Online Appendix to “Power-sharing and Leadership Dynamics in Authoritarian Regimes”

This Appendix fills in the details of some of the formal results in Svolik (2009).

**Proposition 1:** As I argued in the paper, there is no equilibrium in which the dictator uses a pure strategy and the ruling coalition conditions its decision to stage a coup on the observed signal.

In any equilibrium in mixed strategies, (i) the ruling coalition stages a coup with probability $\beta_0$ such that, given the correlation between his actions and the signal $\theta$, the dictator is indifferent between diverting and complying, and (ii) the dictator diverts with probability $\alpha$ such that the ruling coalition is indifferent between staging and not staging a coup after observing a high signal or after observing a low signal, but not both.

Note that the ruling coalition cannot be indifferent between staging and not staging a coup after both a high and low signal: If the dictator chooses such $\alpha$ as to make the ruling coalition indifferent between staging and not staging a coup after observing a high signal, than the ruling coalition will prefer not to stage a coup after observing a low signal. Alternatively, if the dictator chooses such $\alpha$ as to make the ruling coalition indifferent between staging and not staging a coup after observing a low signal, than the ruling coalition will prefer to stage a coup after observing a high signal.

Thus for the ruling coalition, only the actions $(\beta_L = 0, \beta_H > 0)$ and $(\beta_L > 0, \beta_H = 1)$ can be parts of an equilibrium. To obtain the equilibrium action profile, we solve for the indifference conditions.

In the case when $(\beta_L = 0, \beta_H > 0)$, we have

$$
\beta^*_H = \frac{\mu}{\rho [\pi_{Hd}(1 + \mu) - \pi_{Hc}]} \text{ and } \alpha^* = \frac{\pi_{Hc}}{\pi_{Hc} + \pi_{Hd} \left( \frac{1}{1-\rho} - 1 \right)} \text{ . (1)}
$$

To verify that $\beta^*_L = 0$, it must be true that the ruling coalition prefers not to stage a coup after
it observed a low signal,

\[ \rho \leq \Pr(d|L)(1 - \epsilon) + 1 - \Pr(d|L). \] \hspace{1cm} (2)

After substituting \( \alpha^* \) into

\[ \Pr(d|L) = \frac{\pi_{Ld}\alpha^*}{\pi_{Ld}\alpha^* + \pi_{Lc}(1 - \alpha^*)}, \]

inequality (2) can be reduced to

\[
-\frac{(\pi_{Hd} - \pi_{Hc})(1 - \rho)[\rho - (1 - \epsilon)]}{\epsilon(\pi_{Hd} - \pi_{Hc}\pi_{Hd}) - (1 - \rho)(\pi_{Hd} - \pi_{Hc})} \leq 0,
\]

which holds as long as \( \rho \geq 1 - \epsilon \).

In the case when \( (\beta_L > 0, \beta_H = 1) \), the indifference condition implies

\[
\beta_L^{**} = \frac{\pi_{Hd} - \pi_{Hc} - (\frac{1}{\pi_{Hd} - \pi_{Hc}} - \pi_{Hd})\mu}{\pi_{Hd} - \pi_{Hc} - (1 - \pi_{Hd})\mu} \quad \text{and} \quad \alpha^{**} = \frac{1 - \pi_{Hc}}{\pi_{Hd} - \pi_{Hc} + (1 - \pi_{Hc})\left(\frac{\epsilon}{1 - \rho}\right)}.
\]

To verify that \( \beta_H^{**} = 1 \), it must be true that the ruling coalition prefers to stage a coup after it observed a high signal,

\[ \rho \geq \Pr(d|H)(1 - \epsilon) + 1 - \Pr(d|H). \] \hspace{1cm} (3)

After substituting \( \alpha^{**} \) into

\[ \Pr(d|H) = \frac{\pi_{Hd}\alpha^{**}}{\pi_{Hd}\alpha^{**} + \pi_{Hc}(1 - \alpha^{**})}, \]

inequality (3) can be reduced to

\[
\frac{(\pi_{Hd} - \pi_{Hc})(1 - \rho)[\rho - (1 - \epsilon)]}{(\pi_{Hd} - \pi_{Hc})(1 - \rho) + \pi_{Hc}(1 - \pi_{Hd})\epsilon} \geq 0,
\]

which holds as long as \( \rho \geq 1 - \epsilon \).

Moreover, the expected payoff to both the dictator and the ruling coalition is greater in the
equilibrium with \((\beta_L = 0, \beta_H > 0)\) than it is in the equilibrium with \((\beta_L > 0, \beta_H = 1)\). In the equilibrium with \((\beta_L = 0, \beta_H > 0)\), the expected payoff to the dictator is

\[
\frac{b(\pi_{Hd} - \pi_{Hc})(1 + \mu)}{\pi_{Hd} - \pi_{Hc} + \pi_{Hd} \mu},
\]

and it is

\[
\frac{b(\pi_{Hd} - \pi_{Hc})(1 - \rho)(1 + \mu)}{\pi_{Hd} - \pi_{Hc} - \mu(1 - \pi_{Hd})}
\]

in the equilibrium with \((\beta_L > 0, \beta_H = 1)\). The difference between the former and the latter is

\[
(\pi_{Hd} - \pi_{Hc})(1 + \mu)(1 - \rho) \frac{\rho(\pi_{Hd} - \pi_{Hc}) - \mu(1 - \rho \pi_{Hd})}{[\pi_{Hd}(1 + \mu) - \pi_{Hc}][\pi_{Hd} - \pi_{Hc} - \mu(1 - \pi_{Hd})]},
\]

which is positive as long as Assumption 1 in the paper is satisfied.

In the equilibrium with \((\beta_L = 0, \beta_H > 0)\), the expected payoff to the ruling coalition is

\[
\frac{(\pi_{Hd} - \pi_{Hc})[\rho - (1 - \epsilon)] + \pi_{Hc} \rho}{(\pi_{Hd} - \pi_{Hc})[\rho - (1 - \epsilon)] + \pi_{Hc} \epsilon},
\]

and it is \(\rho\) in the equilibrium with \((\beta_L > 0, \beta_H = 1)\). The difference between the former and the latter is

\[
\frac{(\pi_{Hd} - \pi_{Hc})(1 - \rho)[\rho - (1 - \epsilon)]}{(\pi_{Hd} - \pi_{Hc})(1 - \rho) + \pi_{Hd} \epsilon},
\]

which is positive as long as \(\rho \geq 1 - \epsilon\). Thus both the dictator and the ruling coalition prefer the equilibrium in which \((\beta_L = 0, \beta_H > 0)\) to the equilibrium in which \((\beta_L > 0, \beta_H = 1)\). This concludes all proofs associated with Proposition 1.

Proposition 2: Recall that the probability of successful power-sharing is

\[
\Pr(\text{Successful Power-Sharing}) = (1 - \alpha^*)[\pi_{Hc}(1 - \beta_H^*) + (1 - \pi_{Hc})] = (1 - \alpha^*)(1 - \pi_{Hc}\beta_H^*). \quad (4)
\]
As I showed in the paper, in a contested dictatorship with $(\beta_L = 0, \beta_H > 0)$, both the probability that the dictator diverts ($\alpha^*$) and the probability that the ruling coalition stages a coup after observing a high signal ($\beta_H^*$) increase as the balance of power ($b$) shifts in the dictator’s favor. In turn, the probability of successful power-sharing is decreasing in the dictator’s power.

The probability of a successful diversion is

$$\text{Pr}(\text{Successful Diversion}) = \alpha^*[\pi_{Ld} + \pi_{Hd}(1 - \beta_H^*) + \pi_{Hd}\beta_H^*(1 - \rho)] = \alpha^*(1 - \pi_{Hd}\rho\beta_H^*).$$

Substituting $\alpha^*$ and $\beta_H^*$ from (1), we obtain

$$\text{Pr}(\text{Successful Diversion}) = \frac{b\pi_{Hc}(\pi_{Hd} - \pi_{Hc})}{[\pi_{Hd}\epsilon - b(\pi_{Hd} - \pi_{Hc})][\pi_{Hd}(1 + \mu) - \pi_{Hc}]}.$$

Finally, differentiating with respect to $b$, we obtain

$$\frac{\partial \text{Pr}(\text{Successful Diversion})}{\partial b} = \frac{\pi_{Hc}\pi_{Hd}(\pi_{Hd} - \pi_{Hc})\epsilon}{[\pi_{Hd}\epsilon - b(\pi_{Hd} - \pi_{Hc})]^2[\pi_{Hd}(1 + \mu) - \pi_{Hc}]} > 0.$$

Thus the probability of a successful diversion is increasing in the dictator’s power.

**Proposition 3:** By inspection of (1), we see that both $\alpha^*$ and $\beta_H^*$ are decreasing in $\pi_{Hd}$ and increasing in $\pi_{Hc}$. To see that $\alpha^*$ is increasing in $\pi_{Hc}$, differentiate $\alpha^*$ with respect to $\pi_{Hc}$, to obtain

$$\frac{\partial \alpha^*}{\partial \pi_{Hc}} = \frac{\pi_{Hd}\epsilon - b(\pi_{Hd} + \pi_{Hc})^2}{[\pi_{Hd}(\epsilon - b) + \pi_{Hc}]^2} > 0.$$

In turn, the probability of successful power-sharing in (4) is increasing in $\pi_{Hd}$ and decreasing in $\pi_{Hc}$. 

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References