Supplemental Appendix to “Formal Models of Nondemocratic Politics”

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The probability that a member of the winning coalition will be among the $W$ members of the selectorate with the highest realization of the affinity parameters $A_i^c$: The probability that an observation $x$ from a sample of $N$ draws from the standard uniform density is the $k$th largest is

\[
\binom{N-1}{k-1} x^{(N-1)-(k-1)} (1-x)^{k-1}.
\]

If all we know about $x$ is that it is drawn from the standard uniform density, then the probability that $x$ will rank as the $k$th largest observation is

\[
\int_0^1 \binom{N-1}{k-1} x^{(N-1)-(k-1)} (1-x)^{k-1} \, dx.
\]

In turn, the probability that this observation will be at least the $k$th largest observation is

\[
\sum_{i=1}^k \int_0^1 \binom{N-1}{i-1} x^{(N-1)-(i-1)} (1-x)^{i-1} \, dx.
\]

After multiplying and dividing the expression through by $N$, the integrand in each element of this sum can be expressed as the density function of the Beta distribution with the parameters $N - i + 1$ and $i$

\[
\sum_{i=1}^k \frac{1}{N} \int_0^1 \frac{\Gamma(N+1)}{\Gamma(i)\Gamma(N-i+1)} x^{(N-i)} (1-x)^{i-1} \, dx.
\]

Since this density function (by assumption) integrates to 1, the probability that an observation from a sample of $N$ draws from the standard uniform density is at least the $k$th largest is

\[
\sum_{i=1}^k \frac{1}{N} = \frac{k}{N}.
\]
Thus we see that the probability that a member of the winning coalition of size $W$ who considers defecting to the challenger expects to be among the $W$ members of the selectorate of size $S$ with the highest realization of the affinity parameters $A_C^i$ with the probability $\frac{W}{S}$.