifiers. The alternative is to view them as binding situation-variables. Running with an intuitive notion of a situation, worlds are maximal situations, and cases are minimal ones.33 And if we think of adverbial quantifiers in this way, it becomes dubious whether they really have bare occurrences.

Note that (16”’) and (17”’) are possible readings of (16) and (17):

(16”’) In most cases when our dog chases something, it chases your cat.
(17”’) In all cases when our dog does something, it runs.

It is not implausible that these readings come about by partitioning the content of the sentence between the restrictor of ‘in most cases’ (the ‘when’-clause) and its nuclear scope (what comes

31 One might try to argue that frequency adverbs are like ‘somehow’ or ‘everything’, containing an expression whose semantic function is to restrict the domain of quantification to cases. But morphology or etymology yields little support for such a hypothesis. ‘Rarely’, ‘occasionally’, ‘frequently’, etc. are not, and as far as we can tell, never were compounds.

32 See Lewis (1975).

33 For details of a situation-based semantic framework, see Kratzer (1989). For an application to the semantics of frequency adverbs, see Berman (1987) and Heim (1990).
after the coma). These readings aren’t bare. In (16’’) the domain of quantification is restricted by the extension of ‘chase’: the minimal situations in the domain are all chases. Since this expression does occur in (16), in this reading of the sentence the occurrence of ‘usually’ is not bare. Moreover, if we assume a thematically articulated situation-based semantics, we can make a similar point about (17’’). In such a semantics the logical form of an activity verb, such as ‘run’ might be ‘λsλx (AGENT(s, x) ∧ running(s))’ and the logical form of ‘do’ might be ‘λsλx (AGENT(s, x))’. Since the verb ‘do’ is present in (17) – although it is not spelled out on the surface level – ‘always’ does not occur bare in (17) when the sentence is construed as (17’’). All the minimal situations in the domain are doings.

Once we notice that (16’’) and (17’’) are genuine readings, it is no longer clear that (16’) and (17’) are. Given the fact that the domain of quantification in (16) and (17) must be restricted by context, it is hard to distinguish between (17’) and (17’’) in truth-conditional terms. Cases when our dog does nothing don’t count – they are contextually eliminated from the domain. So (17’) and (17’’) are equivalent within the contextually restricted domain. Perhaps we are taken in by this equivalence when we say that (17’) is a reading. Following this train of ideas, one might suggest that instead of (16’), (16’’’) is a possible reading of (16).

34 The partitioning view is fairly mainstream; cf. de Swart (1991), Diesing (1992), Herburger (2000). Among those who see adverbial quantifiers are selective, its main rival is the idea that the restrictor of the quantifier is recovered from context, in particular, from a representation of discourse topic; cf. von Fintel (1994) and (2004). As a referee of the paper has emphasized, if one adopts this alternative view the sentence itself needn’t contain an expression whose extension restricts the domain. The point is well-taken: if von Fintel’s view is right, adverbs of quantification are contextually restricted bare quantifiers. (There is an extra wrinkle here. The domain of the adverbial quantifier is restricted to situations that are world-mates of the evaluation world. When the adverbial quantifier is unembedded, this means that the domain contains only actual situations. This restriction presumably is part of the lexical meaning of adverbial quantifiers. von Fintel does not represent it at the level of logical form but a theory like his that does would not construe adverbial quantifiers as bare.) My point here is a modest one: as long as the partitioning view is a live option they adverbs of quantification cannot serve as clear counterexamples to the claim that natural languages permit no bare quantification.

35 What about sentences like ‘Our dog is always asleep’? Here, the opponent of the bare analysis of ‘always’ should propose the analysis ‘In all cases where our dog is in some state, it is asleep’ and she should assume that stative verbs also receive a situation-based semantics, i.e. the logical form of ‘is asleep’ might be ‘λsλx(IN(s, x) ∧ asleep(s))’.
(16′′′) In most situations where our dog does something, it chases your cat.

If this is all right, neither (16) nor (17) need to involve bare quantification in any of their readings.\(^3^6\) The appearance that they do is due to the fact that the domain is restricted by the extension of an expression in a way that in ordinary contexts is vacuous.

I have surveyed a number of cases of *prima facie* bare quantification in English: quantificational determiners occurring without overt restrictors, compound quantifiers, existentials, predicatively used quantifiers, floating quantifiers, and adverbial quantifiers. Many of the ideas in this section are highly speculative – to defend any one of the semantic proposals would require a lot more work. My aim has been quite narrow: to show that the sorts of examples that naturally come to mind as instances of bare quantifiers in English can be semantically analyzed in a different fashion. I suggest we look for bare quantifiers elsewhere.

4. An example of bare quantification

Quantificational attitude-reports are subject to a *de re/de dicto* ambiguity. In ‘On Denoting’ Russell handles the distinction as a matter of scope: in *de re* reports the quantifier phrase outscopes the attitude verb, while in *de dicto* reports the scope order is reversed. This analysis predicts a two-way ambiguity whenever there is no other scope-bearing element in the sentence. However, there seem to be cases when a third reading is also available.\(^3^7\)

Alex is a somewhat paranoid – he thinks that his neighborhood is full of terrorists. He spends much of his time observing the comings and goings, following people around, making inquiries, and one day he goes to the police. The police officer who interviews him hands him a pile of photographs of people who live in Alex’s neighborhood. When Alex looks at a photograph he is

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\(^3^6\) Here is a test case. Suppose most of the time our dog is asleep, and hence, does nothing. But when she is up pretty much all she does is chase your cat. Is the sentence ‘Our dog usually chases your cat’ true under these circumstances? I think so. If you disagree with me you might be detecting a bare reading.

\(^3^7\) It has been recognized before that quantified attitude reports have more than the usual *de re* and *de dicto* readings. Fodor (1970) claims that ‘Mary wants to buy an inexpensive coat’ is four-way ambiguous. One of the additional readings she proposed has been accepted in the subsequent semantic literature, but the other has not. The readings I will argue for are like the rejected readings. So, one of the offshoots of my discussion in this and the next section is a vindication of Fodor’s four-way distinction. For more on this see Szabó (2010).
asked first whether the person is a terrorist and if he answers affirmatively he is then asked where the person lives. When he is done looking through the photographs he is asked whether there are terrorists in the neighborhood who are not on any of the photographs he has seen. He says that there are not. He is also asked whether he knows how many terrorist he has identified. He says that there were quite a few but he does not know precisely. Fortunately, the police officer took tally. It turns out Alex has identified 17 photographs as showing terrorists, and of those 11 as showing ones that live in the apartment building across the street from him. Let’s assume Alex is sincere and honest in expressing his beliefs. When the police officer who conducted the interview later reports this to his superiors he says the following:

(18) Alex believes that eleven terrorists live across the street from him.

My intuition is that the report is true; I think my judgment is not idiosyncratic. But surely, when the police officer makes his report he is not committing himself to the truth of Alex’s accusations of terrorism. Accordingly, the attitude verb should not be construed as taking narrow scope:

(18a) Eleven terrorists are such that Alex believes that they live across the street from him.

Moreover, since Alex did not keep track of the number of allegations he made, presumably the police officer is not attributing to him a general belief about how many terrorists live across the street from him. But then the attitude verb should not be taken as having wide scope either:

(18b) Alex believes this: eleven terrorists live across the street from him.

We can even incorporate these plausible assumptions into the report itself. Imagine that when the police officer makes his report he has the eleven photographs with him and points at them one after the other as he says this:

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38 I have conducted a survey on a sample of college students. The vignette contained the scenario given here and the sentence (18). I asked the question whether people agree with the police officer’s report. On a scale from 1 to 7 (1 meaning “completely disagree” and 7 “completely agree”) the distribution of responses was as follows: 1 – one, 2 – three, 3 – six, 4 – three, 5 – four, 6 – twelve, 7 – seventy-four.
(18′) Alex believes that eleven terrorists live across the street from him – this guy, this guy, … , and this one. Quite a long list … I am pretty sure he doesn’t realize just how many accusations he has made. Plus, it turns out these people are all perfectly clean.

This seems like a coherent thing to say. There really are eleven people represented by the eleven photographs the police officer is showing, they really are all (let’s assume) perfectly clean, but Alex really does believe of each that he is a terrorist living across the street from him. This much seems enough to guarantee the truth of (18′). Moreover, it is hard to imagine how (18′) could be true if (18) is false. The function of the list after the hyphen is to further specify the content of the belief ascribed, and so it can hardly turn a false ascription into a true one. So, it looks like we should not construe (18) within (18′) as (18a) or (18b). Rather, we should interpret it as (18c):

(18c) Eleven people are such that Alex believes that they are terrorists living across the street from him.

The paraphrase is not perfect: the police officer’s report does not entail that the photographs selected by Alex are of people. Perhaps it would be better to replace ‘people’ with ‘individuals’ or ‘things’ or ‘objects’ but clearly no particular choice of word is forced upon us. There seems to be no word or phrase in (18) that applies to the people on the eleven photographs. Moreover, there is no theoretical motivation to say that there is some unpronounced element at the level of logical form whose extension restricts the domain. Of course, what we have here is but an isolated example – one may wonder when and how such readings arise. I will say something about these questions in the rest of this section and in the next one. For the time being, I tentatively conclude that we have a plausible counterexample to the hypothesis under consideration: within the relevant reading of (18) ‘eleven’ has a bare occurrence.

The bare occurrence here comes about because the police officer’s statement is what I will call a summative report of Alex’s beliefs. Alex’s answers express a number of de re beliefs regarding the people on the photographs and the police officer summarizes those beliefs in his report. The words ‘terrorist’ and ‘lives across the street’ show up in Alex’s answers, so they are to be taken

39 There is, however, a phrase that can be constructed from the words in (18) that describes these people: they all belong to the extension of ‘believed to be terrorists by Alex.’ But there is no reason to think that such a phrase is a constituent of the logical form of (18). I will say more on the significance of this in the last section.
to reflect how Alex thinks of the people on the pictures. The police officer need not think that
either of these words correctly describe any of those people. By contrast, the word ‘eleven’ is the
police officer’s contribution to the report. He is the one keeping tally. Alex need not have any
belief about the number of people he takes to be terrorists across the street.

One might wonder whether the bare reading of (18) should count as de re or de dicto. The
answer depends on how exactly which distinction one has in mind.40 First, there is a syntactic
distinction, according to which a sentence is de re with respect to a certain position just in case
that position is within the scope of an expression of a certain type (modal operator, propositional
attitude verb, etc.) and that position is occupied by a variable bound from outside that scope. It
would be difficult to apply this definition as a test: whether a position is occupied by a variable at
the level of logical form is not something we can pre-theoretically ascertain. Second, there is a
semantic characterization: a sentence is de re with respect to a position just in case it permits
substitution salva veritate at that position. According to the semantic definition summative
attitude descriptions are not a de re with respect to the position occupied by ‘terrorists.’ Even if it
turned out that terrorists are all and only skateboarders it would not follow from the summative
reading of (18) that eleven people are such that Alex believes that they are skateboarders living
across the street. Finally, there is a metaphysical characterization, which says that a sentence is
de re just in case there is some property such that the truth-value of the sentence depends
exclusively on which objects have this property.41 This fits summative readings just fine: (18) is
true or false depending on how many people have the property of being believed to be terrorists
by Alex. So we have a split verdict: the summative reading is semantically de dicto but
metaphysically de re.

How surprising is the existence of summative readings? It depends on your perspective. If you
are a fluent user of Logish – language used in logic and philosophy that has in addition to the

40 For the different ways to draw the de re/de dicto divide, see McKay and Nelson (2008).

41 The version in McKay and Nelson (2008) goes as follows: “An attribution is metaphysically de re with respect to
an object o just in case it directly attributes a property to o.” This does not tell us when sentences are metaphysically
de re, but the problem can be easily fixed. Metaphysically de re sentences are either metaphysically de re
attributions with respect to an object or generalizations of such sentences with respect to that object.
resources of English, the resources of first-order logic as well, you won’t have any difficulty capturing all three readings of (18):\(^{42}\)

\[(18a') \exists_{11} x \left( \text{terrorist}(x) \land \text{BEL}_{\text{Alex}} \text{ lives across the street from Alex}(x) \right)\]

\[(18b') \text{BEL}_{\text{Alex}} \left( \exists_{11} x \left( \text{terrorist}(x) \land \text{lives across the street from Alex}(x) \right) \right)\]

\[(18c') \exists_{11} x \left( \text{BEL}_{\text{Alex}} \left( \text{terrorist}(x) \land \text{lives across the street from Alex}(x) \right) \right)\]

If these regimentations are legitimate, the three readings differ from each other only with respect to scope. The difference between (18a') and (18c'), for example, concerns the question whether ‘terrorist’ is placed outside the scope of the belief operator while remaining within the scope of the quantifier.

But looking at the issue from the perspective of English proper we do have a puzzle. There is some disagreement about the precise labeling of the phrase ‘eleven terrorists’ – some think it is headed by the determiner, while others contend that the head is the noun. But no one would doubt that it is a genuine syntactic unit, and as such it should \textit{prima facie} be treated as semantic unit as well. The standard semantics for quantificational phrases views ‘terrorist’ as the restrictor of the quantifier. In (18c), however, ‘believes’ intervenes between a quantifier and its restrictor: the former but not the latter takes scope over the attitude verb. We need an account of how such a reading comes about.

Standard theories of quantification for natural languages are based on movement or type-shifting.\(^ {43}\) These theories have a common core: they explain scope ambiguity as a result of alternative ways of applying a certain fall-back mechanism that arises due to a semantic mismatch. Simplifying a lot, verbs require arguments of type \(e\) (i.e. expressions whose semantic value is an individual); quantificational phrases are not of that type (being expression that map a pair of one-place predicates to a truth-value, they are of type \(\langle\langle e, t), \langle(e, t), t)\rangle\)), so unless

\(^{42}\) I leave it open whether ‘eleven’ means \textit{exactly eleven} or \textit{at least eleven}. Depending on which interpretation one prefers, ‘\(\exists_{11}\)’ could be defined in a corresponding way.

\(^{43}\) For a canonical movement-based theory, see Heim and Kratzer (1998); for a canonical type-shifting proposal, see Hendricks (1993).
something happens the derivation crashes. In movement-based theories, the quantificational phrase rises to an appropriate site leaving a trace of type $e$ behind. In type-shifting theories, the type of the verb rises, so that the quantifier phrase can combine with it \textit{in situ}. Either way, the quantificational phrase is interpreted as a single unit, so there is no possibility of something intervening between the quantifier and its restrictor.

The point is not that standard theories cannot accommodate bare readings. They can, but they need to bend over backwards. There is a well-known case where even a single word must be split up in the semantics. The phenomenon arises with certain negative quantifiers in German:

(19) Du muss keine Kravatte anziehen.
\vspace{0.5em}
\textit{you must no tie wear}
\vspace{0.5em}
You don’t need to wear a tie.

(19a) $\lnot \exists x \ (x$ is a tie $\land$ MUST (you wear $x$))
(19b) MUST ($\lnot \exists x \ (x$ is a tie $\land$ you wear $x$))
(19c) $\lnot$ MUST ($\exists x \ (x$ is a tie $\land$ (you wear $x$))

The split reading is by far the most natural one; (19a) is out, (19b) is at best marginal. (19c) requires the negation within ‘keine Kravatte’ to split from the phrase through lexical decomposition (‘kein’ = ‘not’ + ‘ein’) or through a higher-order interpretation.\footnote{For the former approach, see Kratzer (1995); for the latter de Swart (2000).} Either way, we need to say something unusual about what brings about the non-standard movement or type-shift. What happens in the bare reading of (18) is analogous: in the semantics ‘eleven’ splits from the rest of the quantifier. Either the determiner moves above the attitude verb leaving its restrictor behind or the type of the determiner shifts while the type of the restrictor remains unchanged.

It is one thing to make an intuitive case for split interpretation and quite another to show how it might actually work. Let me now try to do the latter within a framework involving movement.\footnote{A slight variant of this framework is presented in Szabó (2010).} I am not committed to the details of this story; I present it for the sole purpose of illustration. Those who wish to bypass technicalities may proceed to section 5.
The key idea is that quantifier raising is akin to copying: the syntactic structure of the phrases remains in its original position while another one is adjoined higher up. The quantificational determiner shows up in the higher position leaving a trace below. For the restrictor there are two options: it can move or stay. Whichever position remains unfilled is interpreted vacuously. Finally, there is a special rule that combines the trace of the quantifier with the (possibly null) restrictor creating a restricted trace. The semantic value of a restricted trace is undefined whenever the trace is assigned a value that does not satisfy the restrictor.

Let’s consider a tiny fragment of English. I will abstract away from questions of agreement, tense, and aspect. I will also adopt the (probably too simple) view that ‘terrorist’ and ‘live across the street’ are one-place predicates of type \(\langle e, t \rangle\) and that ‘eleven’ is a quantificational determiner of type \(\langle\langle e, t\rangle,\langle e, t\rangle, t\rangle\). The fragment also has traces indexed by natural numbers \(t_i\), where \(i \in \omega\); their semantic type is \(e\). Semantic values of lexical items are standard (since only traces have assignment-dependent semantic values the superscript is suppressed elsewhere):

\[
\begin{align*}
\langle t_i \rangle^a &= a(i) \\
\langle \text{live across the street} \rangle &= \lambda x_e. x \text{ lives across the street} \\
\langle \text{terrorist} \rangle &= \lambda x_e. x \text{ is a terrorist} \\
\langle \text{eleven} \rangle &= \lambda f_{(e,t)}. \lambda g_{(e,t)}. \text{eleven } x \text{ is such that } f(x) &= 1 \text{ and } g(x) = 1
\end{align*}
\]

Concatenation is interpreted as functional application except in the following two cases:\(^{46}\)

(PA) If \(i\) is an index and \(\sigma\) a sentence then \([i\sigma]\) is a predicate abstract of type \(\langle e, t \rangle\), and

\[
\langle i\sigma \rangle^a = \lambda x_e. \langle \sigma \rangle^a[x/i]
\]

(RT) If \(t_i\) is a trace and \(\nu\) is a nominal expression then \([t_i\nu]\) is a restricted trace of type \(e\), and

\[
\langle t_i\nu \rangle^a = a(i) \text{ if } \langle \nu \rangle(a(i)) = 1; \text{ otherwise undefined.}
\]

In addition, I will assume that the semantics has the unusual feature of interpreting empty syntactic slots that are normally filled by one-place predicates. The semantic value of such a slot is the function \(\lambda x_e. x = x\). This fact makes no difference as long as the syntax does not generate

\(^{46}\) (PA) is from Heim and Kratzer (1998), the second is mine.
expressions with unfilled slots. I assume this is the case as long as we have no movement. But quantifier rising does create syntactic structures with empty slots. The rule has two versions:

\[(QR^\uparrow) \quad [S \xi [DP[\delta]][\psi]] \Rightarrow [S[DP[\delta]][\psi][[t_i][S \xi [[t_i]][\psi]]]] \]
\[(QR^\downarrow) \quad [S \xi [DP[\delta]][\psi]] \Rightarrow [S[DP[\delta]][\psi][[t_i][S \xi [[t_i]][\psi]]]] \]

In the first case, the empty slot is in the lower position; it combines with the trace to form a (vacuously) restricted trace. In the second case, the empty slot is upstairs; it combines with the determiner to form a (vacuously) restricted determiner phrase.

In our tiny fragment it makes no truth-conditional difference which of the two versions of the rule is applied. (Note that in \((20^\uparrow)\) ‘5’ is an index and in \((20^\downarrow)\) ‘8’ is one. Given (PA) you can think of their first occurrences as encoding lambda operators binding the corresponding traces).

\((20)\) Eleven terrorists live across the street.
\[ [S [DP_{D} eleven][N terrorists]][_{VP} live across the street]] \]

\((20^\uparrow)\) \[ [S [DP_{D} eleven][N terrorists]][_{VP} live across the street]] \]
\((20^\downarrow)\) \[ [S [DP_{D} eleven][N terrorists]][_{VP} live across the street]] \]

Since the empty nominal slot is interpreted as the total identity function over individuals, according to (RT) \([[[t_5][N]]^a = a(5)\), and thus according to (PA)
\[ [live across the street]] = [live across the street]. So, \([20] = [20^\uparrow]\). By contrast, \([[[t_8][N terrorist]]^a\) is only defined for those values of the assignment function that are terrorists. So, \([live across the street]] = [live across the street] only if \(a(8)\) is a terrorist; otherwise it is undefined. While these predicates have different semantic values the difference concerns only non-terrorists. But since the semantic rule for ‘eleven’ only cares whether there are eleven terrorists who satisfy the predicate, we have \([20] = [20^\downarrow]\).\(^{47}\)

\(^{47}\) See Szabó (2010) for a discussion of ‘every’ and ‘most’ in this setting. Since the framework allows partial functions, one must be careful in how one states the semantic clauses for quantifiers.
Once we enrich our fragment with the apparatus to handle belief-ascriptions, it will make a difference which quantifier raising rule we apply.

(18) Alex believes that eleven terrorists live across the street.

\[
(18) \quad [s \[dp\{\text{eleven}\}\[n \text{terrorists}]\][vp \text{live across the street}]\
\]

(18↑) \quad [s\[dp\{\text{eleven}\}\[n \text{terrorists}]\]Alex believes that[s[s[t_5]\[n]\][vp\text{live across the street}]])

(18↓) \quad [s\[dp\{\text{eleven}\}\[n \text{terrorists}]\]Alex believes that[B[s[t_8]\[n\text{terrorists}]][vp\text{live across the street}]])

(18) is the usual de dicto reading. In (18↑) the entire phrase ‘eleven terrorists’ is interpreted outside the complement of the attitude verb resulting in the usual de re interpretation. In (18↓) ‘eleven’ is above ‘believe’ but ‘terrorist’ stays below which yields the summative reading.

I think (18) is instructive for three reasons. First, it provides a clearer example of bare quantification than anything discussed in the previous section. Second, it shows that quantifiers can be bare in some but not all their occurrences, and that bare quantifiers can be expressions which form a single phrase with a restrictor. Finally, the case obviously generalizes and thus yields us infinitely many bare quantifiers, rather than just a handful. Exactly how far we can generalize is the topic of the next section.

5. Generalizing the example

Let’s focus on what is crucial about the key example of the last section. We have two scope-bearing expressions in the sentence: a numerical quantifier and a propositional attitude verb. The latter splits the former from its restrictor. The first question to ask is whether the split has anything to do with some special feature of ‘eleven’.

The answer is clearly negative, insofar as intersective quantificational determiners are concerned.\(^{48}\) (21), for example, can be used to make a true report under the circumstances

\(^{48}\) Let \(Q\) be a binary quantificational determiner (i.e. quantifier that combines with a one-place predicate to yield a quantifying phrase that combines with a one-place predicate to yield a quantified sentence). \(Q\) is intersective iff\(^{def}\) whenever everything that is both \(F\) and \(G\) is also both \(F’\) and \(G’\), \(QFG\) is true just in case \(QF’G’\) is.
described above, even if Alex thinks that eleven terrorists across the street are but a pittance. (Perhaps he thinks most neighborhoods have even more terrorists than his own.)

(21) Alex believes that many terrorists live across the street from him.

If the report is summative, ‘many’ is the police officer’s contribution, and the report is true because Alex in fact identified eleven people as terrorists living across the street from him and eleven terrorists across the street are in fact many.

When it comes to non-intersective quantifiers, the empirical situation is more complicated. Imagine that Bob who lives in the same neighborhood also comes to the police and claims that there are a number of terrorists living there. The police officer goes to his supervisor and as they discuss the new development they compare the new accusations with the ones made by Alex. The police officer observes that there is not much agreement between Alex and Bob about where the terrorists are concentrated in the neighborhood. He says:

(22) Alex believes that most of the terrorists live across the street from him.

Remember, Alex has identified 17 people as terrorists and 11 of them as living across the street from him, and he also said that there are no terrorists in the neighborhood who are not on any of the photographs he has seen. Given all that (22) seems true. I admit, however, that intuitions are markedly less clear here than with (21). Moreover, in this case it is very important to pay attention to the contextual domain restriction: (22) comes out true only if we disregard people not in Alex’s neighborhood. While the restriction is presumably not due to the interpretation of any expression within the sentence it makes the example less clean than the previous one.

With the obvious changes in the pattern of responses Alex gave, we can confirm the existence of summative readings involving other quantificational determiners. I predict that the results will be similar as in (21) and (22): the case for summative readings is somewhat stronger if the

49 The survey I conducted regarding this example yielded somewhat mixed results. On a scale from 1 to 7 (1 meaning “completely disagree” and 7 “completely agree”) the distribution of responses was as follows: 1 – five, 2 – eight, 3 – eight, 4 – eight, 5 – eight, 6 – eight, 7 – twenty-two.
quantifier is intersective than if it is not. The only quantificational determiners for which it is
difficult to confirm the existence of a bare reading using versions of the example in the previous
section are ‘a(n)’, ‘some’, ‘at least one’, ‘exactly one’, etc.

(23) Alex believes that some terrorist lives across the street from him.

This is because it is hard to come up with a plausible case when Alex provides the answers
during the interview that assure the truth of the summative reading but plausibly fails to form the
general belief expressed by the complement of the attitude verb. But it can be done. Imagine that
Alex is asked to go through the entire pile of photographs and pick out the ones he thinks are of
terrorists, and then he is asked to go through the entire pile yet again and pick out the ones he
thinks live across the street from him. Assuming Alex is honest, if in fact there is a photograph
Alex has identified as a picture of a terrorist and later as a picture of someone who lives across
the street from him the police officer has all the information he needs to affirm that (23)
expresses a truth. He needn’t worry about the question whether Alex has actually recognized that
he identified someone both as a terrorist and as someone living across the street from him. It is
enough if the police officer himself recognizes this.

I conclude, tentatively, that all English binary quantificational determiners are bare. They can all
be substituted for ‘eleven’ in ‘Alex believes that eleven terrorists live across the street from him’
(adjusting for number, if necessary) and the resulting sentence will have a summative reading. In
the summative reading the quantifier is split from its restrictor and the propositional attitude verb
comes between them, thus there remains no predicate in the sentence that is interpreted outside
the complement of the attitude verbs and could restrict the quantificational domain. It is
important that the presence of the attitude verb has nothing to do with the split – in the small
fragment outlined at the end of the previous section the split comes about as a result of a special
sort of quantifier raising. What the attitude verb does is provide evidence that the quantifier and
the restrictor are not interpreted as a unit. I now want to turn to the question what besides attitude
verbs can yield similar evidence.
It is reasonably clear that some intensional transitives can separate quantifiers and their restrictors. Suppose again that things are as described in the previous section. Let’s say that after he finished with selecting the three photographs Alex is asked to pick out the people on photographs in a line-up. Then it seems that in talking to his superiors the police officer can use (24) to correctly describe what Alex is doing:

(24) Alex is looking for eleven terrorists who live across the street from him.

If this seems doubtful, you can use the same strategy as before to strengthen the relevant intuition. Just imagine that the eleven photographs Alex has selected are at hand, so the officer can expand the report by pointing at them in sequence: ‘Alex is looking for eleven terrorists who live across the street from him – this guy, this guy, … , and this one.’ To screen off the other two readings, as before, we can add: ‘Quite a long list … I am pretty sure he doesn’t realize just how many people he takes to be terrorists across the street from him. Needless to say, all these people are all perfectly clean.’

Do we get split readings with modals? I think so. Suppose you are discussing the upcoming election. There are only two candidates, one of whom would be a disaster. Fortunately, your candidate is up in the polls, so you act like you are taking the outcome for granted. A more reasonable friend might then utter (25) to remind you not to get too cocky:

(25) This election could have two winners.

This sentence has a true reading, where the modal takes neither wide nor narrow scope with regard to the phrase ‘two winners’. It is certainly false that there are two winners such that this election could have them and equally false that the election could end up having two winners. What is true is that there are two people who could each be winners of this election. And this is a loose paraphrase of a split reading.

One might object to the example on the grounds that it employs the verb ‘have’ and the relational noun ‘winner.’ Independently of our concerns ‘The election has a winner’ cannot happily be paraphrased as ‘A winner is such that the election has it.’ But I think the problematic feature can
be dispensed with. Imagine you live in one of those American states where judges are elected. It’s your day in court and it matters a lot who the judge is. The election happened last week, you know there were just two candidates, you know that neither was a judge before the election, you know that just one of them won the election, but you don’t know which one. Before you enter the court room you are talking to your lawyer and you utter (26) to remind her that your trial strategy is contingent on who is presiding over the case.

(26) There are two judges we could face in this court.

It seems to me that you said something true. (As before, the intuition can be boosted if we imagine that you show the photographs of the candidates as you utter the sentence.) And again, the only true reading is the split one.50

A similar pattern can be observed with tense. Helen was consecutively married to Menelaus, Paris, and Deiphobus. We find it natural to report this using (27).51

(27) Helen had three husbands.

But again, tense cannot take either wide or narrow scope with regard to ‘three husbands’. Since the husbands are no more it is false that three husbands are such that each was at some time in the past Helen’s. Since Helen is not guilty of trigamy it is false that at some time in the past Helen had three husbands. The reason (27) is true is that it has a split reading, according to which three individuals are such that in the past they were husbands Helen had. Again, we can drop the use of ‘have’ and the relational noun ‘husband’ and we get almost equally natural examples.

Take a case when the tour guide shows a courthouse that has been in use during the 18th century and she utters (28):

(28) There were fifteen judges who presided in this court.

50 I think ‘Two judges could be presiding in this court’ also has a split reading, although it is somewhat harder to tease it out. I don’t know the reason for the contrast.

51 The example is attributed to Diodorus Crosens by Sextus Empiricus in Against the Physicists 2.98.
The reading where tense takes scope over ‘fifteen judges’ is surely not what she meant, for that says that at some time or other in the past fifteen judges simultaneously presided in the courthouse. The reading where ‘fifteen judges’ takes scope over the past tense is also problematic. It entails the existence of fifteen judges who at some time in the past presided in the court. 

*When* are these judges supposed to exist? If you are a presentist you think quantifiers outscoping tense must quantify over entities that exist *now*, in which case this reading comes out false. If you are an eternalist you probably think the domain of such quantifiers is comprised by past, present and future entities that exist *at some time or other*, in which case you may think this reading is true. But there is a residual difficulty even for the eternalist. They should be reluctant to say that the predicate ‘judge’ applies to long deceased judges just because the word has applied to them at some time in the past. After all, long deceased judges have also died at some time in the past, but applying ‘dead’ to them along with ‘judge’ would yield (29), which seems false.

(29) There were fifteen dead judges who presided in this court.

No presentist can and no sensible eternalist will construe (28) or (29) with the entire quantified phrase taking widest scope. What (28) says is that there are fifteen individuals who in at some time in the past were judges who presided over this court. This is a split reading, and it can easily be true. The same reading of (29) says that there are fifteen individuals who at some time in the past were dead judges who presided over this court – which is false, as it should be.\(^{52,53}\)

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\(^{52}\) Lewis (2004) has used examples like (27) to argue against presentism. The argumentative strategy was to point out that in such a sentence tense should not be construed as taking wide scope and pointing out that the presentist has problems with quantifiers outscoping tense. In Szabó (2007) I responded to Lewis on behalf of the presentist, arguing that the quantifier outscoping tense could be construed as quantifying over states, rather than individuals. I also assumed that the state quantifier is bare.

\(^{53}\) There is considerable disagreement about how modality and tense are to be represented as quantifiers or operators at the level of logical form. Since it is not clear that natural languages have genuine variable-binding (cf. fn. 4) these debates have a problematic presupposition. The arguments for split readings presented here are independent of these issues. What they show is that modality and tense can take scope over the restrictor of a quantified phrase without taking scope over the quantifier.
The merits of recognizing the possibility of split readings is well illustrated by a long-standing disagreement about existential entailments of sentences involving verbs of creation in the progressive. Take (30):

(30) Jack London was building a house.

According to the establishment, (30) does not entail ‘There was a house Jack London was building.’ According to the iconoclasts, it does. Intuition initially sides with the establishment – after all, if there was already a house when Jack was building it, what exactly is the point of his efforts? But the initial intuitions can take a beating once we recognize that we are willing to say of a foundation and some unfinished walls ‘This is what Jack London was building at the end of his life.’

I think the way out of this quandary is to say that the most natural reading of (30) is split. Assume that at the level of logical form the progressive aspect is represented as the scope bearing element PROG. When applied to a sentence PROG yields another sentence that says that some event described by the first is in progress. I leave it open here whether PROG has an analysis in terms of more familiar modal and temporal expressions. What is clear is that PROG creates opacity in its scope: we can describe things in progressive sentences not as they are but as they are becoming. The object of Jack London’s building efforts, for example, is not a house but something that is becoming a house. Nonetheless, it is a perfectly concrete actual thing. In other words, the indefinite article takes scope over PROG but its restrictor does not.

This proposal can also make sense of notorious minimal pairs, like (31a,b) and (32a,b):

(31a) Ned was carving a large block of stone.

(31b) Ned was carving something.

According to the establishment, (31a,b) is a pair, because (31a) entails (31b). According to the iconoclasts, it does not. Intuition initially sides with the establishment – after all, if there was already a block of stone when Ned was carving it, what exactly is the point of his efforts? But the initial intuitions can take a beating once we recognize that we are willing to say of a block of stone ‘This is what Ned was carving at the end of his life.’

I think the way out of this quandary is to say that the most natural reading of (31a) is split. Assume that at the level of logical form the progressive aspect is represented as the scope bearing element PROG. When applied to a sentence PROG yields another sentence that says that some event described by the first is in progress. I leave it open here whether PROG has an analysis in terms of more familiar modal and temporal expressions. What is clear is that PROG creates opacity in its scope: we can describe things in progressive sentences not as they are but as they are becoming. The object of Ned’s carving efforts, for example, is not a block of stone but something that is becoming a block of stone. Nonetheless, it is a perfectly concrete actual thing. In other words, the indefinite article takes scope over PROG but its restrictor does not.

This proposal can also make sense of notorious minimal pairs, like (31a,b) and (32a,b):

(31a) Ned was carving a large block of stone.

(31b) Ned was carving something.

56 Using the technique of restricted traces discussed at the end of section 4, the reading can be represented as \([A_1 []] PROG(Jack London build [t_1 [house]])\).
(31b) Ned was carving a small statue.

(32a) Sam was baking the dough.
(32b) Sam was baking the cookies.

The idea is that the natural readings of (31a) and (32a) are the ones where ‘a large block’ and ‘the dough’ take scope over PROG – a large block of stone was such that Ned was carving it and the dough was such that Sam was baking it. By contrast, the natural readings of (31b) and (32b) are split – a thing was such that Ned was carving it to become a small statue and some stuff was such that Sam was baking it to become the cookies.

The tentative and preliminary result of this brief survey is that a wide variety of scope bearing expressions that create opaque contexts can split an arbitrary binary quantificational determiner from its restrictor. If this is true, bare occurrences of garden-variety quantifiers are fairly common in natural language.

6. An afterthought

At the end of section 3, I mentioned that the question whether natural languages permit bare quantification is connected to the question whether we can quantify over the all-inclusive domain. If no natural language quantifier is bare then in order to quantify all-inclusively, we need an expression whose extension is all-inclusive. While there may be such expressions in natural languages, it is doubtful that they can be made to play the role of restrictor.

English nouns typically employed by philosophers to set an all-inclusive domain certainly fail to do the job. A quick check in the dictionary can tell us that none of ‘thing’, ‘object’ or ‘entity’ applies to people, actions, regions of space, and much else. Moreover, it is unclear how adding adjectives or relative clauses could eliminate this limitation of extension, unless narrow scope negation works as complementation.\(^{58}\) But it does not seem to work that way: a cat or a parrot may be a ‘non-dog’ but a sudden feeling of dizziness is not. ‘Is a dog or not a dog’ does seem to

\(^{58}\) Adding typical adjectives or relative clauses just restricts further the extension of the noun. There are exceptions – e.g. ‘fake diamond’ applies to things that ‘diamond’ does not; ‘president, who is no longer in office’ applies to people that ‘president’ does not, etc. But it is not likely that any such device could turn a noun with limited extension to a nominal expression whose extension is all-inclusive.
have all-inclusive extension, and I would be prepared to say that same about ‘self-identical’ or ‘exist.’ Unfortunately, none of these is a nominal expression and so we get non-sentences when we try to use them as restrictors on ordinary quantifiers. ‘Each is a dog or not a dog is a particular’, ‘All self-identical is particular’ and ‘Every exists is a particular’ are all ill-fated attempts to express the thesis of nominalism in English. The only quantificational devices whose domain can be restricted by a non-nominal expression seem to be adverbs of quantification. But a sentence like ‘Invariably, particulars exist’ can express what nominalists wish to say only if the domain of the adverb is restricted exclusively by the extension of ‘exist’. Whether this is the case is a highly controversial matter – many semanticists think that adverbs of quantification obligatorily range over minimal situations. And some things, like my left toe or the universe, are not minimal situations.

I do not pretend that these brief remarks show that no English word or phrase can simultaneously have all-inclusive extension and serve as the sole restrictor of a quantifier. It is also possible that although English lacks the appropriate expression, other natural languages are more fortunate in this regard. But I do think the difficulties I hinted at should give pause to proponents of absolutely unrestricted quantification. The good news is that if I am correct that bare quantification is commonplace in English, this difficulty disappears. The bad news is that a related one immediately reemerges.

Although the restrictor of the quantifier does not constrain the domain in cases of split quantification, sometimes we can construct a linguistic expression from the constituents of the sentence that does just that.

59 ‘Every existence is a particular’ is a grammatical sentence but it does not say what the nominalist would like to say. I am not my existence (for otherwise my existence would be my existence’s existence) and thus, plausibly, I am not an existence at all. Still, I do exist, and so ‘exist’ applies more broadly than ‘existence’.

60 Thus, if ‘invariably’ is an unselective quantifier, as Lewis (1975) contends, then the desired reading of ‘Invariably, particulars exist’ has the logical form of ‘Invariably x, if x exists then x is a particular.’ If, however, ‘invariably’ selectively quantifies over minimal situations, as in Heim (1990), the logical form is roughly ‘Invariably s, if there is an x that exists in s then the x that exists in s is a particular in some extension of s’. Incidentally, ‘Only particulars exist’ has the same problem as ‘Invariably, particulars exist’ if we assume – as it is standard in the semantic literature – that ‘only’ is a focus-sensitive particle and that ‘Only particulars exist’ contains a phonologically null adverb of quantification.
(33) I could write three papers …
    … because I have three plans and any one of them could be realized.
(33′) I could write three possible papers.

(34) I am writing three papers…
    … because I have three drafts and I am working on each of them.
(34′) I am writing three papers in progress.

(35) I wrote three papers …
    … one after the other, burning each right after I finished it.
(35′) I wrote three past papers.

Philosophers with ontological scruples view these paraphrases with suspicion – they would not concede that (33) quantifies over possible papers, (34) over papers in progress, and (35) over past papers. But it is hard to object to them on any other grounds. In the case of bare readings involving attitude verbs and intensional transitives the restricting expression is even easier to find. When I say ‘You believe three papers are in your drawer’ (based on an elaborate interrogation when I showed you photographs of various items and you had to first identify the ones you think are papers and then among those the ones you think are in your drawer…) I am quantifying over things believed to be papers by you. And when you go to fetch those papers and I say ‘You are looking for three papers in your drawer’ I am quantifying over things you are looking for.

Those who are willing to embrace the existence of merely possible and merely past things, as well as things in progress can paraphrase the split readings using restricted quantification. Moreover, the quantificational domains are restricted to the extension of a linguistic expression – not an expression that can plausibly be taken to be a syntactic constituent within the sentence, but an expression that is nonetheless constructible from those constituents. If these paraphrases are adequate we still lack an example of a proposition expressible in English through bare quantification only. This fact, together with the apparent lack of expressions with all-inclusive
extension should continue to fuel our doubts whether absolute generality in really expressible in a natural language.\textsuperscript{61}

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