Unemployment Risk and Flat Job Ladders
Lessons from the German Reunification

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Abstract
Can a change in labor market regulations improve the allocative efficiency of worker-firm matches? We study this question using the German reunification as a natural experiment which exposed East Germany to Western-style institutions. We show that even 20 years after the reunification, East German workers are significantly less likely to work at the region’s highest-paying firms than workers in the West. This allocational difference explains 25% of the large real wage gap between the two regions. Using matched employer-employee data, we find that the efficiency gap arises because, first, East German workers face a flatter job ladder: when moving job-to-job, the difference between their previous firm’s median wage and the new firm’s median wage is smaller than in the West. Second, East German workers become more frequently unemployed. We show that our results can be generated by a job ladder model in which a higher risk of job termination lowers the incentive for high productivity firms to post vacancies, which flattens the job ladder. Our work highlights that policies shifting a country’s labor market institutions towards Western policies may fail to generate large efficiency gains when they are accompanied by a rise in unemployment.

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1 Introduction

A recent literature has argued that misallocation of resources across firms can explain a sizable fraction of cross-country income differences.\(^1\) In one notable example, Hsieh and Klenow (2009) show that replicating the cross-firm distribution of resources of U.S. manufacturing in India would increase Indian manufacturing productivity by 50%. The result suggests that countries might be able to realize large gains by bringing their economic policies in line with the U.S.. In this paper, we apply the misallocation insight to the labor market, and use the reunification of East and West Germany to study the effects of a sudden shift towards Western-style institutions on the allocation of a country’s workers and on aggregate wages. We show that policies that seek to improve labor market institutions may in fact fail to generate allocative efficiency gains if they are accompanied by a sufficiently large rise in unemployment.

In 1990, the former German Democratic Republic (GDR, the East of Germany) and the Federal Republic of Germany (West Germany) reunited. The reunification exposed East German workers, who previously had limited mobility and earned virtually identical wages within a given sector, to a competitive labor market overnight. Despite a fully integrated labor market, however, East German real wages remain about 30% below West German levels, a fact that has generated considerable research.\(^2\) Our paper examines to what extent this wage gap is due to differences in the allocation of workers to firms. We interpret a firm’s median wage as its inherent measure of productivity, and define allocative efficiency as the correlation between a firm’s median wage and its number of workers, similar to Bartelsman, Haltiwanger, and Scarpetta (2013). We then show, using establishment-level administrative data from the German Federal Employment Agency (BA), that the correlation between firm size and a firm’s median wage is significantly lower in East Germany than in the West even 20 years after the reunification.\(^3\) Moving East German workers towards the region’s higher-paying firms to match the West German worker-firm distribution would increase East German wages by about 8%. The lower allocative efficiency thus explains more than 25% of the wage difference between East and West Germany.

We propose a decomposition of the efficiency gap and show that it could be generated, in an accounting sense, by four possible forces. First, worker mobility in the East may be lower, preventing a reallocation of workers towards firms paying higher wages. We call this the lack of flows hypothesis. Second, only a relatively small share of worker flows could be due to job-to-job transitions towards better firms, or workers that move job-to-job could go to firms

\(^1\)See e.g., Banerjee and Duflo (2005), Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013), and Restuccia and Rogerson (2013).
\(^2\)See e.g., Sinn (2002), Gernandt and Pfeiffer (2008), Smolny (2009), Smolny and Kirbach (2011)
\(^3\)Throughout this paper, we use the terms “firm” and “establishment” interchangeably. The micro data we use are at the level of the establishment.
paying median wages that are similar to their previous employer’s wage level. We call this the flat ladder hypothesis, based on the literature suggesting that workers move up a job ladder when they switch jobs (see, e.g. Burdett and Mortensen (1998)). Alternatively, transitions into unemployment might account for a large fraction of flows, or workers finding a job after an unemployment spell might join firms paying significantly lower median wages than their previous employer. We refer to this as the slippery ladder hypothesis. Finally, workers moving into East Germany might join firms paying low median wages compared to the firms of workers who leave the region. We call this the exit hypothesis.

We investigate the four hypotheses using matched employer-employee data from the BA, which allow us to track the employment history of a large sample of workers for almost two decades. The analysis reveals that the efficiency gap is driven by two of the channels: the flat ladder and the slippery ladder hypothesis. We analyze the flat ladder hypothesis by comparing the median wage of a worker’s origin and destination firms after a job-to-job move, where the wages are normalized by the mean and the variance of the wage distribution in each region to account for distributional differences. We find that the standardized firm-level median wage increases by 9.1% for job movers within East Germany, compared to 11.5% in the West. This finding implies that when switching jobs, East German workers move up relatively less in the within-region distribution of firm wages than workers in the West, which depresses the correlation between firm size and average wage. The channel is compounded by the fact that job-to-job transitions, which lead workers towards higher-paying firms, account for only 60% of job quits in East Germany, while in the West the figure is greater than 70%. Our decomposition shows that setting either of the two components of the flat ladder hypothesis equal to West German levels would completely eliminate the efficiency gap between the two regions.

A second force reducing allocative efficiency in the East is the slippery ladder hypothesis. We find that the share of job quits that is due to transitions into unemployment is about 10 percentage points higher in East Germany than in the West.\footnote{Job-to-job transitions and transitions via unemployment do not sum to one since there is also movement across regions and out of the labor force.} Workers re-joining the workforce after an unemployment spell on average move to firms paying significantly lower median wages than their previous employer, which weakens allocative efficiency. However, this drop in the standardized firm-level median wage is only 9.4% in East Germany, compared to 12.5% in the West. We find that the more frequent movement via unemployment in the East dominates the smaller wage loss, so that the slippery ladder hypothesis reduces allocative efficiency in East Germany. If both the share of flows and the size of the wage drop were set to West German levels, the efficiency gap between the two regions would narrow by about 15%.

The lack of flows or the exit hypothesis contribute little to explaining the lower allocative
efficiency in East Germany. We find that the average flows of workers are in fact higher in East Germany in the decade after the reunification than in the West. For example, in 1993, 34% of all workers in East Germany left their current firm, compared to only 20% in the West. For the exit hypothesis, we document flows from the East towards West German firms amounting to on average 7% of all firm-level inflows, while flows from the West account for only about 4% of firm-level inflows in East Germany. The median wage of new arrivals’ firms is higher than the median wage of leavers’ firms in both regions. However, given the relatively small magnitude of flows, shutting down cross-region flows completely only marginally changes the relative efficiency of the two regions.

To explain our findings, we propose a search and matching model in which low and high productivity firms compete for workers. We impose a simple job ladder by assuming that unemployed workers can only match with low productivity firms, while employed workers can meet both types of firms. This modelling choice is supported by the data, where firms paying higher wages hire a significantly larger fraction of workers from other firms. We shut down movement across regions in our model, and compare East and West Germany as separate economies. The model can qualitatively generate all observed differences between East and West Germany through variation in only one parameter, the probability of exogenous breakdown of a worker-employee relationship. We show that an economy with a higher probability of job separation generates higher unemployment and a lower level of allocative efficiency, matching the characteristics of the East. Importantly, this lower efficiency is generated via the flat ladder and the slippery ladder channels, as in the data. For the flat ladder hypothesis, first, a higher probability of job separation increases the unemployment pool and lowers the fraction of worker flows that is job-to-job. Second, the larger unemployment pool makes it relatively easier for low productivity firms to fill vacancies, which lowers workers’ probability of transitioning from a low to a high productivity firm. This force in turn depresses the expected firm-level wage change from job-to-job switches. In line with the slippery ladder, a higher separation probability also implies a smaller drop in the firm-level median wage for workers that become unemployed, since fewer workers are separated from high productivity firms. The model, as in the data, generates higher total flows in East Germany so that a lack of flows does not explain the lower allocative efficiency. Our model highlights a previously overlooked effect of unemployment on the allocative efficiency of the labor market, which operates via the steepness of the job ladder. A large unemployment pool provides an abundant supply of workers for low productivity firms, leading them to gain employment share at the expense of high productivity ones and weakening allocative efficiency.

We exploit the heterogeneity of the dataset to provide additional support in favor of our mechanism. In both East and in West Germany, more educated individuals are less likely to be
unemployed. As expected, these individuals move up the job ladder the most when they switch jobs.

Our paper relates to the recent work trying to quantify the effects of resource misallocation, such as Hsieh and Klenow (2009), Bartelsman, Haltiwanger, and Scarpetta (2013), or Adamopoulos and Restuccia (2014). While these papers focus on productivity differences across countries, our setup examines whether policy is able to generate efficiency gains within the same country, for the case of the labor market. Recent work by Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez (2015) has similarly studied the consequences of a major policy shift on allocative efficiency by examining the effects of the introduction of the euro on the Spanish manufacturing sector. The strength of our paper is that we can track individuals across firms and over time, which enables us to identify the sources of misallocation at a very granular level.

A number of papers have examined East German convergence after the reunification (e.g., Burda and Hunt (2001), Burda (2006)). In particular, there exists a large literature studying wage differences and wage inequality in East and West Germany (Krueger and Pischke (1995), Hunt (2001, 2002, 2006), Fuchs-Schündeln, Krueger, and Sommer (2010)). Card, Heining, and Kline (2013) quantify the contributions of worker and employer heterogeneity and of the assortativeness of worker-firm matches on West German wage inequality. Our paper is the first to examine the contribution of allocative efficiency to the wage gap between East and West Germany.

Our paper proceeds as follows. In Section 2, we provide evidence that workers in the East German labor market were misallocated prior to the reunification, and provide some historical background. In Section 3, we perform the empirical analysis. We first present some aggregate facts, then compute a measure of allocative efficiency, decompose it into the four possible channels, and calculate their contribution. We develop a model to explain our findings in Section 4 and show that the model’s implications hold in the data. Section 5 concludes.

2 Historical Background

2.1 Misallocation of the Labor Market in the GDR

The policies of the GDR promoted a misallocation of workers. As described in Grünert (1997), policies affected the labor market at the macro level, via the education system and industrial policy, as well as the mobility of workers and wage setting.

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5 Other papers in this literature include Smolny (2009), Smolny and Kirbach (2011), and Kohn and Antonczyk (2013).
At the macro level, the GDR’s economy was centrally planned, with 5-year plans specifying targets for the economy’s output and for desired productivity increases. These objectives determined the number of workers required for each industry. The education system and local organizations then had the objective to qualify and “channel” workers towards these industries. For example, most schools advised students regarding occupational choice to steer them towards target industries, and the number of each firm’s apprenticeship positions had to be approved by local party organizations. Local organizations provided housing, public transport, and other benefits to prevent emigration of workers from their region, or to attract worker inflows if desired. These policies promoted a movement of workers towards politically desired regions and industries.

Labor market policies placed tight restrictions on worker flows. While the GDR’s workers were nominally free to choose jobs, firms were not allowed to lay off workers except for significant disciplinary reasons. Furthermore, individuals had the right by law to have a job in accordance with their qualifications (which was increasingly understood to be the right to have the desired job), and had the right not to be transferred to another workplace if they did not want to. As a result of this, the fraction of employees switching employer in the GDR was only 7% in 1981 and 7.4% in 1986, compared to about 20% in West Germany (Grüner (1997)). Unemployment was extremely low. Worker turnover was seen as undesirable because it made central planning more difficult, and it was argued that workers would lose valuable job-specific knowledge if they switched jobs. In order to tie workers closely to the firm, leadership roles were generally filled through within-firm promotions, and firms were required to designate “reserve managers”, who were developed over time. Additional distortions were generated because many talented workers refused management jobs, since they often paid less than worker positions (Leiby (1999)).

Wages setting was also distorted. The payment of workers was based on a wage level that was centrally fixed for each occupation. Wage differences were mainly used to allocate workers to priority occupations and sectors, usually in agriculture and heavy manufacturing. As a result, job-to-job movement for higher compensation virtually did not exist. While wages were also supposed to provide incentives to raise productivity, bonuses were restricted to not diminish significantly the importance of the centrally agreed wages. Furthermore, seniority rules were an important feature of compensation, with wages usually increasing steadily over the life of a worker.

Overall, these labor market policies clearly distorted the allocation of workers in the GDR. The rigidity of the GDR’s labor market was also reflected in the structure of production. Firms in the GDR were organized in large horizontally and vertically integrated industrial groups, which shielded them from competition. In 1988, industrial production was organized in 173 centrally administered and 95 locally administered groups, which together accounted for close
to 50% of overall employment in the GDR (Lipshitz and McDonald (1990)). The rigid industrial structure further hampered the reallocation of workers towards more productive uses. It also preserved an industrial structure that in the late 1980s was still heavily skewed towards agriculture and heavy manufacturing, while West Germany was increasingly transitioning towards services (Klodt (1990)).

2.2 Reunification and Transition to a Market Economy

The transition of the GDR towards a market economy was extremely rapid. In the late 1980s, the economic stagnation of the GDR fostered growing discontent of its population. Starting in May 1989, East Germans took to the streets in ever greater number to demand greater economic and political freedoms (Apelt (2010)). At the same time, an increasing number of people left the country and fled to West Germany via Hungary and Poland. Given the mounting pressure, the GDR’s communist party eventually allowed East Germans to cross freely into the West from November 7, 1989, thus removing the Berlin Wall (Leiby (1999)). The GDR’s leaders hoped that these steps would end the protests and allow a return to normalcy. However, over the following months, protesters on the street increasingly began to demand reunification (Apelt (2010), Kowalczuk (2010)). This culminated in a resounding victory for the parties standing for fast reunification during the first free elections on March 18, 1990. The election result initiated the reunification process and paved the way to full unity of Germany.

Unification began on July 1, 1990 with full monetary, economic, and social union of the two Germanys. At this point, the new East German government enacted rapid capitalist reforms, introducing the regulations and institutions of a market economy to the GDR. These included for example the West German commercial code and federal taxation rules, as well as a reform of the labor market which imposed Western-style institutions (Leiby (1999)). At the same time, the West German Deutschmark (DM) became the legal currency of both halves of Germany. Wages and salaries were converted from Ostmark into DM at a rate of one-to-one, as were savings up to 400DM. While the currency reform implied an East German wage level of about 1/3 the West German level, in line with productivity, the switch meant that East German firms immediately lost export markets in Eastern Europe, since customers there could not pay in Western currency. Additionally, East German customers switched to Western products, which were of much higher quality than East German ones (Smolny (2009)). West German unions negotiated sharp wage increases in many East German industries, which were not in line with productivity gains but driven by a desire to harmonize living conditions across the country (Burda and Hunt (2001), Smolny (2009)). As a consequence, East German unit labor costs rose sharply, and output and employment collapsed (Burda and Hunt (2001)). This trend was
further exacerbated by the break-up and transfer of unproductive East German conglomerates to private owners, who usually downsized or closed plants.\footnote{This transfer was done via the *Treuhandanstalt*, a public trust, which was set up by the West German government to manage and ultimately sell the GDR’s public companies. West German were initially slow to invest into East German firms. Eventually, most firms were sold at very steep discounts to the highest bidder, usually West German firms, which were often motivated by subsidies and had little interest in keeping their acquisitions alive (Leiby (1999)).}

The rapid shift towards a market-based economy based on Western regulations should provide ample scope for reallocation of workers. We document in the next section that convergence of real wages remains incomplete, and then investigate to what extent this fact is driven by a lower allocative efficiency in East Germany.

3 Empirical Analysis

3.1 Aggregate Facts

We begin our analysis by examining several aggregate variables for East and West Germany. While the facts reported are generally well-known in the literature (e.g., Burda and Hunt (2001), Smolny (2009)), they provide the motivation to look at the micro dynamics of the labor market, which represent the main contribution of the paper. The data are compiled by the individual states’ statistical offices in every year, and are available for the time period since 1991 for East Germany and since 1970 for the West.\footnote{http://www.vgrdl.de/Arbeitskreis_VGR/} We also obtain unemployment data from the Federal Employment Agency. Throughout the paper, we aggregate across states excluding Berlin to construct series for East Germany and for West Germany.\footnote{We exclude Berlin because it cannot be attributed to either East or West Germany, since it was divided between the two before 1990.}

Economic performance of East Germany is still significantly below West German levels. In Figure 1a, we show that a considerable fraction of the initial GDP per capita gap between East and West closed between 1991 and 1995. However, convergence stopped in 1995, and 60% of the initial gap is still present today. In Appendix A, we propose a standard growth accounting exercise to show that the persistent gap is almost entirely due to differences in total factor productivity. As emphasized in the literature, micro level distortions can show up at the aggregate level as difference in TFP (e.g., Restuccia and Rogerson (2008)). Hence, the large TFP gap motivates the search for differences in allocative efficiency across East and West.\footnote{However, due to data availability, at the micro level we do not compute measures of productivity but rather focus on wage differences as the exogenous firm level primitives. For this reason our findings in the micro data do not map directly into this decomposition.}

Turning to the labor market, in Figure 1b we compute real wage per worker in East and West
Germany. The series map closely our results for GDP per capita, and show that a significant gap still remains between the two regions. In 2010, the average real wage per worker was about 30% higher in West Germany than in the East. Wage convergence basically stopped in 1995. At the same time, East German unemployment is considerably higher than in the West throughout the period after the reunification. Figure 2 shows that unemployment in East Germany increases sharply after the reunification in 1990, and is about twice as high than in the West by the late 1990s. As discussed in Section 2, this development was the result of profound structural adjustments in the East German economy that led to the closure and reorganization of many firms.

The findings highlight that, from an economic perspective, East and West Germany still look very different. This outcome arises despite the fact that they have been exposed to the same policies and regulations for the past 20 years. We investigate one possible source of the lack of convergence, namely the differences in the efficiency of the labor market, as represented by the allocation efficiency of workers to firms.

3.2 Micro Facts: Data

Our micro analysis uses confidential establishment-level data and matched employer-employee data provided by the German Federal Employment Agency (BA) via the Institute for Employment Research (IAB). The establishment-level data are obtained from the Establishment History Panel (BHP).\[^{10}\] This dataset contains a 50% sample of all establishments in Germany with at least one employee liable to social security on the 30th June of a given year. The data are

\[^{10}\]We use the version of the BHP that is stratified by size.
based on mandatory annual social security filings. Government employees and the self-employed are not covered. Similarly, marginal part-time employees were exempt from social security until 1999, and are therefore not in the dataset until that year.\footnote{Marginal part-time employees are employees with person group code 109, “Marginally paid employees according to § 8 Paragraph 1 No. 1 SGB IV”, and person group code 209, “Marginally paid employees which are paid according to the household check procedure according to § 28a Paragraph 7 SGB IV”.} The underlying population of workers comprises about 80\% of the German working population. The data are recorded as annual cross-sections since 1975 for West Germany and since 1991 for East Germany, which we combine to form a panel covering about 650,000 to 1.3 million establishments per year.

An establishment in the BHP is defined as a company’s unit with at least one worker liable to social security operating in a distinct region and industry. Since several plants of the same company may operate in the same region and industry, the establishments in the BHP do not always correspond to economic units such as a plant (Hethey-Maier and Schmieder (2013)). Throughout the rest of this paper, we use the terms establishment and firm interchangeably to refer to these entities. For each such establishment, the dataset contains information on the establishment’s location, number of employees, employee structure by education, age, and occupation, and the wage structure. The BHP also tracks the inflows and outflows of employees by establishment in every year. Inflows are defined as employees who are working at an establishment at the reference date of June 30 in one year but not at the reference date of the previous year. Outflows are defined accordingly. The data also cover establishment entry and exit.\footnote{These are defined based on the first and the last record of an establishment, refined using the classification by Hethey-Maier and Schmieder (2013).} We only use full-time employees in all of our analyses unless otherwise noted.

The matched employer-employee data are provided via the longitudinal version of the Linked Employer-Employee Dataset (LIAB). The dataset is a combination of a subset of the establishment information from the BHP, the IAB Establishment Panel (a representative survey of...
German establishments) and individual-level information from the Integrated Employment Biographies (IEB). The IEB contains employment information and socio-economic characteristics of all individuals that were employed subject to social security or received social security benefits since 1993. These data are based on social security filings and benefits claims. The LIAB links the individual-level data to information on establishments for a representative sample of individuals. This sample is drawn by taking all establishments that are in the IAB Establishment Panel survey in any year between 2000 and 2008, and selecting all the individuals who were employed in one of these establishments at least one day during the time period from 1999 to 2009. The entire employment history of these individuals is then drawn for the period 1993-2010, including spells at other establishments, unemployment spells, etc., with exact beginning and end dates for each spell. The LIAB sample covers about 1.9 million individuals working for between 2,700 and 11,000 establishments per year. In addition to establishment information, the dataset records an individual’s education, residence, year of birth, occupation and daily wage. As with the BHP, we keep only full-time employment spells.

We first derive a simple measure for allocative efficiency. We then decompose allocative efficiency into four terms to illustrate the potential sources of these differences, and investigate these sources.

### 3.3 Allocative Efficiency

We interpret an establishment’s median wage as its inherent measure of productivity, and treat it as an exogenous characteristic throughout the paper. Our measure of allocative efficiency is based on the idea that if two firms are identical except that one of them has higher productivity, reflected in the payment of a higher wage, then all workers should move to this firm. In the real world, labor market frictions or decreasing returns to scale may prevent such an extreme allocation. However, a region’s allocative efficiency is higher the larger the share of workers that is employed at those firms paying a relatively high wage, as these firms are assumed to be the high productivity firms. Our measure of efficiency does not depend on the wage level, which, as documented above, is significantly lower in the East. Instead, it focuses on the correlation between firm size and wages.

To obtain some intuition of the shape of the worker-firm distribution, we first use the BHP to construct quartiles of the firm-level wage distribution in each year, for East and West Germany separately, and compute the fraction of workers that is employed by firms in each of these quartiles. All wages and employee counts are based on full-time employees only. The average fraction of workers employed in each of the quartiles is shown in Figure 3a for the years 1991-1995, and in Figure 3b for the years 2006-2010. The figures highlight that within both
periods, allocative efficiency in West Germany is higher, since a larger proportion of workers is employed by firms which pay a higher than average wage. For example, in the period 2006-2010, 49% of East German workers are employed by firms in the fourth quartile of the East German wage distribution, while in West Germany the figure is 59%. Allocation efficiency has increased slightly over time in both regions, but the gap does not change.

To make the comparison quantifiable, we propose a simple decomposition in the spirit of the work by Bartelsman, Haltiwanger, and Scarpetta (2013). Define $\bar{w}$ as the weighted average of the firm-level median wages, where the weighting is based on the number of employees in each firm

$$\bar{w} = \frac{1}{N} \sum_{j \in J} w_j n_j.$$ 

Here, $N$ is the total number of workers, $J$ is the total number of firms, $w_j$ is median wage paid at firm $j$ (observed in the BHP), and $n_j$ is number of employees at firm $j$. Notice that, to ease notation, we omit region or time subscripts, but the decomposition is calculated for each
region-year bin. We rewrite the average of firm-level wages as follows

\[
\bar{\bar{w}} = \frac{1}{N} \sum_{j \in J} w_j n_j = \bar{w}_u + \frac{1}{N} \sum_{j \in J} (w_j - \bar{w}_u) (n_j - \bar{n}),
\]

where \( \bar{w}_u \) is the average unweighted firm-level median wage, \( \bar{w}_u \equiv \frac{1}{J} \sum_{j \in J} w_j \), and \( \bar{n} \) is the average firm size, \( \bar{n} \equiv \frac{1}{J} \sum_{j \in J} n_j \). The average firm-level wage can thus be expressed as the sum of the unweighted average wage and a covariance term which increases if more workers are concentrated at firms paying a high wage. We define

\[
\rho \equiv \frac{1}{N} \sum_{j \in J} (w_j - \bar{w}_u) (n_j - \bar{n}) \quad \bar{w}_u
\]

so that

\[
\bar{\bar{w}} = (1 + \rho) \bar{w}_u.
\]

The variable \( \rho \) is a measure of allocation efficiency, since it measures by how much the weighted average firm-level wage is higher than the unweighted one due to the fact that high paying firms employ more workers. In Figure 3c, we plot this variable over time for both East and West Germany. We find that allocation efficiency is consistently higher in West, with the gap decreasing only slightly over time, mainly because of the first years following reunification. Efficiency has been increasing in both East and West Germany since the late 1990s without affecting the absolute difference. Averaging across 1993-2010, we find that West Germany’s allocative efficiency is about 30% higher than in the East over the period.

To quantify the importance of the gap in allocative efficiency, we compute a simple counterfactual exercise and examine by how much the average wage in East Germany would increase if it was brought up to the same allocative efficiency as in the West. From equation (3), \( \bar{\bar{w}}_E = (1 + \rho_E) \bar{w}_{uE} \) and \( \bar{\bar{w}}_W = (1 + \rho_W) \bar{w}_{uW} \), and hence the counterfactual wage in East Germany under the West German efficiency can be computed as

\[
\bar{\bar{w}}_{E,W} = (1 + \rho_W) \bar{w}_{uE}.
\]

Figure 3d plots the percentage wage gain, \( \frac{\bar{\bar{w}}_{E,W} - \bar{\bar{w}}_E}{\bar{\bar{w}}_E} \). The figure shows that after a steep drop in gains during the first year after the reunification, the implied wage gain from moving to the West German allocation stays roughly constant at 8% throughout the whole period. Since the total gap in average wage between East and West is approximately 30%, this counterfactual shows that 25% of the gap can be accounted for by differences in allocation efficiency.

An important assumption in the previous exercise is that the wage distribution is exogenous.
Thus, the previously derived figures only hold exactly if we could replicate in East Germany the same allocation of workers across firms as in the West without affecting the distribution of firm wages. While this is a strong assumption, we nonetheless believe that the exercise is valuable in providing an estimate of the quantitative implications of allocation efficiency. In Section 4, we use a model to provide a more detailed analysis in which we allow for the wage distribution as an endogenous object.

Our finding of a large and persistent gap in allocative efficiency challenges the view that exposing a country to the same set of policies and regulations is sufficient to eventually reach the efficient distribution. Differences across the two regions persist over time. To make progress in understanding the sources of the gap, we now decompose the evolution of allocative efficiency into four terms. These highlight the potential drivers of the lack of convergence.

### 3.4 Decomposition of Allocative Efficiency

To be explicit about the dynamics of $\rho_t$, we let $t$ indicate the month of the observation. We assume that an individual firm’s median wage evolves according to

$$w_{j,t+1} - \bar{w}_u = \gamma (w_{j,t} - \bar{w}_u) + \epsilon_{j,t+1},$$

where $\gamma$ captures the reversion of firm-level wages towards the mean and $\epsilon_{j,t+1}$ is a mean zero error term that is uncorrelated with the size of the firm. This equation captures changes in the individual firm’s quality: while the aggregate wage distribution remains fixed, firms change position within that wage distribution. We assume that the number of firms in each region remains constant.

To derive the decomposition, we note that three types of worker movement affect allocation efficiency within a given region: workers can move firm-to-firm within the same region, they can move from unemployment into a job or in the reverse direction within the region, or they can move into and out of the labor force. Transitions out of the labor force include relocations between East and West Germany, but also include retirements, etc. For the purposes of our decomposition, we consider a steady state where inflows and outflows into and from unemployment and into and out of the labor force are exactly balanced and where the share of each type of flows in total flows is constant. Define $f$ as the share of within-region job-to-job flows in total flows in a given period, let $u$ be the share of within-region transitions into and out of unemployment, and $l$ be the share of flows into and out of the labor force, including cross-region transitions. Define $m$ as the fraction of workers in the labor force transitioning in one of the three ways. We can then show that the allocative efficiency in steady state is given by
\[
\rho = \frac{1}{1 - \gamma} \left[ \sum_{(A)} f \frac{\Delta \bar{w}^f}{\bar{w}_u} + u \frac{\Delta \bar{w}^u}{\bar{w}_u} + l \frac{\Delta \bar{w}^l}{\bar{w}_u} \right],
\]

where we define \( \Delta \bar{w}^f \) as the average difference in a job-to-job transition between the destination firm’s median wage level and the origin firm’s median wage level, and \( \Delta \bar{w}^u \) and \( \Delta \bar{w}^l \) are defined analogously for flows via unemployment and flows via out of the labor force. The derivation of this equation is presented in Appendix B. Note that the equation is based on firm-level wages, rather than individual-level wages, since the former capture the productivity of the firm based on our assumptions.

Equation (6) highlights the four effects influencing the steady state allocative efficiency within a region. Term (A) reflects the extent of overall worker movement. Terms (B), (C), and (D) capture the efficiency effect of movements between firms, from and to unemployment, and into and out of the labor force. Based on this decomposition, we propose four hypotheses which could drive the lack of convergence between East and West Germany. First, if there is little mobility in the labor market, reallocation is small (term (A)). We call this the Lack of Flows Hypothesis. Second, allocative efficiency in the East would be diminished if the share of firm-to-firm transitions is low, or if the median wage of destination firms is not much higher than the median wage of origin firms (term (B)). We refer to this as the Flat Ladder Hypothesis, based on the literature suggesting that workers move up a job ladder when they switch jobs (see, e.g. Burdett and Mortensen (1998)). If the firm-level wage change is small, workers switching jobs move to firms with relatively similar productivity to their origin firm, and hence such movements do not contribute much towards improving allocative efficiency. The third effect that could reduce allocative efficiency is that many workers could move into and out of unemployment, and these transitions could be associated with a lower wage of destination firms relative to origin firms (term (C)). We call this the Slippery Ladder Hypothesis, since movements into unemployment would cause workers to move down the job ladder. Finally, workers moving out of the labor force, for example due to moves across regions, could on average be working for firms paying a high wage relative to workers joining the labor force (term (D)). This effect would weaken the correlation between firm size and average wages. We refer to it as the Exit Hypothesis. Using the matched employer-employeeed data from the LIAB, we can empirically quantify the contribution of each of these four forces to allocative efficiency. We show that most of the efficiency gap between East and West Germany is due to the flat ladder and the slippery ladder hypothesis.
Lack of Flows Hypothesis (A)

To investigate the lack of flows hypothesis, we use establishment-level BHP data to examine whether worker flows in East Germany are large following reunification. If workers do not move, convergence in allocative efficiency cannot take place. In each year we compute the total number of workers joining a firm (inflows), and the total number of workers leaving a firm (outflows) in East and West Germany, and divide these by the total number of employees in the labor force in the previous year. Figure 4a shows that this ratio of worker inflows is initially significantly higher in East Germany compared to West Germany. In 1993, the first year for which we have reliable data for East Germany, 30% of East German workers join a new firm, compared to only 19% of workers in West Germany. Similarly, 35% of East German workers leave a firm in 1993 (Figure 4b). The outflows converge to West German levels only around the mid-2000s. Since around one third of workers move in the initial period following reunification, our results suggest that a lack of flows is not responsible for the missing convergence in allocative efficiency. In Section 3.5, we compute the exact contribution of flows to efficiency in equation (6).

One driver behind the increased flows in East Germany could be firm entries and exits. Outflows could be so high because of firm exits, and inflows could be elevated due to entries. We therefore recompute the figures using only firms that are not entrants or exiter in the current year. Figures 15 and 16 in Appendix D show that inflows and outflows are still elevated in East Germany even when only continuing firms are considered. Our findings are also not

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13The spike of inflows in 1999 is most likely due to the regulatory change that required firms to report marginal part-time workers from then on. While we exclude marginal part-time workers from our analysis, the change made it relatively cheaper for firms to employ marginal part-time employees as regular employees, causing these employees to switch status and be recorded as inflows. Women in West Germany were often hired as marginal employees, while East German women were often hired as regular employees. Figure 19 in the Appendix documents that indeed there was a large “inflow” of female workers in West Germany in 1999.
driven by workers of a specific occupation class (Figures 17-18 in Appendix D).\textsuperscript{14}

**Flat Ladder Hypothesis (B)**

The flat ladder hypothesis consists of two parts. First, the impact of job-to-job movements on allocative efficiency could be low because few workers move job-to-job within their region (term $f$ in equation (6)). Second, increases in allocative efficiency could be low because workers switching jobs move to firms that pay only slightly higher median wages than their previous employer (term $\Delta \bar{w}^f$). We examine each of these two components in turn, using the matched data.

To compute the share of within-region job-to-job flows in all worker flows, we record for every worker in each month whether he is employed, unemployed, or out of the labor force. We also include identifiers recording whether the worker is in East or in West Germany, based on the location of the worker’s firm if the worker is employed and the location of the worker’s unemployment agency if he is unemployed. For transitions out of the labor force, we focus only on movements across regions, where the East-West classification is based on the location of the origin firm for outflows and on the location of the destination firm for inflows. Our computation misses movements into and out of the labor force which are due to retirement, schooling, etc., which we assume balance out in steady state.\textsuperscript{15} We compute job-to-job transitions as cases where a worker employed at a given firm is recorded as working for a different firm in the next month. This methodology is likely to understate true job-to-job movements, as workers taking a short break with a movement into unemployment or out of the labor force are not recorded.\textsuperscript{16} We calculate within-region job-to-job flows as share of total flows between jobs and unemployment, either within or across region. We average the flows across months for each year.

Figure 5a highlights that a significantly smaller fraction of each year’s inflows in East Germany is from another job than in the West: during the 1990s, job-to-job transitions account for on average about 58% of worker inflows in East Germany, compared to 70% in West Germany. Thus, less than two thirds of workers joining a firm in the East are job-to-job movers. We obtain a similar result for the share of outflows that are due to job-to-job movements (Figure 5b).

To compute the wage gap, we study workers’ job-to-job transitions within a given region, holding the identity of the worker fixed. We consider wage changes over time for a given worker

\textsuperscript{14}This is based on the B1 code reported in an employee’s employment notification (“occupational status”). Since these flows are not available for full-time workers only, we compute them for all workers excluding marginal employees.

\textsuperscript{15}We plan to examine this in more detail.

\textsuperscript{16}We find that allowing for a gap between jobs of up to four months does not alter our results significantly.
to avoid potential selection issues due to differences across workers. For each transition, we compute the difference in the normalized, firm-level median wage between the worker’s origin firm and his destination firm, where we use a normalized wage to account for distributional differences across the two regions. This wage is calculated by first computing the distribution of establishment-level log median wages in both East and West Germany in each year, dropping wages that are below the 1st or above the 99th percentile. We then subtract from each establishment’s log wage the average log wage of the establishment’s region in that year, and divide by the standard deviation of the wage distribution. Finally, we average across transitions and across years. This standardized variable captures the ranking of establishments within each region. The first set of bars in Figure 6 shows that job-to-job moves in East Germany are also associated with on average lower normalized wage gains than in the West. A job-to-job movement in East Germany leads to a standardized firm-level wage increase by about 9.1%, compared to 11.5% in the West. Thus, based on this measure, the firm-level wage increase associated with switching jobs is smaller in East Germany than in the West, and hence job switchers in the East move up the job ladder by less.

The second set of bars in Figure 6 presents the average wage gaps based on firm-level wages that are not normalized. Within-region job-to-job movements are on average associated with an increase in the firm-level median wage of 4.4% in East Germany and 5.3% in the West. The third set of bars, based on the change of a worker’s individual wage, shows a similar picture. This variable depends on characteristics that do not necessarily reflect the quality of the establishment, such as the worker’s bargaining power, his responsibilities on the two jobs, etc.

Overall, we find that job-to-job flows are a smaller share of overall flows in East Germany, and the wage gap associated with moving is smaller. This suggests that the flat ladder hypothesis
is an important driver behind East Germany’s inferior allocative efficiency. We compute the exact contribution of this hypothesis in (6) in Section 3.5.

**Slippery Ladder Hypothesis (C)**

The analysis of the slippery ladder hypothesis proceeds along the same steps as for the flat ladder hypothesis. Allocative efficiency could be reduced either because a larger share of workers move into or out of unemployment in the East (term $u$ in equation (6)), and such transitions are associated with a reduction in the firm-level median wage, or because workers moving through unemployment to a new firm experience a reduction in the firm-level median wage that is larger in East Germany than in the West (term $\Delta \bar{w}^u$).

We compute the share of within-region unemployment-to-job transitions similar to job-to-job movements, using workers that are unemployed in one month and then employed in the same region in the next. We average across months in each year. Figure 7a shows that inflows from unemployment are significantly higher in East Germany than in the West. In an average year in our sample, inflows from unemployment account for 32.8% of inflows in the East, compared to 22.9% in the West. The flipside of this result is that job-to-unemployment transitions as a share of outflows are also significantly higher in the East (Figure 7b).

We next compute, for workers going through an unemployment spell between two jobs, the difference in the firm-level median wage between the pre- and the post-unemployment firm. This wage difference is calculated using the same three measures as before, holding the identity of the worker fixed. The first set of bars in Figure 8 shows that after transitioning through unemployment, the average worker’s next establishment pays a standardized firm-level median wage that is 9.4% lower than the previous wage in East Germany, compared to 12.5% in the West. Thus, going into unemployment is actually more harmful in the West. Using the raw firm-
Figure 7: Flows from and to unemployment

(a) Unemployment-to-job flows as a share of total inflows

(b) Job-to-unemployment flows as a share of total outflows

Figure 8: Wage changes from job-to-job movements via unemployment, within region

level median wages without the normalization, we find that transitions through unemployment imply a firm-level wage decline by 1.7% in the East and 3.7% in the West. Workers also suffer a significant drop in their individual wage when going through unemployment.

Overall, unemployment has a strong negative effect on firm-level wages in both East and West Germany. However, while more workers transition through unemployment in the East, these workers also move down by less in the distribution of firm-level median wages. We show in Section 3.5 that the first effect dominates, so that the slippery ladder channel reduces allocative efficiency in the East relative to the West.

Exit Hypothesis (D)

To study the exit hypothesis, we focus on workers leaving the labor force due to movement across regions. Empirical evidence suggests that there has been substantial migration from
East to West Germany following reunification (e.g., Burda (2006)). Based on equation (6), East Germany’s allocative efficiency could be reduced as a result of substantial flows out of the labor force, captured by $l$, if workers joining the region on average move to firms paying a lower median wage than the firms of leavers.

Similar to before, we calculate our inflow-based measure of cross-region flows as the share of total inflows that are due to cross-region job-to-job and unemployment-to-job transitions. Figure 9a documents that the share of inflows into West German firms from the East has been around 7% over the sample period, while inflows into East German firms from the West have been around 3-4% of inflows during the 1990s. These transitions include both migration as well as commuting. Similarly, cross-border outflows as a share of total outflows are significantly higher for East Germany than for the West (Figure 9b). Interestingly, during the early 1990s, West-East flows as a share of total outflows were higher than flows in the other direction. This result arises because of the significant firm-to-firm and firm-to-unemployment flows in East Germany during the early 1990s, as documented before.

We begin our analysis of the wages of cross-regional movers by focusing first on individual workers’ experiences, as before, and compare the firm-level median wage in a worker’s origin firm in his initial region to his destination firm in the other. We average these figures across transitions and over time. The first set of bars in Figure 10a highlights that workers moving West experience a reduction in the normalized firm-level median wage by 23%. Note that the normalization is conducted with respect to two different distributions. The figure shows that East German workers moving West on average transition to a firm with a substantially lower rank in the West German wage distribution than the rank of their origin firm in the East. On the other hand, workers moving East experience an increase in the normalized firm-level
median wage by 40%, and hence move to a relatively better East German firm. Once we take into account the different wage levels in the two regions, however, we find that the non-standardized firm-level median wage of workers moving from East to West increases significantly, while workers moving from West to East move to firms paying a lower median wage (second set of bars in Figure 10a). The behavior of individual-level wages is similar (third set of bars). These results highlight that East-West movers are able to secure substantial wage gains. However, since the wage level in the West is higher, they appear to be less discerning when choosing a new job, and on average move to firms that have a worse position in the West relative to their East German origin firm. The experience is reversed for West-East movers. We compute wage changes similarly for workers who were unemployed in their initial region and then move across regions for a new job. The results are qualitatively similar to the case of direct job-to-job movements across regions, but scaled down (Figure 10b).

To implement the efficiency decomposition (6), we compare the firm-level wage differential for movers to and leavers from a region, respectively, holding the region fixed, rather than the identity of the worker. From this exercise, we find that workers moving into East Germany move to firms whose average firm-level median wage is 1.4% higher than the wage at the average leaver’s firm. The corresponding figure for West Germany is 2.8%. The figures indicate that new arrivals in both regions move to relatively better firms than leavers, improving allocative efficiency. However, this mechanism is stronger in the West.
3.5 Computing the Decomposition

We now quantitatively examine the relative importance of the four hypotheses for the allocative efficiency gap. To obtain steady state values of the flows, we average across all years in the sample. Since inflows and outflows do not exactly balance, we report efficiency measures using both a calculation based on inflows and one based on outflows. We also re-compute the wage changes using wages in levels rather than in logs in order to exactly implement the decomposition formula. Table 1 summarizes the estimated values of the individual components from that calculation. Using these values, we compute the ratio of allocative efficiencies across the two regions as

$$\frac{\rho_W}{\rho_E} = \frac{1 - \gamma_W}{1 - \gamma_W} \cdot \frac{m_W}{m_E} \left[ f_W \frac{\Delta \bar{w}_f}{\bar{w}_{u,W}} + u_W \frac{\Delta \bar{w}_u}{\bar{w}_{u,W}} + l_W \frac{\Delta \bar{w}_l}{\bar{w}_{u,W}} \right].$$

We calibrate the mean reversion parameter $\gamma_W$ to match the level of allocative efficiency in West Germany in 2010, which is approximately $\rho_W = 0.40$. Given our values for the other components, this implies $\gamma_W = 0.99$. Assuming that $\gamma_E = \gamma_W$ and using the ratio of unweighted average wages observed in the data, $\bar{w}_{u,E}/\bar{w}_{u,W} = 0.72$, our exercise yields $\rho_E = 0.30$ using the inflow-based measure and $\rho_E = 0.34$ based on outflows. Thus, we obtain a ratio of relative efficiencies of 1.32 based on inflows, and of 1.18 based on outflows. This result is close to the empirically observed, actual aggregate gap of $\rho_W/\rho_E = 1.3$ found in Section 3.3 (first row of Table 2), highlighting that our decomposition is able to match the aggregate figure.

We can now assess the relative importance of the four hypotheses by shutting down the different channels in our decomposition. Row (2) of Table 2 documents the effect of eliminating
the lack of flows channel by setting $m_E = m_W$. We find that such a change would actually increase the efficiency ratio, confirming that the higher flows in the East improve the efficiency of the allocation, all else equal. In row (3), we study the effect of the flows component of the flat ladder hypothesis. We increase the fraction of job-to-job flows in the East to West German levels, and simultaneously reduce flows between jobs and unemployment so that the fractions still sum to one. We find that this change completely eliminates the efficiency gap. This result highlights that the relative inefficiency of the East German worker allocation is partially due to East German workers transitioning too often via unemployment. In row (4), we keep the flows at their original levels, and instead impose the same job ladder in both regions by setting the firm-level wage increase from job-to-job transitions in the East equal to the West German level, $\Delta \bar{w}_E^f / \bar{w}_{u,E} = \Delta \bar{w}_W^f / \bar{w}_{u,W}$. This change has an even larger effect and increases East German allocative efficiency significantly above the West German level. Thus, setting either of the two components of the flat ladder hypothesis equal to West German levels would remove the efficiency gap. Row (5) combines these two effects.

To study the slippery ladder hypothesis, in row (6) we first replicate the previous argument and show that setting employment flows equal to West German levels would completely eliminate the gap. In row (7), we investigate the wage effect of unemployment by setting $\Delta \bar{w}_E^u / \bar{w}_{u,E} = \Delta \bar{w}_W^u / \bar{w}_{u,W}$. Since wage losses in West Germany are higher than in the East, this exercise actually worsens the efficiency gap. Setting both flows and wage changes equal to West German levels, we find a decline in the efficiency gap by about 5-25%, dependent on whether the inflow- or outflow-based measure is used (row (8)). Thus, overall the slippery ladder channel has a small adverse effect on efficiency, but this effect is only driven by the flows component.
Finally, we consider the impact of the exit hypothesis by setting $l_E = l_W = 0$, thus removing cross-border flows. We find that the effect of this exercise on allocative efficiency is even smaller than for the slippery ladder channel, and close to zero using the outflow-based measure. The reason for this outcome is that cross-border flows represent only a relatively small share of total flows. We therefore conclude that the gap in allocative efficiency between East and West Germany is mainly driven by the flat ladder channel, or alternatively by the flows component of the slippery ladder channel.

4 A Model of Misallocation and Unemployment

To illustrate the mechanism that drives the connection between higher unemployment and lower allocative efficiency in East Germany, we develop a stylized search model of the labor market. We extend the benchmark search framework of Mortensen (1982) and Mortensen and Pissarides (1994) by introducing two types of firms with different productivities and imposing a job ladder: we assume that workers in unemployment can find a job only at low productivity firms, while workers at low productivity firms can move to both low and high productivity firms. We then vary a single parameter, the rate of job separation, and show that the empirical results can be generated if the rate of job separation is exogenously higher in the East than in the West.

Our main assumption is that the job ladder is rigid in the sense that unemployed workers must move to a low productivity firm first. This assumption is supported in the data. Figure 11 plots the share of firms’ hires that are job-to-job, as opposed to from unemployment, in an average year for the four quartiles of the firm-level median wage distribution in both East and West Germany. The figure shows that the share of job-to-job hires increases significantly with a firm’s median wage. For firms in the lowest wage quartile in East Germany, the share of hires that come from another job is only 58% (West: 66%), and rises monotonely to 83% (West: 83%) at the top quartile. Our model imposes that these high productivity firms only hire workers from other jobs.

4.1 Setup

4.1.1 Environment

We model each region (East and West Germany) in isolation. We describe thus the environment for one region and then perform comparative statics exercise that can be interpreted as comparing East and West.
Let time be continuous. A region is populated by a mass one of homogenous workers and a continuum of firms. There exist two types of firms: high productivity ones with productivity $p_H$, and low productivity ones with productivity $p_L < p_H$. We exogenously impose a job ladder: workers from unemployment finding a job can transition only to $p_L$ firms, while from $p_L$ firms they can switch jobs both to $p_L$ or to $p_H$ firms. We assume that there is directed on-the-job search, with employed workers looking for vacancies separately in the markets of low and high productivity firms, respectively. Since all high productivity firms are identical, workers at these firms do not search for jobs.

The other features of the environment follow the standard Mortensen-Pissarides model. Firms produce output using a constant returns to scale technology, equal to $p_L$ and $p_H$, respectively, for each worker. We can therefore write the firms’ value functions on a per-worker basis, since there are no scale effects. To hire a worker, firms need to post vacancies. We denote by $v_H$ vacancies posted by $p_H$ firms and by $v_L$ vacancies posted by $p_L$ ones. We assume that posting a vacancy carries a flow cost that is proportional to the level of productivity of the firm, for example because staff employed in hiring new workers cannot be used in production. Hence, vacancy costs are $\chi p_L$ for $p_L$ firms and $\chi p_H$ for $p_H$ firms, where $\chi$ is a constant.

On the worker side, workers can be either unemployed, employed at low productivity firms $p_L$, or employed at high productivity firms $p_H$. Unemployed workers earn a flow benefit $b$, which can be interpreted either as utility from leisure or as unemployment insurance. Workers employed at $p_L$ firms earn wage $w_L$, while workers employed at $p_H$ firms earn wage $w_H$. We call the (endogenous) fractions of unemployed workers, workers employed at firms $p_L$, and workers employed at firms $p_H$ respectively, $\phi_U$, $\phi_L$, and $\phi_H$. Market clearing requires $1 = \phi_U + \phi_L + \phi_H$.

Workers and firms meet in a frictional labor market. Unemployed workers and low productivity firms meet based on the matching function $m_U (\phi_U, v_L) = \phi_U^{\alpha_U} v_L^{1-\alpha_U}$. By assumption, unemployed workers cannot move to high productivity firms. Workers at low productivity firms costlessly search for other jobs by searching for vacancies separately in the low productivity
and the high productivity market. We assume that workers finding a match always switch jobs, regardless of the destination firm’s type. On-the-job search leads to a match with another low productivity employer based on the matching function \( m_L(\phi_L, v_L) = \phi_L^{\alpha_L} v_L^{1-\alpha_L} \). Workers at low productivity firms meet high productivity employers with \( m_H(\phi_L, v_H) = \phi_L^{\alpha_H} v_H^{1-\alpha_H} \). Those workers employed by high productivity firms do not search on-the-job, since they cannot find a better employer. We define tightness in each of the markets by \( \theta_U = \frac{\nu_L}{\phi_L} \), \( \theta_L = \frac{\nu_L}{\phi_L} \), and \( \theta_H = \frac{\nu_H}{\phi_L} \), respectively. Then, the rate at which workers move out of unemployment is given by \( \theta_U q_U(\theta_U) = \frac{m_U(u,v_L)}{\phi_U} \); the rate at which workers move from \( p_L \) to other \( p_L \) firms is given by \( q_L(\theta_L) = \frac{m_L(\phi_L, v_L)}{\phi_L} \), and the rate at which workers move from \( p_L \) to \( p_H \) firms is given by \( q_H(\theta_H) = \frac{m_H(\phi_L, v_H)}{\phi_L} \).

Vacancies at low productivity firms are filled either from unemployment or based on transitions from other low productivity firms. Thus, vacancies \( v_L \) are filled at rate \( \theta_U q_U(\theta_U) + \theta_L q_L(\theta_L) \), where the first term represents vacancies filled from unemployment and the second term represents vacancies filled from other \( p_L \) firms. The rate at which \( v_H \) vacancies are filled is given by \( \theta_H q_H(\theta_H) \). Last, jobs at \( p_L \) and \( p_H \) firms are exogenously terminated at rate \( \lambda \), so that at rate \( \lambda \) people transition back to unemployment. Individuals discount future earnings at rate \( r \). We restrict our analysis to steady states.

### 4.1.2 Value Functions

Our environment is recursive and generates a set of value functions. Let \( U \) be the worker’s value of being unemployed, and \( V_L \) and \( V_H \) be the value of vacant positions in low and high productivity firms, respectively. Similarly, we denote by \( J_L \) and \( J_H \) a worker’s value of being in a low and in a high productivity firm, and by \( W_L \) and \( W_H \) the value of a low and of a high productivity firm with a filled position, respectively.

The worker’s value of being unemployed is given by

\[
rU = b + q_U(\theta_U)(W_L - U).
\]

Workers earn the flow benefit of being unemployed, and transition to a low productivity firm with a probability \( q_U(\theta_U) \) that depends on market tightness.

For employed workers, the value functions are given by

\[
rW_L = w_L + \lambda (U - W_L) + q_L(\theta_L)(W_L - W_L) + q_H(\theta_H)(W_H - W_L).
\]

\[^{17}\text{This assumption can be justified through some opacity in the matching market. That is, before accepting the job workers assign a positive probability that the new firm is of type } p_H. \text{ As long as this probability is bigger than zero even for } p_L \text{ firms, then workers would always accept a firm to firm movement.}\]
and
\[ rW_H = w_H + \lambda (U - W_H). \] (10)

Workers earn a flow wage and transition to unemployment at rate \( \lambda \). Workers in low productivity firms can additionally transition firm-to-firm at respective rates \( q_L (\theta_L) \) and \( q_H (\theta_H) \). However, for workers moving between low productivity firms, the expected change in the value is zero.

On the firm side, the value functions of low and high productivity vacancies are
\[ rV_L = -\chi p_L + (\theta_U q_U (\theta_U) + \theta_L q_L (\theta_L)) (J_L - V_L) \] (11)
and
\[ rV_H = -\chi p_H + \theta_H q_H (\theta_H) (J_H - V_H). \] (12)

Firms pay the flow costs of posting a vacancy. Low productivity firms hire both from unemployment and from other low productivity firms. High productivity firms hire only from low productivity firms.

For filled jobs, the firms’ value functions are
\[ rJ_L = p_L - w_L + \lambda (V_L - J_L) \] (13)
and
\[ rJ_H = p_H - w_H + \lambda (V_H - J_H). \] (14)

A filled job produces net flow output given by the difference between productivity and wage. Jobs are terminated at rate \( \lambda \) and become again a vacancy.

### 4.1.3 Wage Setting and Free Entry

To close the model we need to define a wage setting process and an entry cost. We follow the literature and assume that wages are set based on Nash bargaining between the worker and the firm, and that there is free entry of firms, for both \( p_L \) and \( p_H \) vacancies. Additionally, in the steady state equilibrium (which we focus on) the flows into and out of unemployment and into and out of employment at \( p_L \) firms must balance. The environment thus reduces to a system of seven equations in seven unknowns. The seven unknown equilibrium objects are \( v_L, v_H, \phi_U, \phi_L, \phi_H, w_H, \) and \( w_L \), with the equilibrium restrictions given by two wage equations from Nash Bargaining, two free entry conditions, two Beveridge curves for flow balance, and one market clearing condition. This system can be solved numerically. In Section C, we describe in detail the solution steps.
4.2 Constructing the Empirical Moments in the Model

We use our model to demonstrate that this simple setup can replicate the patterns observed in the data by varying only one parameter, the rate of exogenous job termination $\lambda$. Differences in this parameter across East and West Germany are the exogenous driving force of the model. A higher $\lambda$ in East Germany could for example be justified by the fact that firms in East Germany face competition by more productive West German firms, or it could be due to less experience by East German entrepreneurs in running firms. We construct the same moments in the model as in the allocation efficiency decomposition discussed in Section 6. We then simulate our model and show that these moments change in response to variation in $\lambda$ in the same way as in the data. In particular, the efficiency differences we find are driven by the flat ladder and the slippery ladder hypothesis. Throughout the section, we assume that flows into and out of the labor force are equal to zero.

We now briefly discuss how the empirical moments are constructed in the model, before turning to the simulation.

**Total Flows $m$** The model contains four types of flows. First, workers can move from unemployment to low productivity firms. These flows are given by $m_U(u, v_L) = q_U(\theta_U)u$. Second, workers can transition between low productivity firms, with total flows $m_L(\phi_L, v_L)$, and they can move from low productivity to high productivity firms, with total flows $m_H(\phi_L, v_H)$. Finally, total flows into unemployment are given by $\lambda(\phi_L + \phi_H)$. While in the data we calculated total flows either using an inflows-based measure or an outflows-based measure, given our steady state assumption both of these moments must be equal in the model. In the following, we construct the moments based on the inflows-based measure. Total inflows are given by

$$m = m_U(u, v_L) + m_L(\phi_L, v_L) + m_H(\phi_L, v_H).$$

**Fraction of Hires from Employment $f$** The fraction of hires from employment can be computed by dividing the flows coming from other firms by all inflows. Thus, we have:

$$f = \frac{m_L(\phi_L, v_L) + m_H(\phi_L, v_H)}{m_U(u, v_L) + m_L(\phi_L, v_L) + m_H(\phi_L, v_H)}.$$

**Fraction of Hires from Unemployment $u$** Since we assume that there are no flows into and out of the labor force, the fraction of hires from unemployment is simply given by $u = 1 - f$. 
Wage Change for Firm to Firm Transitions $\Delta w^f$ The average wage change for job-to-job transitions is given by

$$\Delta w^f = \frac{q_L (\theta_L) (w_L - w_L) + q_H (\theta_H) (w_H - w_L)}{q_L (\theta_L) + q_H (\theta_H)} = (w_H - w_L) \frac{q_H (\theta_H)}{q_L (\theta_L) + q_H (\theta_H)},$$

where the first term drops out because transitions between low productivity firms are not associated with a wage increase, and the second term is the wage gain from transitioning to a high productivity firm multiplied by the probability of such a move. Notice that in the model any wage increase is linked to an increase in the firm productivity.

Wage Change for Firm to Firm Transitions through Unemployment $\Delta w^u$ We can also compute the average wage change in moving from a job to another through unemployment. Based on our assumptions, workers coming from unemployment always move to a low productivity firm, paying wage $w_L$. Their pre-unemployment firm is either of low productivity or of high productivity, with shares $\frac{\phi_L}{\phi_L + \phi_H}$ and $\frac{\phi_H}{\phi_L + \phi_H}$, respectively. Thus, the wage change is

$$\Delta w^u = w_L - \frac{\phi_L w_L + \phi_H w_H}{\phi_L + \phi_H} = (w_L - w_H) \frac{\phi_H}{\phi_L + \phi_H}. \quad (15)$$

Both wage changes $\Delta w^f$ and $\Delta w^u$ consist of two components. First, they depend on the average wage gap between low productivity and high productivity firms. A higher wage gap raises the average gain from firm-to-firm transitions, and increases the loss for workers moving through unemployment. Second, the wage changes depend on a probability measure. For direct job-to-job transitions, this probability reflects the likelihood with which job-to-job transitions lead workers to a high productivity firm, rather than to another low productivity one. For transitions through unemployment, the probability captures the likelihood that the worker was employed at a high productivity firm before the unemployment spell, since after unemployment all workers re-enter the labor market at a low productivity employer. Last, note that empirical variation in the normalized wage change across the two regions corresponds in the model only to a change in the probability of moving to high productivity firms, since our computation of normalized wage in the data changes fixes the term $w_L - w_H$.

Allocative Efficiency $\rho$ Finally, we calculate the allocative efficiency. From equation (1), we have that $\rho = \frac{w}{\bar{w}_u} - 1$, where $\bar{w}$ is the weighted average wage and $\bar{w}_u$ is the unweighted average. We can compute both of these objects in the model using wages and the mass of workers at low and high productivity firms. Thus,
\[
\rho \equiv \frac{\bar{w}}{w_u} - 1 = \frac{\phi_L w_L + \phi_H w_H}{\frac{\phi_L + \phi_H}{2} w_L + \frac{1}{2} w_H} - 1.
\]

Notice that this expression is increasing in the fraction of workers \( \phi_H \) working at high productivity firms.

### 4.3 Comparative Statics from Changing the Risk of Job Termination

We conduct a numerical exercise to highlight that the model is qualitatively able to generate the facts observed in the data if East Germany has a higher risk of job termination \( \lambda \). Specifically, we compute the previously described summary statistics for different values of \( \lambda \) and show that the empirically documented differences between East and West Germany are replicated in the model. The results are displayed in Figure 12. For each statistic, we report the percentage deviation relative to the baseline value of \( \lambda \). While the parameter values are arbitrary and the scope of the exercise is thus qualitative at this point, we plan to quantitatively estimate the model in the future. We now describe one by one each of the eight panels in the figure, linking them to the empirical facts on East and West Germany previously documented.

The top left panel illustrates how unemployment varies with the separation rate. Naturally, a high risk of job termination increases unemployment. This is consistent with Figure 2, which shows higher unemployment in East Germany. The second panel in the first row shows that allocation efficiency decreases with \( \lambda \). When termination occurs more frequently, more workers are allocated to the low productivity firm since they move into unemployment more often. The model thus generates a lower allocative efficiency in East as found in the data.

We next examine the drivers behind the differences in allocative efficiency, following the decomposition steps in the data. Consistent with the data, lower allocation efficiency is not driven by total flows \( m \), which are in fact increasing in \( \lambda \) (first panel in the second row). When workers are more frequently separated from firms, there is more churning of workers between firms and unemployment, and hence higher flows. Thus, the lack of flows hypothesis does not contribute to lower allocative efficiency in East Germany.

We observe instead that our model is consistent with the flat ladder hypothesis. As \( \lambda \) rises, a smaller fraction of people is hired directly from other firms, as more workers are hired from unemployment (second panel in row 2). This channel reduces \( f \), consistent with a lower value in East Germany. At the same time, workers moving firm-to-firm on average have a smaller wage increase (first panel in row 3). We find that this result is driven both by a smaller probability that transitioning workers move to a \( p_H \) firm, and by a lower wage gap between \( w_H \) and \( w_L \) firms. This can be seen by looking at the two panels in the last row of Figure 12. The left panel
Figure 12: Model Simulation in Percentages

shows that the gap \( w_H - w_L \) is decreasing in \( \lambda \), and the right panel shows that \( \frac{q_H(\theta_H)}{q_L(\theta_L) + q_H(\theta_H)} \) decreases as well. It is important to notice that the monotonicity of the wage gap in \( \lambda \) is a consequence of our choice of parameter values in this exercise, and does not necessarily need to hold in general. Our result is consistent with the finding in the data that upon a firm-to-firm movement workers in East Germany see a lower increase in both normalized and absolute wages. As \( \lambda \) increases, the value of high productivity jobs relative to low productivity jobs decreases. This result arises because it becomes easier to lose a worker, and for high productivity firms this effect is not compensated by an increase in the likelihood of hiring new workers, since \( p_H \) firms do not hire from unemployment directly. For low productivity firms, on the other hand, it becomes easier to hire workers due to the large pool of unemployed. The relative drop in the value of high productivity jobs relative to low productivity jobs reduces the relative number of
vacancies posted by high productivity firms and thus the flows between these two types of firms.

Regarding the slippery ladder hypothesis, the model also generates a larger fraction of hires from unemployment as the separation rate rises, as can be seen from the right panel in the second row and the fact that by construction $u = 1 - f$. For wages, we find that a higher rate of separation makes $\Delta w^a$ smaller in absolute terms (second panel in row 3). This outcome matches the empirical fact that in East Germany workers transitioning via unemployment take a smaller wage loss than in the West. The model generates this result by the fact that if $\lambda$ is large, few people work at $p_H$ firms, and thus on average individuals have a small wage loss after an unemployment spell. The result is tightly linked to the efficiency of the allocation. In an environment in which workers climb up the ladder and fall down due to unemployment, a more efficient allocation implies that workers have more to lose when falling off the ladder.

Overall, the model matches qualitatively all the observed differences between East and West Germany by changing one parameter, the probability of an exogenous breakdown of a worker-employee relationship, and hence the riskiness of jobs. We next briefly discuss how our assumption that high productivity firms do not hire from unemployment could be relaxed. Then we provide a validation exercise exploiting heterogeneity across different groups within East and West Germany. The next step, on which we are currently working, is to examine the quantitative properties of the model in an enriched framework.

4.4 Model Discussion and Validation

The model revolves around one main assumption, that is, high productivity firms cannot hire directly from unemployment. While we have shown above that empirically firms paying a higher wage hire more job-to-job, our assumption is arguably too strong, imposed to keep the framework parsimonious and to highlight its main properties. The strict nature of the assumption however is not necessary to obtain the results. In fact, the key mechanism of the model is that the relative value of posting a high-productivity vacancy decreases in the unemployment rate. This outcome would arise as long as the percentage change in market tightness generated by an increase in unemployment is higher for low productivity firms, which would be the case as long as firms are able to hire only from unemployment or from firms that are equally or less productive than themselves.\footnote{In fact, we can replicate the results above in a slightly different setting in which $p_H$ firms can hire from unemployment, from other $p_H$ firms, and from $p_L$ firms, while $p_L$ firms cannot hire from $p_H$ firms. Results are available upon request.} Most ladder models satisfy this weaker assumption.

The model’s main insight is to provide a link between the unemployment rate and the steepness of the job ladder, i.e., the likelihood that a worker moves to a better productivity
firm in a job-to-job transition. If the model mechanism is sound, then the same correlation – between a lower risk of job termination and a steeper job ladder – should hold for groups characterized by different unemployment rates within each region, as long as they operate in different labor markets. We next use this insight to provide further validation of the model.

If firms post vacancies that are specific to different education groups, we can exploit the variation in unemployment rates across more and less educated individuals. We use the matched employer-employee data to sort workers into four groups, based on their highest level of education. We then compute the average annual unemployment rate for each of these groups from the sample of workers in our dataset. Figure 13a shows these unemployment rates for East Germany and for West Germany, respectively. On average, less educated workers face a higher unemployment rate, both in East and West Germany. Given this result, our model predicts that workers with a low education should face a flatter job ladder - even upon a firm-to-firm movement. Figure 13b shows that more educated workers face indeed a steeper job ladder when moving job-to-job, that is, the gap in terms of normalized wage between the new and old firm is larger. This finding provides support for our suggested mechanism.

5 Conclusion

Our paper has documented a new set of empirical facts. More than 20 years after the German reunification, workers are still less efficiently allocated to firms in East than in West

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19 These groups are 1) workers with at most intermediate school education (around 9 years of education), 2) workers with secondary school leaving certificate (Abitur) (13), 3) workers with a degree from a university of applied sciences (15), and 4) workers with a university degree (17).
Germany. Our finding shows that access to the same labor market regulations may not be sufficient to achieve convergence in allocative efficiency within a reasonable time period. We then use matched employer-employee data to decompose the East-West efficiency gap into its main components. We show that the lower allocative efficiency in the East is due to workers being more likely to fall of the job ladder by becoming unemployed (slippery ladder hypothesis) and due to workers facing, upon a job-to-job movement, a smaller increase in their employer’s productivity, as measured by the firm-level median wage (flat ladder hypothesis). Last, we show that these findings can be replicated within a stylized job ladder search model in which the risk of job termination is higher in East than in West Germany, thus leading high productivity firms to post fewer vacancies, which in turn decreases allocative efficiency. The model, although stylized, highlights a previously overlooked role of unemployment, that is, its effect on the steepness of the job ladder: a large unemployment pool provides an abundant supply of workers for low productivity firms, making them gain employment share at the expense of high productivity ones. This channel reduces the allocative efficiency of the labor market. If the unemployment effect is sufficiently strong, allocative efficiency might even decline.

We believe that these results are of interest beyond the specific case of the German reunification. First, a recent literature has highlighted that a sizable fraction of cross-country differences in aggregate productivity is driven by lower allocative efficiency in developing countries. Our work provides a cautionary tale relevant for this literature. We show that exposing developing countries to Western-style regulations might not be sufficient to reap the large gains from reallocating resources. Second, more generally, we highlight a new channel through which unemployment may harm an economy and have long-lasting effects via its negative impact on allocative efficiency.

Our work leaves open two obvious directions in which we are currently working. First, to expand the simple model to allow for a quantitative exploration of the data. Second, to understand the deep drivers of the higher unemployment risk in East Germany and how (and whether) they are connected to the disruptive transformation from a planned economy to a market economy.
References


Appendix

A Growth Accounting

We perform a standard accounting exercise to decompose the gap in GDP per capita between East and West Germany into its different components, which will enable us to trace out how they change over time. We follow the literature and assume an aggregate Cobb-Douglas production function, with elasticities to labor and capital equal to $1 - \alpha$ and $\alpha$. We set, as usual, $\alpha$ equal to $\frac{2}{3}$. Aggregate GDP in East and West in a year $t$ is therefore given by

$$Y_{E,t} = A_{E,t}K_{E,t}^{\alpha}N_{E,t}^{1-\alpha}$$
$$Y_{W,t} = A_{W,t}K_{W,t}^{\alpha}N_{W,t}^{1-\alpha},$$

where we observe in the data provided by the federal statistics offices, for each year and separately for East and West Germany, employment $N$, capital $K$ and GDP $Y$. We can so use the formula above to compute the implied total factor productivity term, $A$. We rewrite the previous equation in per capita terms, that is

$$y_{E,t} = A_{E,t}k_{E,t}^{\alpha}n_{E,t}^{1-\alpha}$$
$$y_{W,t} = A_{W,t}k_{W,t}^{\alpha}n_{W,t}^{1-\alpha},$$

where $y \equiv \frac{Y}{L}$, $k \equiv \frac{K}{L}$, $n \equiv \frac{N}{L}$ and $L$ is total population, also observed in the data. Last, we decompose the percentage difference in GDP per capita between West and East into its three components, that is

$$\log y_{W,t} - \log y_{E,t} = \log A_{W,t} - \log A_{E,t} + \alpha (\log k_{W,t} - \log k_{E,t}) + (1 - \alpha) (\log n_{W,t} - \log n_{E,t}).$$

GDP p.c. gap | TFP gap | Capital p.c. gap | Workers p.c. gap

In Figure 14 we plot each component of this decomposition. The initial convergence in GDP per capita is both due to a convergence in capital per capita and in TFP. However, virtually all the current gap between East and West Germany is due to a lower level of TFP in East.
Figure 14: Decomposition of the difference in GDP per capita
B Derivation of the Decomposition Formula

Using the covariance component of equation (1) as a measure of allocative efficiency and the assumption that the wage distribution and the number of firms are fixed, we have that

\[
\rho_{t+1} = \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (\gamma(w_{j,t+1} - \bar{w}_u) + \epsilon_{j,t+1}) (n_{j,t} - \bar{n}_t + \Delta n_{j,t+1} - \Delta \bar{n}_{t+1}),
\]

(16)

where \(\Delta\) indicates the change from one period to the next, \(N_{t+1}\) is the total number of workers in period \(t+1\), and the other variables are defined as before. Define \(n_{j,t+1}^{f+}\) as the number of workers moving to firm \(j\) from another firm between periods \(t\) and \(t+1\), \(n_{j,t+1}^{f-}\) as the number of workers moving from \(j\) to another firm, and \(n_{j,t+1}^{u+}\) and \(n_{j,t+1}^{u-}\) as the number of workers moving to firm \(j\) from unemployment and from \(j\) into unemployment, respectively. Let \(n_{j,t+1}^{l+}\) and \(n_{j,t+1}^{l-}\) be moves from out of the labor force to firm \(j\) and in the reverse direction. Since \(\epsilon_{j,t+1}\) is mean zero and uncorrelated with size, equation (16) can be re-written as

\[
\rho_{t+1} = \gamma \frac{N_t}{N_{t+1}} \rho_t + \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) \left( n_{j,t+1}^{f+} - n_{j,t+1}^{f-} + n_{j,t+1}^{u+} - n_{j,t+1}^{u-} + n_{j,t+1}^{l+} - n_{j,t+1}^{l-} \right),
\]

(17)

which can be equivalently written as:

\[
\rho_{t+1} - \gamma \frac{N_t}{N_{t+1}} \rho_t = \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{f+} - \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{f-}
\]

\[
+ \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{u+} - \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{u-}
\]

\[
+ \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{l+} - \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{l-}.
\]

(18)

Since \(n_{j,t+1}^{l+} = n_{j,t+1}^{l-}\), this can be simplified to

\[
\rho_{t+1} - \gamma \frac{N_t}{N_{t+1}} \rho_t = \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) \left( n_{j,t+1}^{f+} - n_{j,t+1}^{f-} \right)
\]

\[
+ \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{u+} - \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{u-}
\]

\[
+ \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{l+} - \frac{1}{\bar{w}_u} \frac{1}{N_{t+1}} \sum_{j \in J} (w_{j,t+1} - \bar{w}_u) n_{j,t+1}^{l-}.
\]

(19)

Let \(N_{t+1}^{f}\) be the total number of workers moving firm-to-firm, \(N_{t+1}^{u+}\) be the total number of people joining new firms from unemployment, \(N_{t+1}^{u-}\) be the total number of people leaving firms for unemployment, and \(N_{t+1}^{l+}\) and \(N_{t+1}^{l-}\) be the total number of people moving into and out of the labor force, respectively.
Multiply both sides by $N_{t+1}/N_t$ and let $i$ index workers to obtain

$$
\frac{N_{t+1}}{N_t} \rho_{t+1} - \gamma \rho_t = \frac{1}{\bar{w}_u} N_{t+1} \left[ \sum_{i \in F^+_{t+1}} w_{i,t+1} - \sum_{i \in F^-_{t+1}} w_i \right] \\
+ \frac{1}{\bar{w}_u} N_{t+1}^{-u} \sum_{i \in F^+_{t+1}} (w_{i,t+1} - \bar{w}_u) - \frac{1}{\bar{w}_u} N_{t+1}^{-1} \sum_{i \in F^-_{t+1}} (w_{i,t+1} - \bar{w}_u) \\
+ \frac{1}{\bar{w}_u} N_{t+1}^l \sum_{i \in F^+_{t+1}} (w_{i,t+1} - \bar{w}_u) - \frac{1}{\bar{w}_u} N_{t+1}^l \sum_{i \in F^-_{t+1}} (w_{i,t+1} - \bar{w}_u), \quad (20)
$$

where $F^+_{t+1}$ is the set of workers that move to a firm from another firm and the other sets are defined analogously. We define $\bar{w}_{i,t+1}^{+} = \frac{1}{N_{t+1}} \sum_{i \in F^+_{t+1}} w_{i,t+1}$ as the average over the firm-level median wages of destination firms for workers coming from another firm, and use similar definitions for the other wages. Furthermore, define $N_{t+1}^u = \min \{ N_{t+1}^{u+}, N_{t+1}^{u-} \}$, and let $\Delta N_{t+1}^u = N_{t+1}^{u+} - N_{t+1}^{u-}$, which is possibly negative. Similarly, let $N_{t+1}^l = \min \{ N_{t+1}^{l+}, N_{t+1}^{l-} \}$ and $\Delta N_{t+1}^l = N_{t+1}^{l+} - N_{t+1}^{l-}$. Then we obtain

$$
\frac{N_{t+1}}{N_t} \rho_{t+1} - \gamma \rho_t = \frac{N_{t+1}^f}{N_t} \Delta^w \bar{w}_{i,t+1}^{f} \\
+ \frac{N_{t+1}^u}{N_t} \Delta^w \bar{w}_{i,t+1}^{u} + \frac{\Delta N_{t+1}^u}{N_t} \left\{ \frac{1}{\{N_{t+1}^{u+} \geq N_{t+1}^{u-} \}} (\bar{w}_{i,t+1}^{u+} - \bar{w}_u) + \frac{1}{\{N_{t+1}^{u+} < N_{t+1}^{u-} \}} (\bar{w}_{i,t+1}^{u-} - \bar{w}_u) \right\} \\
+ \frac{N_{t+1}^l}{N_t} \Delta^w \bar{w}_{i,t+1}^{l} + \frac{\Delta N_{t+1}^l}{N_t} \left\{ \frac{1}{\{N_{t+1}^{l+} \geq N_{t+1}^{l-} \}} (\bar{w}_{i,t+1}^{l+} - \bar{w}_u) + \frac{1}{\{N_{t+1}^{l+} < N_{t+1}^{l-} \}} (\bar{w}_{i,t+1}^{l-} - \bar{w}_u) \right\}. \quad (21)
$$

where $\Delta^w \bar{w}_{i,t+1}^{f}$ is the average change in wage from moving job-to-job, $\Delta^w \bar{w}_{i,t+1}^{u}$ and $\Delta^w \bar{w}_{i,t+1}^{l}$ are the average wage changes from moving jobs via unemployment and via out of the labor force, and 1 are indicator functions.

We consider a steady state where inflows and outflows from unemployment are equal, $N_{t+1}^{u+} = N_{t+1}^{u-}$, and total flows $\Delta N_{t+1}$ as a fraction of the number of employed are constant:

$$
m = \frac{\Delta N_{t+1}}{N_t}. \quad (22)
$$

We assume that in steady state the share of each type of flows in net flows is constant. Thus, define $u = \frac{N_{t+1}^u}{\Delta N_{t+1}}$, $f = \frac{N_{t+1}^f}{\Delta N_{t+1}}$, $l = \frac{N_{t+1}^l}{\Delta N_{t+1}}$, and let $\frac{u}{m} = \frac{\Delta N_{t+1}^u}{\Delta N_{t+1}}$. Then, since $N_{t+1} = N_t + \nu N_t$ and in steady state $\rho$ is constant, equation (21) becomes

$$
(1 + v - \gamma) \rho = m \left[ u \frac{\Delta^w \bar{u}^u}{\bar{u}_u} + f \frac{\Delta^w \bar{u}^f}{\bar{u}_u} + l \frac{\Delta^w \bar{u}^l}{\bar{u}_u} \right] + v \left\{ \frac{1}{\{u > 0\}} \left( \frac{\bar{w}^u - \bar{w}_u}{\bar{w}_u} \right) + \frac{1}{\{u < 0\}} \left( \frac{\bar{w}^u - \bar{w}_u}{\bar{w}_u} \right) \right\}, \quad (22)
$$

42
where $\Delta \bar{w}^u = \bar{w}^{u^+} - \bar{w}^{u^-}$, and the other average wages are defined analogously. If inflows and outflows from the labor force are also exactly balanced, then we obtain

$$\rho = \frac{1}{1 - \gamma} \left[ u \frac{\Delta \bar{w}^u}{\bar{w}_u} + f \frac{\Delta \bar{w}^f}{\bar{w}_u} + l \frac{\Delta \bar{w}^l}{\bar{w}_u} \right],$$

which is equation (6).
C Solution of the Model

We start from the set of value functions

\[ r_U = b + q_U (\theta_U) (W_L - U) \]
\[ rW_L = w_L + \lambda (U - W_L) + q_L (\theta_L) (W_L - W_L) + q_H (\theta_H) (W_H - W_L) \]
\[ rW_H = w_H + \lambda (U - W_H) \]
\[ rJ_L = p_L - w_L + \lambda (V_L - J_L) \]
\[ rJ_H = p_H - w_H + \lambda (V_H - J_H) \]
\[ rV_L = -\chi p_L + (\theta_U q_U (\theta_U) + \theta_L q_L (\theta_L)) (J_L - V_L) \]
\[ rV_H = -\chi p_H + \theta_H q_H (\theta_H) (J_H - V_H) \]

Next, we follow the literature and impose free entry. Since we have two types of firms, we have free entry for both of them, hence \( V_H = V_L = 0 \). Using free entry we can rewrite, using the definition of \( V_H \) and \( V_L \),

\[ J_L = \frac{\chi p_L}{\theta_U q_U (\theta_U) + \theta_L q_L (\theta_L)} \]
\[ J_H = \frac{\chi p_H}{\theta_H q_H (\theta_H)} \]

while \( J_L \) and \( J_H \) become

\[ J_L = \frac{p_L - w_L}{r + \lambda} \]
\[ J_H = \frac{p_H - w_H}{r + \lambda} \]

Next, we use Nash bargaining, with parameter \( \beta \), to set equilibrium wages. Nash bargaining implies that

\[ W_H - W_L = \beta (W_H - W_L + J_H) \]

next note that

\[ W_H - W_L = \frac{w_H - w_L}{r + \lambda + q_H (\theta_H)} \]

which we can substitute above together with the free entry condition to get

\[ w_H = (1 - \beta) \frac{r + \lambda}{r + \lambda + \beta q_H (\theta_H)} w_L + \beta \left( \frac{r + \lambda + q_H (\theta_H)}{r + \lambda + \beta q_H (\theta_H)} \right) p_H \]
Similarly by Nash bargaining we get that
\[ W_L - U = \beta (W_L - U + J_L) \]
and notice that
\[ W_L - U = \frac{w_L - b + q_H(\theta_U)(W_H - W_L)}{r + \lambda + q_U(\theta_U)} \]
we then get
\[ w_L = (1 - \beta) \frac{r + \lambda}{r + \lambda + \beta q_U(\theta_U)} \left( b - q_H(\theta_H) \left( \frac{w_H - w_L}{r + \lambda + q_H(\theta_H)} \right) \right) + \beta \left( \frac{r + \lambda + q_U(\theta_U)}{r + \lambda + \beta q_U(\theta_U)} \right) p_L \]
which we leave in implicit form to ease interpretation.

Next, we need to calculate the Beveridge curves, which are two, since we must take into
account stability both in unemployment and in the number of people employed at firms \( p_L \). Let \( \phi_H \) be the mass of workers employed at firms \( p_H \), it is immediate to see that \( 1 - u = \phi_L + \phi_H \).
We get the following two accounting equations
\[ \dot{\phi}_U = \lambda(\phi_L + \phi_H) - q_U(\theta_U)\phi_U \]
\[ \dot{\phi}_L = q_U(\theta_U)u - \phi_L(q_H(\theta_H) + \lambda) \]
where notice that the flows from \( \phi_L \) into \( \phi_U \) do not show into the accounting equation since are both a positive and a negative flows which cancel out. In steady state we get the Beveridge curves
\[ \phi_U = \frac{\lambda}{\lambda + q_U(\theta_U)} \]
\[ \phi_L = \frac{q_U(\theta_U)}{(q_H(\theta_H) + \lambda)} \phi_U \]
Overall, we need to solve for seven unknowns, \( u_L, u_H, \phi_L, \phi_H, \phi_U, w_H, w_L \) and we have 7 equilibrium restrictions given by: 2 wage equations, 2 free entry conditions, 2 Beveridge curves,
and 1 market clearing condition. That is,

\[
\begin{align*}
  w_H &- (1 - \beta) \frac{r + \lambda}{r + \lambda + \beta q_H (\theta_H)} w_L - \beta \left( \frac{r + \lambda + q_H (\theta_H)}{r + \lambda + \beta q_H (\theta_H)} \right) p_H = 0 \\
  w_L & - (1 - \beta) \frac{r + \lambda}{r + \lambda + \beta q_U (\theta_U)} \left( b - q_H (\theta_H) \left( \frac{w_H - w_L}{r + \lambda + q_H (\theta_H)} \right) \right) - \beta \left( \frac{r + \lambda + q_U (\theta_U)}{r + \lambda + \beta q_U (\theta_U)} \right) p_L = 0 \\
  \phi_H - \frac{\lambda}{\lambda + q_U (\theta_U)} & = 0 \\
  \chi_H - \frac{q_U (\theta_U)}{(q_H (\theta_H) + \lambda) \phi_U} & = 0 \\
  1 - \phi_U - \phi_L - \phi_H & = 0
\end{align*}
\]

This system is solved numerically for each parameter value of \( \lambda \) for which the result are reported.
D Additional Figures

Figure 15: Full-time worker inflows (continuing establishments)

Figure 16: Full-time worker outflows (continuing establishments)
Figure 17: Worker inflows by occupation class

(a) Unskilled blue collar workers
(b) Skilled blue collar workers
(c) Master craftsmen
(d) White collar workers
Figure 18: Worker outflows by occupation class

(a) Unskilled blue collar workers

(b) Skilled blue collar workers

(c) Master craftsmen

(d) White collar workers

Figure 19: Inflows of females