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Engines of growth: Domestic and foreign sources of innovation

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Abstract

We examine productivity growth since World War II in the five leading research economies: West Germany, France, the United Kingdom, Japan, and the United States. Data on the capital–output ratio suggest that these countries grew as they did because of their ability to adopt more productive technologies, not because of capital-deepening per se. We use a multicountry model of technological innovation and diffusion which nests the cases of endogenous and exogenous growth to simulate the growth of the five countries, given initial productivity levels in 1950 and research efforts in the subsequent four decades. Based on plausible assumptions about 'technology gaps' that existed among these countries in 1950 the exogenous growth version of the model explains their growth experiences more successfully. Specifically, the simulations capture the magnitude of the slowdown in German, French, and Japanese productivity growth and the relative constancy of U.K. and U.S. growth.

JEL classification: F43; O14; O31; O34; O40

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1. Introduction

The revival of interest about what drives national growth rates has spawned a number of controversies: What causes output per worker to differ among countries: differences in capital per worker or differences in available technology? Are sources of growth primarily national or global in origin? Why have productivity levels among the richest countries converged in the post war period and why has productivity growth slowed down? Will a constant research effort suffice to deliver ongoing technological progress, or will sustaining

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productivity growth require putting ever expanding resources into coming up with new ideas?[2]

Whether growth is primarily driven by factor accumulation or by technology, the issue remains as to whether sources of growth are primarily foreign or domestic in origin. If capital accumulation is the key to growth, while international capital markets are highly segmented, countries must rely on their own savings to finance investment. A country with an initially lower level of capital has to finance the investment needed to catch up with its neighbors on its own, which could take a long time. Moreover, a country with a lower savings rate than its neighbors will never catch up, condemned to a permanently lower relative level of output per worker. In contrast, with a high degree of capital market integration a backward country can catch up rapidly by borrowing from abroad. Cross-country differences in savings still imply permanent differences in national levels of GNP per worker, as low savers find themselves in debt to high savers, but levels of GDP per worker will soon converge.

Taking the alternative view that technological innovation and diffusion drive national growth rates, if innovations are applicable only at home, a country must innovate on its own to raise total factor productivity. A backward country has to be more innovative than its neighbors in order to catch up with them, and catching up is likely to take a long time. But if innovations are easy to adopt regardless of where they came from, a technologically backward country can catch up rapidly by absorbing the most advanced technologies, and an innovative country gains little relative advantage in terms of factor productivity.[3]

Our purpose here is to examine some evidence on these issues. We begin by showing that capital deepening provides at best an incomplete explanation of the growth in manufacturing productivity of the leading economies since World War II. We then examine what role technological innovation and international diffusion play in explaining why countries grew as they did.

We adopt a specific model of international technology diffusion taken from Eaton and Kortum (1997). Various parameterizations yield special cases with different implications for growth and convergence. The model nests the cases of exogenous and endogenous growth. If the productivity of researchers grows over time in proportion to the stock of existing knowledge, then a given research effort can sustain permanent growth in productivity. If research productivity rises less fast, however, then sustaining ongoing productivity growth requires putting ever more resources into research.

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[2]Differences in human capital may also be crucial to a country's growth prospects, as argued by Lucas (1988), Barro (1991). One view is that human capital facilitates the adoption of new technology (Benhabib and Spiegel, 1994). An alternative view treats human capital as another factor of production, in which case accumulation of human capital per worker raises effective labor per worker (Mankiw et al., 1992 and Barro et al., 1995). Thus, human capital can play a role in either the technological-adoption explanation or the capital-deepening explanation of differences in productivity growth.

[3]We do not review these controversies in detail. Romer (1994), Grossman and Helpman (1994), Barro and Sala-I-Martin (1995) provide excellent discussions. Coe and Helpman (1995), Eaton and Kortum (1996), Eaton and Kortum (1997), Evenson and Englander (1994) examine the empirical content of models of international technology diffusion.

In our earlier work we chose parameters for each version of the model by fitting it to a cross section of data on productivity, research, and patenting from the five leading research economies (the United States, Japan, Germany, France, and the United Kingdom) for the late 1980s. In doing so we assumed that by then these countries had achieved a steady state in which manufacturing productivity was growing on average at a common rate, since by that point their growth rates were fairly similar. The two versions of the model fit the data equally well, with very similar implications about the international transferability of technology.

In the current paper we see how well each version of the model explains manufacturing productivity growth in the same five countries from 1950 to 1990, a period that began with the countries growing at very different rates. Since the assumption that these countries were in steady state throughout the period is obviously way off the mark, we interpret their growth experiences in terms of the out-of-steady-state behavior of the model. The state variables governing the model's dynamics are productivity levels and the pools of ideas from home and abroad that individual countries have yet to adopt. We initialize our model by setting productivity levels at their actual 1950 values. Of course, we do not know the size of the pools of knowledge available to these countries in 1950. We make the simple assumption that the pools are proportional to what they would be if 1950 were a steady state. Using the parameter estimates from our previous paper we calibrate one additional parameter governing the overall level of the pools.

We find that the model predicts growth rates after 1950 that are quite close to actual ones. To fit the post war experience, the pools available to the United States must be about two times larger relative to U.S. productivity in 1950 than they would be if 1950 were a steady state, in the case of exogenous growth, and about six times larger with endogenous growth. The exogenous version of the model fits rather better, delivering a root mean square error of about 7.6 percent, as opposed to 13.6 percent for the case of endogenous growth.

Since Japan and the three European countries were much further behind the United States in 1950 than they are now (or are in the steady state of our model) these pools are very large relative to productivity levels for these other countries. Either version of the model picks up the moderate and relatively constant rate of U.S. productivity growth. It also explains the rapid growth of Japan, Germany, and France in the 1950s and 1960s in terms of the large pools of ideas available to them. As these pools shrink relative to these countries' levels of productivity, growth slows to rates more like that in the United States. It also tells this story for the UK, thus capturing its experience in the first three decades after World War II. It, however, fails to predict the U.K. growth revival of the 1980s. The exogenous growth story provides a further reason why growth slowed down: As the stock of knowledge grew relative to the labor force, new ideas became harder to find. Hence this version relies less on the size of the initial stocks of unused ideas to explain why growth was initially so high.

Is it plausible to think that Japan and Europe had great potential to grow after World War II by adopting foreign technology? We propose two arguments. First, the United States was clearly a technological leader, even before World War II, and was therefore a great source of technology for others to adopt.[4] Second, the war effort itself produced knowledge about how to apply a number of new technologies.[5] Civilian applications of these technologies were left to be exploited after the war.

We proceed as follows. Section 2 below examines data on productivity growth and capital-deepening in manufacturing for the five leading research economies. Section 3 presents a particular model of technology diffusion and discusses its implications for growth under various parameter values. In Section 4 we consider how well the two variants of our model explain the growth of these countries between 1950 and 1990. We also simulate how the post-war growth experience would change had technology diffusion patterns been different than they were. Section 5 concludes the discussion.

2. Productivity and capital: A glimpse at the data

Table 1 summarizes what happened to value added per hour worked in manufacturing in the United States, Japan, Germany, France, and the United Kingdom over the four decades since World War II. We use a measure of productivity in the manufacturing sector for several reasons. First, most innovations are used in manufacturing. Second, we do not want

Table 1
Value added per hour in manufacturing

Year	Germany	France	UK	Japan	US
(levels, relative to the US in 1990)					
1950	0.12	0.12	0.12	0.04	0.32
1970	0.45	0.42	0.30	0.26	0.57
1980	0.68	0.64	0.37	0.47	0.71
1990	0.86	0.91	0.66	0.78	1.00
(annual rates of growth, percent)					
1950–1970	6.5	6.3	4.5	9.5	3.0
1970–1980	4.0	4.1	2.3	6.1	2.1
1980–1990	2.4	3.6	5.8	5.1	3.4
1950–1990	4.9	5.1	4.3	7.6	2.9

Sources: Value added per hour in manufacturing in each country relative to the United States is from van Ark (1995). Value added per hour in U.S. manufacturing is from Lysko (1995), BLS (1991).

[4]To quote Nelson and Wright (1992), "The process of global diffusion and adoption of American methods would surely have continued, however, either by imitation or by direct foreign investment, if it had not been interrupted by World War II (page 1945).... The United States came out of World War II bouyant, with technological capabilities extended by wartime experience (p. 1950)." Techniques of mass production were a source of U.S. dominance prior to World War II but were at first slow to diffuse. Womack et al. (1991) claim that "Much of the European economic miracle of the 1950's and 1960's was nothing more than a belated embrace of mass production (pages 234–235)."

[5]Examples include: innovations in aluminum fabrication stemming from war-time production of aircraft in Germany (Peck, 1962), magnetic recording in Germany, guided rockets in Germany, jet engines in England, radar in the United States, silicone products in the United States, and titanium in the United States (Jewkes et al., 1969).

to count productivity growth brought about by labor reallocation from agriculture to manufacturing or from manufacturing to services. Third, the land share in manufacturing is low compared with that in services, so that land availability plays a much lesser role in determining labor productivity. We use internationally comparable data compiled at the University of Groningen as part of the International Comparisons of Output and Productivity project.[6]

Several things stand out. During the 40-year period there were only two reversals of relative position: between France and Germany for second place and between the United Kingdom and Japan for last place. Nevertheless, growth rates over the entire period were very different, with Japan at 7.6 percent, France at 5.1 percent, Germany at 4.9 percent, the United Kingdom at 4.3 percent, and the United States at 2.9 percent. Germany, Japan, and France experienced a slowdown of growth throughout the period. For the United States and United Kingdom, however, the slowdown in the 1970s was reversed, with the 1980s representing the period of fastest growth. Consistent with the convergence hypothesis, the country that led in 1950, the United States, had the lowest growth rate over the entire period while Japan, the country with the lowest productivity in 1950, grew the fastest. The range of growth rates in the most recent period is much smaller, but their ranking is less supportive of convergence. It is true that productivity growth in the two laggards was highest, with U.K. productivity growing at 5.8 percent and Japanese productivity growing at 5.1 percent. U.S. productivity growth rose to 3.4 percent, however, ahead of West Germany, which grew at only 2.4 percent.

Turning to capital as an explanation of these differences in productivity levels and growth rates, the first six columns of Table 2 report data on the ratios of capital-to-labor and of capital-to-value added in manufacturing constructed from van Ark and Pilat (1993) and the U.S. Bureau of Labor Statistics, BLS (1991) for three of the five countries.[7] Again, levels are reported relative to the United States in 1990.

Capital–labor ratios do vary positively with value added per hour worked, both across countries and over time. However, this does not imply that capital-deepening was a source of productivity differences. In fact, if capital is internationally mobile then differences in capital–labor ratios are driven by differences in technology, as capital moves to take advantage of its higher marginal product in countries where technology is more advanced.

[6] van Ark and Pilat (1993) describe in detail how the data are constructed. We take the van Ark (1995) measures of manufacturing value added per hour in each country relative to the United States in 1950, 1970, 1980, and 1990. We multiply these figures by value added per hour in the U.S. manufacturing sector, from Lysko (1995). We use data on U.S. manufacturing value-added per hour from the U.S. Bureau of Labor Statistics (1991), to extend Lysko's series from 1956 back to 1950. The data in Lysko (1995) are preferred because they correct the index-number bias that was present in the Bureau of Labor Statistics' fixed-weight measures. The Bureau of Labor Statistics has since corrected this problem but their revised series for the manufacturing sector is not compatible with van Ark's data since it is based on the concept of net manufacturing output rather than manufacturing value added. The data are normalized by the level of value added per hour in the United States in 1990. Growth rates are compounded continuously.

[7] We back out the ratio of capital-to-labor and capital-to-value added for Germany and Japan relative to the United States from the van Ark and Pilat (1993) tables of relative total factor productivity levels, the labor share used to construct them, and relative value added per hour. Since the relative value added per hour is not available for 1960, we use a geometric interpolation to estimate it using the available data in 1950, 1955, and 1965. We then scale these using the inverse of an index of value added per unit of capital from BLS (1991).

Table 2
Capital in manufacturing

Capital-labor ratios				Capital-value-added ratios				
(indexed to 1990 US)				(indexed to 1990 U.S.)			(indexed to 1990)	
Year	Germany	Japan	US	Germany	Japan	US	Germany	France
1950	0.11		0.25	0.91		0.78		
1955		0.06	0.30		0.90	0.78		
1960	0.16	0.04	0.34	0.63	0.52	0.82	0.75	0.87
1973	0.45	0.23	0.49	0.84	0.70	0.72	0.90	0.84
1979	0.64	0.41	0.63	0.94	0.92	0.88	0.94	0.89
1990	0.99	0.77	1.00	1.15	0.99	1.00	1.00	1.00

Sources: Capital-hours ratios and capital-value-added ratios for German and Japanese manufacturing relative to the United States in each year are from van Ark and Pilat (1993). These same ratios for U.S. manufacturing are from BLS (1991) and Lysko (1995). Capital-value-added ratios for Germany and France (indexed to 1 in 1990) are from Lysko (1995). The latter two series are only comparable over time.

However, with perfect international capital mobility, Harrod-neutral differences in technology do not affect the ratio of capital to *value added.* In contrast, if differences in labor productivity result from differences in savings rates among countries with identical technologies then higher labor productivity should be associated with a strictly higher ratio of capital-to-value added. Thus, the behavior of the capital-value-added ratio can distinguish between capital-based and technology-based explanations of productivity differences[8]

In fact, as shown in Table 2, we find little association between productivity and the capital–value added ratio. Indeed, while the capital–value added ratio has fluctuated, the data largely confirm the Kaldor (1961) view that the capital–output ratio is constant during the process of growth. Consider the variation over time first. While there has been a small increase in capital–value-added ratios, the rate of increase has been about the same in each of the three countries. Thus, capital-deepening does not explain why Japanese productivity grew much faster than U.S. productivity.

Looking across countries in any given year strengthens the argument even further. In 1950 U.S. manufacturing productivity was more than two and a half times that of Germany and nearly 8 times that of Japan. Assuming a Cobb–Douglas production function with a capital share of 1/3 and identical technology, the U.S. capital–value-added ratio should

[8]Consider two countries r and p, in each of which value added is produced at constant returns to scale. Value added per worker in country i is given by $Q_i = F(K_i, A_i)$ where F is a homogenous function common to the two countries, K_i is the capital stock per worker in country i, and A_i represents the Harrod-neutral level of technology in country i. Say that $Q_r > Q_p$. One possibility is that $K_r > K_p$ while $A_r = A_p = A$. But the diminishing marginal product of capital would then imply that $K_r/F(K_r, A) > K_p/F(K_p, A)$, that is, a strictly higher capital-value added ratio in r. Another possibility is that $A_r > A_p$ while capital mobility establishes the condition that $F_K(K_r, A_r) = F_K(K_r/A_r, 1) = F_K(K_p/A_p, 1) = F_K(K_p, A_p)$. An implication is that $K_r/A_r = K_p/A_p$. In this case, while, as before, $K_r > K_p$, that is, the ratio of capital-to-physical labor is higher in r, now $K_r/F(K_r, A_r) = 1/F(1, A_r/K_r) = 1/F(1, A_p/K_p) = K_p/F(K_p, A_p)$, that is, the two countries have the same ratios of capital-to-value added.

have been more than 6 times that of Germany and more than 60 times that of Japan. But the evidence from the nearest years for which we have data indicates differences nowhere near these magnitudes.[9]

While we only examine data for three countries, Japan and Germany are two where pure capital-deepening is often held to be an important source of growth after World War II.[10] From this perspective, the relative constancy of the capital–value added ratios in the manufacturing sectors of all three countries is key evidence against the view that different rates of capital-deepening explain why developed countries experienced different growth rates.[11] Instead, we turn toward patterns of innovation and diffusion to understand what happened.[12]

3. A model of innovation and international technology diffusion

We now present a multicountry model of international technology diffusion that treats capital as perfectly mobile across countries. Under particular parameter values the model reduces to the following special cases:

Equal rates of diffusion within and between all countries: If all countries have access to the same technologies then some other factor is needed to explain differences in productivity across countries, even though technology may be the source of growth in world productivity. An implication, noted by Lichtenberg (1992), is that cross-country differences in national levels of productivity should then be unrelated to cross-country differences in national research expenditures. Instead Lichtenberg finds that countries that spend more on research attain higher levels of productivity.

No diffusion between countries: At the opposite extreme, each country might use only those technologies discovered at home. In this case, except by coincidence, levels of productivity across countries will diverge. Endogenous growth models in which a country's productivity depends only on its own rate of investment in research have this implication. This implication is not, however, consistent with evidence that relatively backward countries make use of the technologies of their more advanced neighbors, that is,

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[9]With a much broader set of countries, King and Levine (1994) find some evidence that capital–output ratios rise with productivity. Nevertheless, their data require a capital share of at least 2/3 for cross-country differences in this ratio to account for observed productivity differences. Moreover, like Kaldor, they find no evidence that capital–output ratios in individual countries grow as countries become more productive.

[10]Lysko (1995) provides additional evidence on the evolution of capital–output ratios for Germany and France, reported in the last two columns of Table 2. While there is some upward movement, the extent of capital-deepening is too slight to explain the high income growth of these countries relative to the United States. (Note that these figures are indexed to 1990, and therefore cannot be compared across countries.)

[11]This evidence is consistent with Mankiw et al. (1992) finding that productivity is positively correlated with the investment rate. In our view, that correlation is driven by underlying differences in technology.

[12]King and Rebelo (1993) point out the implausibly large interest rate differentials implied by a capital-deepening explanation of productivity differences. Barro et al. (1995) address this issue by inroducing internationally immobile human capital as a third factor of production. Nonetheless, their model still implies that the capital–output ratio should rise with productivity.

'technological catch-up.'[13] It also fails to explain why inventors frequently take out patents in a number of different countries.

A common pool of technology adopted at different rates in different countries: A more general possibility is that all research outcomes enter a common pool which individual countries can tap. Countries may differ, however, in their ability to draw on this pool.[14] Under this specification, countries' *levels* of productivity converge to a common level only if they exploit the common pool at the same rate. Even if they do not, however, national *growth rates* of productivity will converge if a more backward country finds that a greater fraction of the ideas in the pool are worth adopting. An advantage of this approach is that it can explain steady-state differences in productivity levels while technological spillovers nevertheless occur. A deficiency is that, like equal diffusion, it implies that a greater national research effort confers no particular national advantage, contrary to the Lichtenberg (1992) results.

Diffusion rates depend on the countries involved: The most general specification makes the rate of diffusion specific to the source and destination of the innovation. If it is the case that diffusion is more rapid within than between countries, then a country can attain a higher relative level of productivity by doing more research. Faster adoption of technology also raises productivity. Productivity growth rates again may be equalized since backward countries have a larger backlog of ideas to adopt.[15]

Our model contains the following components:

3.1. *Production*

Output in country n is produced by combining intermediate inputs subject to a constant-returns-to-scale Cobb–Douglas production function,

$$Y_{nt} = \exp\left(\int_0^1 \ln[Z_{nt}(j)X_{nt}(j)]dj\right),$$

where $X_{nt}(j)$ is the quantity of input j produced at time t in country n and $Z_{nt}(j)$ is the quality of that input. There are $n = 1 \ldots N$ countries. They trade the homogeneous output but not the inputs.[16] To produce any input at rate x requires factor services at rate x. We assume that the same Cobb–Douglas function of capital and labor is used to produce each input. As in Grossman and Helpman (1991b), productivity differences across countries arise solely

[13]Economic historians have appealed to this notion to explain the spread of the industrial revolution in the nineteenth century. See, for example, Gerschenkron (1962), Fagerberg (1994). Nelson and Phelps (1966) provide an early model of diffusion from an advanced to a backward country. Helliwell and Chung (1991) report recent evidence supporting the view that low-productivity countries benefit from high-productivity ones.

[14]Parente and Prescott (1994) take this approach in relating a country's level of productivity to its willingness (promoted by low tax rates) to adopt new technologies from around the world. Benhabib and Spiegel (1994) also take this approach in a model in which high levels of human capital promote adoption.

[15]This case is considered in Eaton and Kortum (1996), Eaton and Kortum (1997), Park and Brat (1995), Ben-David and Loewy (1995).

[16]By assuming that inputs are not traded, we ignore some interesting terms-of-trade effects as are illustrated in Johnson and Stafford (1993).

from differences in the quality of inputs. Within a country the quality of inputs improves over time as a consequence of research performed domestically and abroad. We use the index,

$$A_{nt} = \exp\left(\int_0^1 \ln Z_{nt}(j)dj\right)$$

to aggregate the qualities of individual inputs into a single measure of overall productivity.[17]

3.2. *Ideas*

Quality improvements result from new ideas. There are three dimensions to an idea: (i) its quality, (ii) its sector of application, and (iii) the time until it diffuses to each country.

An idea's quality is a random variable Q drawn from the Pareto distribution,

$$F(q) = \Pr[Q < q] = 1 - q^{-\theta}.$$

A given idea has the same quality no matter where it is used.

An idea applies only to one out of a continuum of inputs. The input j to which the idea applies is drawn from the uniform distribution on [0,1].

An idea takes time to learn and to apply to a specific purpose. If an idea is discovered at time t in country i then it diffuses to country n at time $t + \tau_{ni}$, for $n = 1, 2 \ldots N$. We assume that the marginal distribution of the diffusion lag from country i to country n is exponential with parameter ϵ_{ni}, that is, $\Pr[\tau_{ni} \leq x] = 1 - e^{-\epsilon_{ni}x}$.[18] Thus ϵ_{ni} is the speed of diffusion from country i to country n and ϵ_{ni}^{-1} is the mean diffusion lag. As ϵ_{ni} goes to infinity diffusion becomes instantaneous while as ϵ_{ni} approaches 0 diffusion becomes arbitrarily slow. In the limiting case of $\epsilon_{ni} = 0$ there is no distribution of the diffusion lag since ideas from country i are never adopted in country n.

Different restrictions on the ϵ_{ni}s capture the various special cases of diffusion that we discussed above. If there is equal diffusion among all countries then $\epsilon_{ni} = \epsilon$ for all n, i. No diffusion between countries means that $\epsilon_{ni} = 0$ for $n \neq i$. If countries draw on a common pool at different rates then, $\epsilon_{ni} = \epsilon_{n.}\epsilon_{.i}$, where $\epsilon_{n.}$ reflects country n's ability to absorb technology and $\epsilon_{.i}$ reflects country ι's ability to make it available. In general, however, diffusion rates can depend on specific countries.

We assume that country i produces new ideas at rate $E_{it} = \alpha_{it} L_{it}^{1-\beta} R_{it}^{\beta}$ where R_i is research employment in country i, L_i is the workforce there (including researchers), and α_i

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[17] This index actually equals total factor productivity if factors are evenly divided among production of the individual inputs, as would occur if they were allocated by a central planner. In our model, however, different inputs are subject to different degrees of monopoly power, so that more factors are allocated to inputs for which the mark-up over cost is lower. Nevertheless, as we show in Eaton and Kortum (1997), total factor productivity is proportional to this index in a decentralized equilibrium.

[18] The distribution of the diffusion lags across destination countries need not be independent. Hence if a particular invention is absorbed particularly quickly by country n then it might be more likely to be absorbed quickly by country m as well.

and $0 \leq \beta \leq 1$ are parameters governing the productivity of researchers. If $\beta < 1$ then countries face decreasing returns in the intensity with which they perform research.[19]

At any time t country n can draw upon the pool of unexploited ideas from country i. As country i undertakes research it generates a flow into this pool of E_{it} while, as country n adopts them, it depletes the pool in proportion to its size at rate ϵ_{ni}. We introduce N^2 state variables, η_{ni}, representing the stock of ideas from country i that have not yet diffused to country n. Since $\dot{\eta}_{nit} = E_{it} - \epsilon_{ni}\eta_{nit}$,

$$\eta_{nit} = \int_{-\infty}^{t} e^{-\epsilon_{ni}(t-s)} E_{is} ds$$

is the size of the pool of ideas from country i that country n has not yet drawn upon. Finally, we introduce N state variables, μ_n, representing the stock of ideas that have diffused to country n. Ideas flow to country n from the stocks of undiffused ideas,

$$\dot{\mu}_{nt} = \sum_{i=1}^{N} \epsilon_{ni}\eta_{nit}, \tag{1}$$

where $\mu_{nt} = \int_{-\infty}^{t} \dot{\mu}_{ns} ds$.

3.3. *The technological frontier*

We distinguish between the concepts of diffusion and adoption. If $\epsilon_{ni} > 0$, then every idea from country i will eventually diffuse to country n. But some ideas will not be adopted. Only the best available idea for each input in each country is actually used. Thus, for each country n, $Z_{nt}(j)$ represents the highest quality idea that has diffused to country n in sector j by time t. A new idea diffusing there will be adopted if and only if its quality exceeds $Z_{nt}(j)$.

The technological frontier in country n at time t represents the quality of the ideas being used in each sector. The position of this frontier is conveniently summarized by a distribution function, $H_n(z|t)$, representing the fraction of sectors with quality below z. As we show in Appendix A.1, this distribution is given by:

$$H_n(z|t) = e^{-\mu_{nt} z^{-\theta}} \tag{2}$$

which depends only on the stock of diffused ideas μ_{nt} regardless of when these ideas were adopted for production or where they came from.[20]

An idea of quality q that has diffused to country n will be adopted there with probability $H_n(q|t)$. Therefore, integrating over the probability density of possible qualities $F'(q) = \theta q^{-(\theta+1)}$, we obtain the probability of adoption, $\int_1^{\infty} H_n(q|t)F'(q)dq = \mu_{nt}^{-1}$.

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[19]Let U be the an individual's talent for research. We imagine U being drawn from a probability distribution, $\Pr(U \leq u) = G(u)$. Suppose that a researcher with talent U has ideas at rate $\alpha\ \beta\ U$. If individuals with the most talent for research actually become researchers then the talent u^* of the least talented researcher solves $1 - G(u^*) = R/L$. The aggregate rate of idea production is therefore, $\alpha\beta L \int_{u^*}^{\infty} udG(u)$. We obtain the functional form used in our model if the talent distribution is Pareto, $G(u) = 1 - u^{-1/1-\beta}$.

[20]Here and below we use approximations that become arbitrarily close (in percentage terms) as μ becomes large. See Appendix A.1 or, for a more formal analysis, Kortum (1997).

We define the inventive step of a newly diffused idea as the percentage improvement in quality that it brings about if adopted. The average inventive step of a new idea, conditional on its adoption, is $\int_z^\infty \ln(q/z)F'(q)/[1-F(z)]dq = \theta^{-1}$.

Productivity growth in country n is the product of three terms: (i) the rate of arrival of newly diffused ideas, (ii) the probability that an idea will be adopted, and (iii) the average inventive step of the ideas that are adopted. Combining our expressions for each, $\dot{A}_{nt}/A_{nt} = \frac{1}{\theta}\dot{\mu}_{nt}/\mu_{nt}$. Integrating this equation yields the following relationship between our index of productivity A_{nt} and the stock of ideas that have arrived in country n by time t:

$$A_{nt} = c\mu_{nt}^{1/\theta}, \tag{3}$$

where c is a constant derived in Appendix A.2.

A country with a lower level of productivity adopts a higher proportion of the ideas coming its way, $\mu_{nt}^{-1} = c^\theta A_{nt}^{-\theta}$. In this sense the model captures the idea of technological catch-up whereby a country with a lower level of productivity can grow faster by taking advantage of the ideas of others. Indeed, given the rate at which ideas diffuse into a country, its productivity growth is inversely related to its level of productivity,

$$\frac{\dot{A}_{nt}}{A_{nt}} = \frac{c^\theta}{\theta}\dot{\mu}_{nt}A_{nt}^{-\theta}.$$

However, one country's level of productivity may continue to lag behind another's if fewer ideas diffuse to it. We now turn to the dynamics of this diffusion process.

3.4. Productivity dynamics

The system of differential equations:

$$\dot{\mu}_{nt} = \sum_{i=1}^{N} \epsilon_{ni}\eta_{nit},$$

$$\dot{\eta}_{nit} = E_{it} - \epsilon_{ni}\eta_{nit} \tag{4}$$

describes how the vector of state variables evolves over time given paths for the forcing variables, E_{it} (reflecting research effort and the work force in each country) and the N^2+N initial conditions for the state variables.

We now specify how the productivity of researchers (in coming up with ideas) varies across countries and over time. We assume that research productivity depends both upon the country's own stock of knowledge and upon the world stock of ideas. Specifically, we assume that:

$$\alpha_{it} = \alpha\left(\frac{\mu_{it}}{\overline{\mu}_t}\right)\overline{\mu}_t^\gamma$$

where α is a constant term, $\overline{\mu}_t = \sum_{i=1}^N \mu_{it}$, and $\gamma \le 1$.

If $\gamma = 1$ our model exhibits endogenous growth as in the R&D-based growth models of Romer (1990), Grossman and Helpman (1991a), Grossman and Helpman (1991b), Aghion and Howitt (1992). In these models the existing stock of knowledge raises research productivity to the extent that constant research effort can generate perpetual growth of knowledge. If $\gamma < 1$ then, as the stock of knowledge increases, innovations become harder to find. Perpetual growth of knowledge requires ever increasing research effort, as in the single-country model of Jones (1995), Kortum (1997). A key feature of either version of the model is that, as long as no set of countries are completely technologically isolated, all countries converge to a common growth rate, with potentially different relative levels of productivity. That is, both versions of the model deliver a steady state with parallel growth. (Appendix A.3 describes how we solve the model.)

4. Numerical simulations of the model

We now simulate the two cases of our model to see how well they fit the post-war productivity performance of Germany, France, the United Kingdom, Japan, and the United States (shown above in Table 1). We use estimates of the parameters taken from Eaton and Kortum (1997). In that paper, we chose parameters so that the steady state of the model matched data on recent productivity growth and on relative productivity levels, research employment, and international patent applications in the late 1980s. We used no information on how these economies evolved over the postwar period.

We appeal to the relative similarity of growth rates among these countries in recent years to justify our assumption of a steady state with parallel growth in the last decade. In contrast, the widely disparate growth experiences of these countries in the 1950s and 1960s forces us to abandon any attempt to interpret this earlier period in terms such a steady state. What we do here is to look at how well the out-of-steady-state dynamics of our model trace out what happened given the initial conditions dealt out by the Depression and World War II, and the subsequent research effort of these countries.

4.1. *Parameter values*

The parameter values that we use are shown below in Table 3. One set of parameters is for the endogenous growth case and the other is for the exogenous growth case. The two sets of parameters are very similar except for the research spillover parameter which must be one in the endogenous growth case and less than one in the endogenous growth case.

The diffusion parameters ϵ_{ni} are crucial for our exercise here since they determine how quickly ideas spread. Our estimates were based on an extended version of the model that we used to fit patent applications by inventors from each of the five countries taken out in each of the five countries.[21] To economize on parameters we restricted $\epsilon_{ni} = \epsilon_{n.}\epsilon_{.i}\epsilon_H$, where $\epsilon_{n.}$ reflects country n's ability to absorb technology, $\epsilon_{.i}$ reflects country i's ability to provide it, and $\epsilon_H \geq 1$ allows for faster diffusion to the home country if $n = i$ ($\epsilon_H = 1$ if $n \neq i$). To make them easier to interpret, we multiply the appropriate diffusion parameters in Table 3

[21] Eaton and Kortum (1997) incorporate the decision to patent based on: (i) the distribution of the diffusion lag, (ii) the inventive step, (iii) market size, and (iv) the strength of intellectual property protection. Taking the last three factors into account we infer the first.

Table 3
Parameter values

Definition	Symbol	Endog. Growth	Exog. Growth
Parameter of research spillover	γ	1	0.40
Parameter of search distribution	θ	1.87	1.85
Parameter of research productivity	α	0.000495	0.000446
Parameter of talent distribution	β	0.18	0.16
Diffusion factor from:			
Germany	$\epsilon_{.1}$	0.93	0.93
France	$\epsilon_{.2}$	0.28	0.28
UK	$\epsilon_{.3}$	0.58	0.56
Japan	$\epsilon_{.4}$	1.17	1.20
US	$\epsilon_{.5}$	0.21	0.21
Diffusion factor to:			
Germany	$\epsilon_{1.}$	0.19	0.20
France	$\epsilon_{2.}$	0.22	0.22
UK	$\epsilon_{3.}$	0.07	0.07
Japan	$\epsilon_{4.}$	0.11	0.12
US	$\epsilon_{5.}$	0.10	0.10
Diffusion factor domestic	ϵ_H	17.7	17.8

The parameter we present as α is equal to the value of α/J from Eaton and Kortum (1997). The parameter J indicates the range of inputs used in a country, but in the present paper, where we do not model the incentives for research, it can be subsumed in α. The magnitude of this parameter is based on researchers and workers being measured in millions.

for the endogenous growth case to produce the implied matrix of diffusion rates (this matrix is very similar in the exogenous growth case).

	Source				
Destination	Germany	France	UK	Japan	US
Germany	3.13	0.053	0.110	0.222	0.040
France	0.205	1.09	0.128	0.257	0.046
UK	0.064	0.019	0.708	0.081	0.015
Japan	0.102	0.031	0.064	2.28	0.023
US	0.093	0.028	0.058	0.117	0.372

The diffusion rates imply diffusion lags in the range of 5 to 20 years for most country pairs. Diffusion lags are at most a few years for the home country.

4.2. *Driving variables*

Our theory attributes the production of new ideas in each country over time to research employment as well as to the size of the work force. The latter proxies for the quality of the talent pool from which researchers are selected. Although research employment was endogenous in Eaton and Kortum (1997), here we condition on the actual path of research. Hence we examine how well the model predicts productivity conditional on the R&D that actually occurred, and not on how well the model predicts R&D itself.

The number of workers in each country from 1950–1990 is taken from Summers and Heston (1991).[22] Our proxy for researchers is business enterprise employment of R&D scientists and engineers, taken from the 1992 OECD STIU Data Base.[23] Since research conducted under government contracts is often not related to productivity enhancement, we multiply private research employment in each country by the fraction of industry R&D expenditures that are privately funded there.[24]

We combine the data on workers and researchers (in millions) to construct our index of innovative potential in each country from 1950 to 1990, $R_{it}^{\beta}L_{it}^{1-\beta}$. The following table shows this index in the years 1950, 1970, and 1990 using $\beta = 0.18$, which is right for the endogenous growth case (β is a bit smaller in the exogenous growth case).

	Germany	France	UK	Japan	US
Innovative potential in 1950	6.3	4.8	7.8	9.6	19.6
Innovative potential in 1970	8.6	6.2	8.4	17.5	30.4
Innovative potential in 1990	10.7	8.2	9.4	24.1	45.4

In 1990, the United States had about twice the innovative potential of Japan, which itself had two to three times the innovative potential of either France, Germany, or the United Kingdom. Over time, Japan registers the most rapid increase in innovative potential, followed closely by the United States. An implication is that innovative potential was more evenly distributed among these five countries in 1950 than in 1990.[25]

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[22]We extend the worker data through 1989 and 1990 using an updated version of the Summers and Heston dataset.

[23]Industry employment of researchers in the United States in 1989 and 1990 is from *Research and Development in Industry*, National Science Foundation, 1994. Research employment in the 1950s in the United States is from *Employment of Scientists and Engineers: 1950–1970*, Bureau of Labor Statistics, 1972. Other missing OECD data on research employment were interpolated. For countries other than the United States we extrapolate back to fill in missing data during the 1950s and early 1960s. (In Germany, the OECD series begins in 1967, in France 1964, in the United Kingdom 1968, and in Japan 1963.)

[24]These fractions, based on OECD data in 1988, are 0.885 for Germany, 0.791 for France, 0.835 for the United Kingdom, 0.984 for Japan, and 0.668 for the United States. We do not take account of any changes in these fractions of privately funded R&D over time.

[25]We have also looked at a more direct measure of innovative potential, extracted from the patenting performance of these countries. According to the model of patenting in Eaton and Kortum (1997), patent applications in country n from inventors in country i in year t are $P_{nit} = \alpha R_{it}^{\beta}L_{it}^{1-\beta}\bar{\mu}_t^{\gamma-1}e_{nit}$ where e_{nit} is constant in steady state (it depends, among other things, on the rate of diffusion between country i and n). Guided by this equation, we analyze patent applications in each of the five countries from inventors in each of the five countries(annually from 1951 to 1990). Under the identifying assumption that the time variation in e_{nit} is due either to changes in the propensity to patent in particular countries or to an overall increase in the propensity to patent abroad, we are able to extract the component $R_{it}^{\beta}L_{it}^{1-\beta}\bar{\mu}_t^{\gamma-1}$. In the case of endogenous growth this is an index of innovative potential while in the case of exogenous growth it should rise more slowly than an index of innovative potential. What do we learn relative to the measure in the table? Looking across countries in 1990, the index based on patents tells the same story as the worker and researcher data for the United States, France and the United Kingdom. Relative to these countries the patent index raises the position of Japan by 25 percent and that of Germany by a factor of 2. As for the changes in innovativeness between 1950 and 1990, the two indices are again similar in France, the United Kingdom, and the United States, but the patent index shows Germany's innovativeness increasing more slowly over time while it shows Japan's increasing much more rapidly over time. To summarize, the patent data suggest that Germany is much more innovative and that Japan has only recently become a big innovator.

Given the values of innovative potential in 1990, in both the endogenous and exogenous growth cases the parameters in Table 3 are calibrated to generate a steady state in which total factor productivity in all countries grows at 1.8 percent per year, the mean of total factor productivity growth in the U.S., German, and French manufacturing sectors averaged over 1979–1990, from Lysko (1995). For the exogenous growth case, the value of $\bar{\mu}$ in 1990 reconciles this productivity growth rate with a two percent per year growth rate in research effort and the labor force. (This second figure represents a compromise between actual labor force growth and actual growth of research employment in our five countries.) These steady states predict close to actual levels of total factor productivity relative to the United States in 1990 (since the parameters of the model were estimated to match these data, along with patent and research data, as closely as possible). Our data on total factor productivity levels, shown below, are transformations of the data on value added per hour in Table 1 under the assumption of a common interest rate over time and across countries and a capital elasticity, $\phi = .3$, a figure reported in Lysko (1995).

	Germany	France	UK	Japan	US
Total factor productivity in 1990	0.90	0.94	0.75	0.84	1.00

4.3. *Initial conditions in 1950*

To simulate a dynamic path of productivity we need initial conditions (in 1950) for the vector of stocks of diffused ideas, μ_{1950} (relative to their 1990 values), and the vector of stocks of undiffused ideas η_{1950}, which together constitute initial values of the state variables. Obtaining initial conditions for μ is straightforward: We derive them from actual 1950 productivity levels using our estimate of θ. How to initialize the vector of stocks of undiffused ideas is less obvious. Of course any attempt at quantifying the stocks of undiffused ideas is highly speculative. To get some idea, we set the *relative* values of the elements of η equal to the values determined by a 1950 steady state, that is, the steady state obtained by setting innovation potentials of the five countries at their 1950 levels and assuming labor force growth of 0 percent (endogenous growth) or 2 percent (exogenous growth). Having thus determined the relative magnitudes of the elements of η we then scale the overall magnitude of the η vector in order to maximize the fit of the model, that is, we minimize:

$$\sum_{n \in C} \sum_{t \in T} (\ln A_{nt} - \ln \hat{A}_{nt})^2,$$

where $C = Germany, France, UK, Japan, US$, $T = 1970, 1990$, and $\hat{A}$ is productivity simulated from the model, as described below.

The exogenous growth case fits best when η is scaled up by a factor of 1.9 while the factor for the endogenous growth case is 5.9 (a factor of 1 would put the ηs in their 1950 steady state position relative to the actual 1950 value of μ in the United States). With growth exogenous, growth in the early years gets an extra kick from the low level of $\bar{\mu}$ relative to its 1950 steady-state level. New ideas in this period were easy to come by. Hence this version of the model does not require very large pools of existing but unadopted ideas to explain the productivity boom in the early period. In both the exogenous and endogenous growth cases, the pools of unaccessed ideas relative to existing stocks of knowledge are much larger for Japan and the Europeans than for the United States, explaining why these countries grew much faster than the United States in the early years.

4.4. Baseline simulation

We simulate how each version of the model tracks productivity from 1950 to 1990. (Appendix A.4 describes technical details.) Table 4 reports results for the endogenous growth case while Table 5 reports them for exogenous growth. Figs. 1 and 2 graph the

Table 4
Productivity simulation: Endogenous growth case

Year	Germany		France		UK		Japan		US	
	data	model	data	model	data	model	data	model	data	model
(productivity levels)										
1950	0.23	0.23	0.23	0.23	0.23	0.23	0.10	0.10	0.45	0.45
1970	0.57	0.55	0.55	0.53	0.43	0.53	0.39	0.49	0.68	0.67
1980	0.76	0.66	0.73	0.65	0.50	0.63	0.59	0.61	0.79	0.77
1990	0.90	0.79	0.94	0.78	0.75	0.74	0.84	0.74	1.00	0.91
(annual rates of growth, percent)										
1950–1970	4.5	4.3	4.4	4.3	3.1	4.2	6.7	7.9	2.1	2.0
1970–1980	2.8	1.9	2.9	1.9	1.6	1.8	4.3	2.2	1.5	1.4
1980–1990	1.7	1.8	2.5	1.9	4.0	1.6	3.5	2.0	2.4	1.6

The columns labeled *data* are total factor productivity, while the columns labeled *model* are the simulated values.

Table 5
Productivity simulation: Exogenous growth case

Year	Germany		France		UK		Japan		US	
	data	model	data	model	data	model	data	model	data	model
(productivity levels)										
1950	0.23	0.23	0.23	0.23	0.23	0.23	0.10	0.10	0.45	0.45
1970	0.57	0.53	0.55	0.53	0.43	0.46	0.39	0.44	0.68	0.71
1980	0.76	0.68	0.73	0.69	0.50	0.58	0.59	0.60	0.79	0.86
1990	0.90	0.85	0.94	0.87	0.75	0.72	0.84	0.76	1.00	1.03
(annual rates of growth, percent)										
1950–1970	4.5	4.1	4.4	4.3	3.1	3.6	6.7	7.4	2.1	2.3
1970–1980	2.8	2.6	2.9	2.6	1.6	2.3	4.3	3.0	1.5	1.9
1980–1990	1.7	2.2	2.5	2.3	4.0	2.1	3.5	2.5	2.4	1.8

The columns labeled *data* are total factor productivity, while the columns labeled *model* are the simulated values.

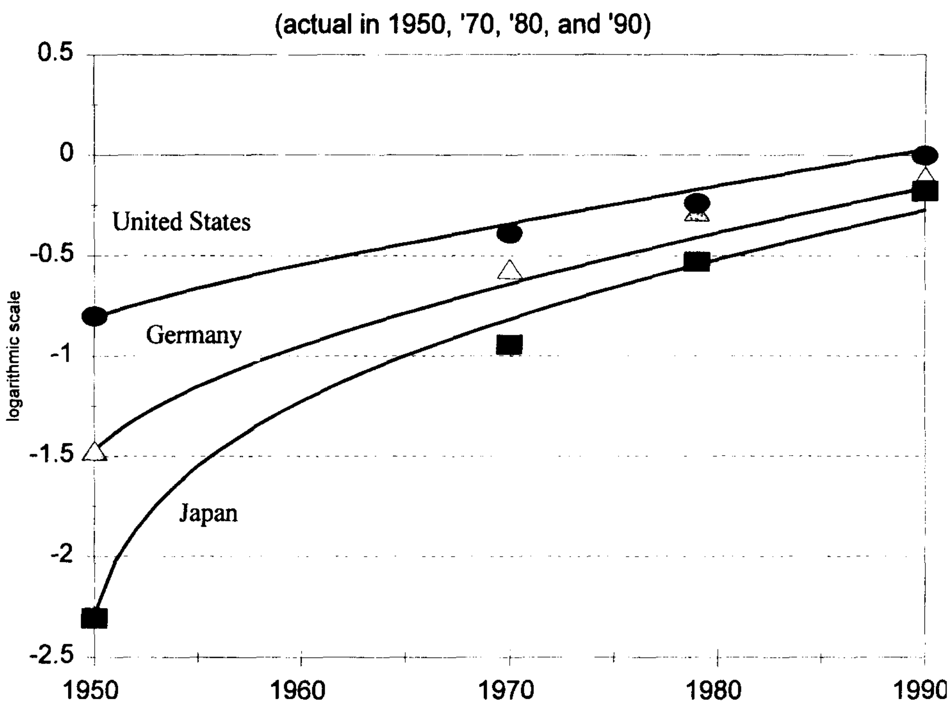

Fig. 1. Simulated productivity (actual in 1950, 1970, 1980 and 1990).

results for the exogenous growth case. The tables repeat the TFP *data* (from Table 1) in columns adjoining the values simulated by the *model* for each country. In 1950, data and model are equal by construction (given how we chose the 1950 value of μ). For the subsequent years, both versions of the model pick up the overall slowdown in productivity, and the much faster growth in Japan and the continental European countries compared with the United States in the early part of the period. The exogenous growth version of the model does substantially better, however, with a root mean square error of 7.6 percent compared with 13.6 percent for the endogenous growth case. The endogenous growth case has trouble explaining the rate of convergence to the steady state, making it happen too soon. Hence it overpredicts growth in the early period and underpredicts it later on. The exogenous growth case allows for a more gradual productivity slowdown, relying less on the exhaustion of large pools of available ideas in the beginning and more on the decline in research productivity that occurred because ideas were becoming harder to find.

By 1990, data and model productivity levels remain surprisingly close, with the largest deviation being France where model productivity is 14% below actual productivity. Model productivity also captures the slowdown in productivity growth seen in the data for Germany, France, and Japan. As productivity levels in these countries approach levels in the United States, they benefit from a smaller fraction of U.S. ideas. Furthermore, over time the large pools of undiffused ideas fall, relative to levels of technology, and approach their steady-state levels. In contrast, the United States gets a slight boost to growth as its technology level declines relative to the others.

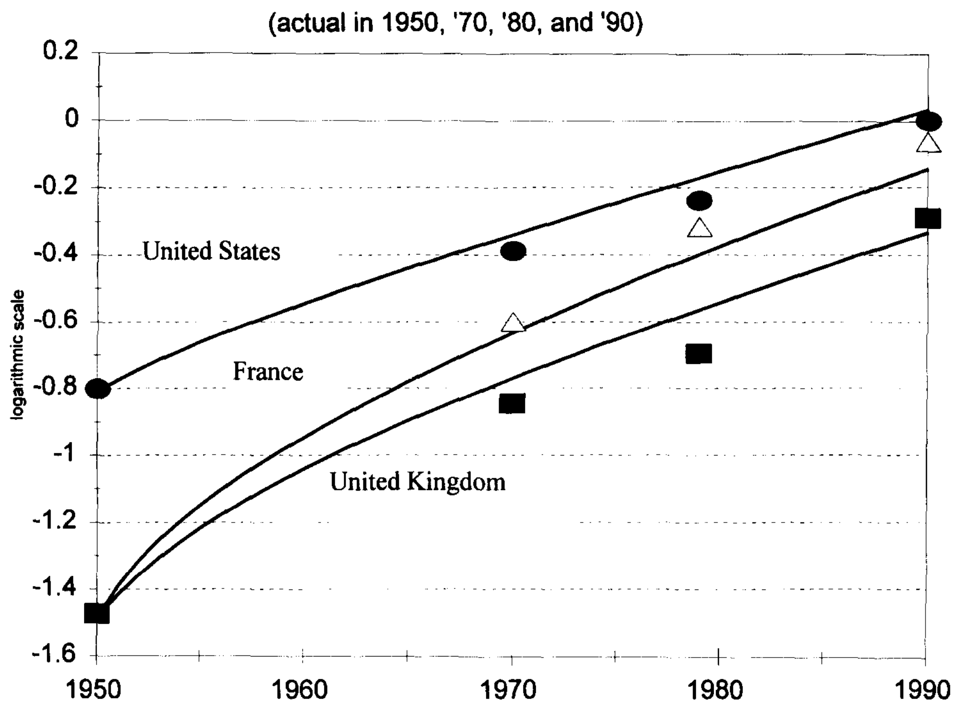

Fig. 2. Simulated productivity (actual in 1950, 1970, 1980 and 1990).

In Eaton and Kortum (1997) we chose parameters under the assumption that productivity in our five countries had reached a steady state by the late 1980s. In fact, that assumption is roughly consistent with our out-of-steady-state simulations. In the endogenous growth case, productivity growth in 1990 had settled down to between 1.6 and 1.9 percent in our five countries (close to the steady state value of 1.8 percent) and relative productivity levels had all moved to within 8 percentage points of their steady-state positions. In the exogenous growth case, growth in 1990 ranged between 1.8 percent and 2.3 percent, with France and Japan remaining a little more than 10 percent below their steady-state position.

4.5. *Alternative scenarios*

We now consider what would have happened under various alternative assumptions about how technology diffuses. In Table 6 we show how each alternative alters the simulated level of productivity in each country by 1990. Since it is simpler, we focus on the case of endogenous growth.[26] In all cases the initial 1950 values of the state variables

[26] In particular, we do not have to worry about how isolation affects the drag imposed by the world stock of ideas on research in an isolated country.

Table 6
Alternative scenarios: Productivity levels in 1990

Year	Germany	France	UK	Japan	US
Data:					
1950	0.23	0.23	0.23	0.10	0.45
1990	0.90	0.94	0.75	0.84	1.00
Simulations:					
Baseline	0.79	0.78	0.74	0.74	0.91
Alternatives:					
Complete isolation	0.26	0.25	0.26	0.13	0.64
Complete integration	1.25	1.23	1.32	1.27	1.26
United States isolated	0.47	0.45	0.50	0.44	0.64
Japan isolated	0.70	0.69	0.65	0.13	0.81

The *baseline* simulation is the same as in Table 4. In *complete isolation* we cut off diffusion between countries. In *complete integration* we scale up diffusion between countries by the same factor we use to scale up diffusion within countries. In *United States isolated* we cut off diffusion between the United States and other countries while in *Japan isolated* we cut off diffusion between Japan and other countries.

are the same as in the baseline simulation. In examining these alternatives we continue to assume that the innovative effort in each country remained on its historical path.

In the first alternative, *complete isolation*, we cut off diffusion between countries ($\epsilon_{ni} = 0 \, \text{for} \, n \neq i$) but we do not alter the diffusion rate within countries. Eliminating international diffusion has a devastating effect on productivity growth, with productivity in Germany, France, and the United Kingdom at only about one-third of the baseline level by 1990. Japan fares much worse. The United States does relatively well since a sizable amount of research takes place within its borders.

The second alternative, *complete integration*, goes to the opposite extreme by removing the bias against innovations diffusing between countries relative to the speed at which they diffuse at home.[27] In this case, productivity is higher than the baseline level by 25 percent or more by 1990. Productivity levels become very tightly clustered.

In the third alternative, *U.S. isolated*, we eliminate diffusion of technology between the United States and the other four countries. All countries are hurt: the United States because it obtains no ideas from abroad and the others because they obtain no ideas from the United States. The United States, of course, ends up with the same level of productivity as with *complete isolation*. The other countries do relatively better since they still share ideas amongst each other, but productivity is about two-thirds below the baseline. In the fourth alternative, *Japan isolated*, we cut off diffusion of technology between Japan and the other four countries. Japan has the same level of productivity as with *complete isolation* while the others are a little over 10 percent behind where they would otherwise be. A natural conclusion is that the free flow of ideas from either the United States or Japan to other industrialized countries has made an important contribution to world growth.

[27]Thus, we set $\epsilon_H = 17.7$ even for $n \neq i$.

5. Conclusions

What can we conclude about the controversies posed in the introduction? The behavior of capital–output ratios indicate that capital accumulation by itself cannot explain differences in manufacturing productivity in the major industrial countries over the last four decades. In this paper we examine the alternative view: that patterns of innovation and technology diffusion explain these differences.

As for the issue of foreign vs. domestic sources of growth, we conclude that growth is primarily the result of research performed abroad. We find that even the largest country, the United States, would have grown less than half as much if it had been isolated from the rest of the world. These findings are consistent with historical accounts.[28]

Our model implies that, with international technological mobility, economies will converge to a steady state with parallel growth. Based on the initial conditions of 1950, both the exogenous and endogenous growth versions of our model track the convergence of post-war manufacturing productivity in Germany, France, the United Kingdom, Japan, and the United States, and the overall slowdown in growth. The exogenous growth case captures the timing much better, and requires less extreme assumptions about initial technology gaps. Nevertheless, the performance of either version of the model leads us to interpret this period as one of partial technological convergence from the great technological disparity left by World War II.

Acknowledgements

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Appendix

Mathematical appendix

A.1 The distribution of the technological frontier

Ideas are adopted in a sector with quality z at a stochastic rate $\dot{\mu}_{nt}z^{-\theta}$. The probability that no idea is adopted in the time interval $[t, t+dt]$ is thus $e^{-\dot{\mu}_{nt}z^{-\theta}dt}$. Therefore,

$$H_n(z|t+dt) = H_n(z|t)e^{-\dot{\mu}_{nt}z^{-\theta}dt}$$

or,

$$\frac{\partial \ln H_n(z|t)}{\partial t} = -\dot{\mu}_{nt}z^{-\theta}.$$

[28]For an example of the importance of foreign technology, see Mueller's (1962) account of the foreign inventions underlying Du Pont's innovations.

Solving this differential equation, with the two initial conditions: (i) $\lim_{s\to-\infty} H_n(z|s) = 1 \forall z \geq 1$ and (ii) $\lim_{s\to-\infty}\mu_{ns} = 0$, yields the cumulative distribution function for the technological frontier.

A.2 The geometric mean of the technological frontier

The log of the productivity index is simply,

$$\ln A_{nt} = \int_1^\infty \ln z dh_n(z|t).$$

Changing the variable of integration to $x = \mu_{nt} z^{-\theta}$,

$$\ln A_{nt} = \theta^{-1} \int_0^{\mu_{nt}} \ln(\mu_{nt}/x) e^{-x} dx = \theta^{-1} \ln \mu_{nt} (1 - e^{-\mu_{nt}}) - \theta^{-1} \int_0^{\mu_{nt}} \ln x e^{-x} dx.$$

For large μ_{nt} we have an arbitrarily good approximation,

$$\ln A_{nt} = \theta^{-1} \ln \mu_{nt} - \theta^{-1} \int_0^\infty \ln x e^{-x} dx.$$

The Laplace transform of $-\psi - \ln t$ is $s^{-1} \ln s$, where ψ is Euler's constant. Evaluating the Laplace transform at $s = 1$ implies,

$$\int_0^\infty \ln x e^{-x} dx = -\psi.$$

This gives us the desired result that,

$$\ln A_{nt} = \theta^{-1} \ln \mu_{nt} + \psi/\theta.$$

A.3 Solving the model

The state of the whole system at any time t can be summarized by the $N^2 + N$ vector of state variables

$$y_t = [\eta_{11t} \ldots \eta_{1Nt} \ldots \eta_{N1t} \ldots \eta_{NNt}, \mu_{1t} \ldots \mu_{Nt}]',$$

representing the size of the N^2 pools of unexploited ideas and the size of the N stocks of ideas that have diffused. For either version of the model we must solve the differential equations (Eq. (4)) as a system. Define $\bar{E}_{it} \equiv \alpha \bar{\mu}_t^{\gamma-1} R_{it}^{\beta} L_{it}^{1-\beta}$, so that:

$$\dot{\eta}_{nit} = \bar{E}_{it} \mu_{it} - \epsilon_{ni} \eta_{nit}.$$

For the case in which $\gamma = 1$ the system of differential equations is,

$$\dot{y}_t = \Delta_t y_t, \tag{5}$$

where, y was defined above as the $N^2 + N$ vector of state variables and,

$$\Delta_t = \begin{bmatrix}
-\epsilon_{11} & & & & & & 0 & \bar{E}_{1t} & & 0 \\
& \ddots & & & & & & & \ddots & \\
& & -\epsilon_{1N} & & & & & 0 & & \bar{E}_{Nt} \\
& & & \ddots & & & & & \vdots & \\
& & & & -\epsilon_{N1} & & & \bar{E}_{1t} & & 0 \\
& & & & & \ddots & & & \ddots & \\
0 & & & & & & -\epsilon_{NN} & 0 & & \bar{E}_{Nt} \\
\epsilon_{11} & \cdots & \epsilon_{1N} & & & & 0 & 0 & & 0 \\
& & & \ddots & & & & & \ddots & \\
0 & & & & \epsilon_{N1} & \cdots & \epsilon_{NN} & 0 & & 0
\end{bmatrix}$$

These equations also describe the evolution of the system for $\gamma < 1$ conditional on a path of $\overline{\mu}_t$. A description of the full system then requires an additional equation describing the evolution of the world's stock of knowledge:

$$\dot{\bar{\mu}}_t = \sum_{i=1}^{N} \dot{\mu}_{it}.$$

In either case, convergence to a steady state with constant productivity growth requires that the $\bar{E}_{it}$ themselves converge to constants. If they were to keep growing then growth in the stocks of diffused ideas would eventually accelerate and productivity growth would approach infinity. If $\gamma = 1$ a steady state requires constant labor forces, constant fractions of which are doing research. If $\gamma < 1$ then a steady state can emerge even if labor forces grow at an exponential rate n, as long as a constant fraction does research.

To examine the properties of a steady state, consider what happens if we set $\bar{E}_{it} = \bar{E}_i$, so that $\Delta_t = \Delta$. Under certain restrictions on the ϵ_{ni}s, which we discuss below, the steady state exhibits parallel growth with each state variable growing at the same rate, g, including $\overline{\mu}$. Thus, $gy_t = \Delta y_t$ where g is the largest eigenvalue of Δ. If $\gamma = 1$ then g is the endogenous growth rate of knowledge. If $\gamma < 1$ then the ratio of the world stock of knowledge to the total labor force moves toward a constant level which delivers an eigenvalue $g = n/(1-\gamma)$, the rate required to ensure that the $\overline{E}_i$s are constant.

Given the value of g, we can analyze a simpler N-dimensional system to gain some intuition about what determines relative productivity levels. In steady state, $\eta_{nit} = \bar{E}_i\mu_{it}/(g + \epsilon_{ni})$. Therefore, the *steady-state* vector of relative stocks of diffused ideas, μ^*, must satisfy:

$$g\mu^* = \Lambda\mu^*,$$

where the matrix Λ has elements $\Lambda_{ni} = \epsilon_{ni}\bar{E}_i/(g + \epsilon_{ni})$. A country's relative productivity depends on the rate at which it gets ideas, particularly from those countries producing the most ideas. If diffusion rates within and between all countries are equal, productivity levels will be equal.

What restrictions on the ϵ_{ni}s ensure that countries converge to parallel growth? Frobenius' theorem guarantees that if the matrix Λ is *indecomposable* then its largest eigenvalue is positive and has associated with it a strictly positive eigenvector. This eigenvalue, the Frobenius root, is the growth rate to which the system eventually converges and the associated eigenvector (defined up to a scalar multiple) gives the relative levels to which the state variables (the pools of diffused and undiffused ideas) converge.

In order for Λ to be indecomposable, no rearrangement of its rows or columns allows it to be represented as block semidiagonal, that is, it must be impossible to switch columns or rows to write it as:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix}$$

where Λ_{11} and Λ_{22} are square matrices.

Say that Λ were decomposable so that it could be represented this way. Then the block of countries corresponding to Λ_{22} would be isolated from the rest and (assuming that Λ_{22} is itself indecomposable) we could solve for its Frobenius root (call it λ_2) which would determine the rate at which the isolated block of countries would eventually grow. Consider then the block of countries corresponding to Λ_{11}. Assuming that Λ_{11} is also indecomposable then, left on their own, these countries would eventually converge to a growth rate determined by the Frobenius root of Λ_{11} (call it λ_1). One possibility is that $\lambda_1 < \lambda_2$, meaning that the isolated set of countries on its own would grow faster than the other block on its own. If Λ_{12} contains strictly positive elements then the 'technology gap' between the two blocks would eventually grow to a point at which the isolated block would boost the growth rate of the other countries; λ_2 would then determine the rate at which the whole world would eventually grow. We could think of this isolated block, then, as the 'engine of growth' for the world economy. If, however, $\Lambda_{12} = 0$, then both blocks are isolated, and converge to the growth rates determined by their respective Frobenius roots. This is also the outcome if $\lambda_1 > \lambda_2$ since in this case the isolated block eventually becomes so small relative to the connected block that it does not influence growth in the connected block. The implication, then, is that the world converges to a common growth path if there are sufficient direct or indirect spillovers among all countries.

This argument holds even though the elements of Λ themselves depend upon g, and, for $\gamma < 1$, upon $\bar{\mu}$: An element of Λ is zero if and only if $\epsilon_{ni} = 0$. Hence a steady state with divergent growth can only emerge when some countries never receive ideas from abroad. An extreme example of isolation occurs if diffusion within countries is instantaneous while across countries it is nonexistent. In this case the stock of diffused ideas in country n grows at rate $g_n = \bar{E}_n$, hence productivity growth is $\bar{E}_n/\theta$. As in many models of endogenous growth, a country's productivity growth would then depend on only its own level of research activity.

A.4 Simulating the Model

To track what our model predicts about what happened between 1950 and 1990, we assume that stocks are evaluated at the beginning of the year and that employment is constant throughout the year. The solution to Eq. (5) in Appendix A.3 is then:

$$y_t = A_{t-1} \begin{bmatrix} e^{\lambda_{1t-1}} \\ \vdots \\ e^{\lambda_{Mt-1}} \end{bmatrix},$$

for $t = 1951 \ldots 1990$. With y_{1950} determined as described above, we can iterate forward using the equation $A_{t-1} = x_{t-1}D_{t-1}$ where x_{t-1} is a matrix with columns equal to the eigenvectors of Δ_{t-1} (and associated eigenvalues, $\lambda_{1t-1} \ldots \lambda_{Mt-1}$) and D_{t-1} is a diagonal matrix with diagonal elements given by $(x_{t-1})^{-1}y_{t-1}$. The matrix Δ_{t-1}, for $t = 1951 \ldots 1990$, is constructed from the parameters of the model and inventive potentials in year $t-1$. The last 5 elements of y_t form the vector of stocks of diffused ideas in year t, $_t$. An index of labor productivity in country n in year t is simply $A_{nt} = \mu_{nt}^{1/\theta}$, for $n = 1 \ldots 5$. In the case of exogenous growth we approximate $\overline{\mu}_t$ as $\sum_{i=1}^{N} \mu_{it-1}$. (By using the previous year to calculate the drag on research productivity imposed by the existing world stock of knowledge the model is linear in the exogenous as well as endogenous growth case.)

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