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Model for the low-temperature transport of Bi-based high-temperature superconducting tapes

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A grain-structure model is used to obtain the low-temperature transport properties of polycrystalline Bi-based high-temperature superconducting tapes, including the magnetic-field-dependent critical current. The grain structure is regarded as resembling a brick wall, such that the net horizontal supercurrent passes from brick to brick chiefly through the horizontal junctions between bricks. A high-field critical-current plateau is predicted, assuming inhomogeneous Josephson junctions between highly anisotropic superconducting grains.

I. INTRODUCTION

The critical-current density J_c of polycrystalline high-temperature superconducting Bi 2:2:1:2 and 2:2:2:3 tapes shows an extraordinary magnetic-field dependence at low temperatures.[1-3] In the most dramatic example, a Bi 2:2:2:3 tape studied by Sato *et al.*[2] shows a zero-field current density J_{c0} of 3×10^5 A/cm^2, which, in fields above $H_0\approx1$ T applied parallel to the tape plane, drops to about $J_{c1}\approx1\times10^5$ A/cm^2, this level persisting up to the highest measured fields ($H_1=23$ T).

To explain this high-field plateau, Tenbrink, Heine, and Krauth[1] invoked a conventional bulk-pinning argument, ignoring the possibility of grain-boundary weak links. Jin *et al.*[3] also argued that the presence of the high-field J_c implies that the grain-boundary links in the Bi materials are for some reason "mild" or "strong." Yet Dimos, Chaudhari, and Mannhart[4] showed a strong reduction of the critical-current density in $YBa_2Cu_3O_7$ across grain boundaries in which the *ab* planes are tilted or twisted with respect to each other. The similarity of the $YBa_2Cu_3O_7$ and the Bi materials suggests that the grain boundaries in Bi materials might behave similarly and act as weak links. If so, can one find an alternative explanation for the high-field J_c? In this paper we do this, based on a grain-structure model for J_c. This "brick-wall" model[5] is shown in Fig. 1. Each brick represents a crystal grain, and all the crystals are assumed oriented along a common *c* axis normal to the tape plane, while the orientation of the *a* and *b* axes of the grains is random in the tape plane. The thickness D of the grains is small compared with the length $2L$ along the principal tape axis, while their width W is comparable to the length. This model closely resembles the experimentally observed microstructure.[1,2]

Because of the weak links we suppose that electrical contact between the short sides of adjacent grains can be neglected in the presence of a magnetic field. At the same time, we assume that, because of adequate flux pinning, current flow along the *ab* plane within the grains does not play a limiting role in determining the overall current density. The supercurrent density flowing down the tape will weave around the obstacles to flow, as shown schematically in Fig. 1. We assume that the linear dimensions of the Josephson junctions are smaller than the Josephson penetration depth, so that we are in the "small-Josephson-junction" limit, where self-field effects can be neglected. Thus the net supercurrent will be limited by the Josephson critical-current density j_c across the *c*-axis twist boundaries. We use j to describe *the current density along the c axis* in contrast to J, which describes *the net current density along the tape* in the *ab* plane. In such a model the field and temperature dependence of j_c and J_c are the same; they are related by $J_c=j_cL/D$. We assume that the zero-field Josephson critical-current density $j_0^{\dagger}$ of the *c*-axis twist boundaries is much smaller than the corresponding value j_0 characterizing the Josephson coupling of the CuO layers inside the grains.

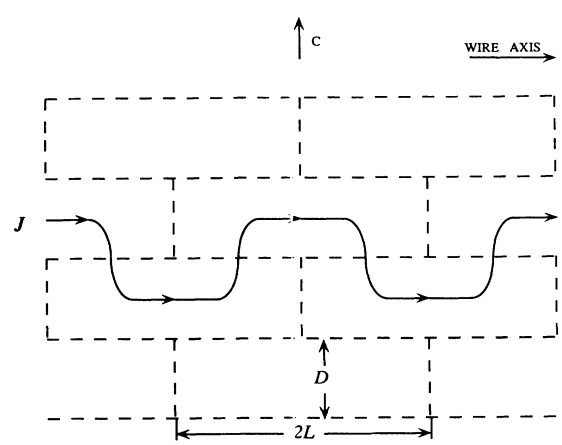

FIG. 1. Brick-wall model. The length of each superconducting brick is $2L$ and the thickness is D.

In the following the dependence of macroscopic critical currents of the system on magnetic field H parallel to the ab plane is considered, and the dependence on the perpendicular field is discussed. The concepts here include (1) the generalization of the dependence of the Josephson current on applied field from the standard case of weak magnetic fields ($H < H_{c1,g}$) to the case of strong fields ($H \gg H_{c1,g}$) and (2) treating the small-junction limit in the case of highly anisotropic superconducting grains, including the case of inhomogeneous junctions. Here we use $H_{c1,g}$ to denote the lower critical field of the superconducting grains that form the junction.

II. DEPENDENCE OF CRITICAL CURRENT ON PARALLEL MAGNETIC FIELD

To study the effect of the mixed state on the properties of the Josephson junction, we obtain first the equation for the gauge-invariant phase difference across the junction in an applied parallel magnetic field $H_0 \gg H_{c1,g}$. The superconductors initially are assumed to be thick ($D \gg \lambda_{ab}$) and long ($L \gg \lambda_c$), the penetration depths of the superconductors are λ_{ab} and λ_c, and the junction is along the ab plane (xy plane).

Integrating $\mathbf{A}$ along the path shown in Fig. 2 and using the expression for the current density inside the superconductors, $\mathbf{j}_k = -(c/4\pi\lambda_{ab}^2)[(\phi_0/2\pi)\nabla\Phi_k + \mathbf{A}]$, we obtain the relation

$$\frac{\phi_0}{2\pi}\nabla\varphi = \mathbf{H}\times\hat{\mathbf{z}}d - \frac{4\pi\lambda_{ab}^2}{c}[\mathbf{j}_1^{(s)} - \mathbf{j}_2^{(s)}], \tag{1}$$

where $k = 1,2$ denote the superconductors forming the junction, $\hat{\mathbf{z}}$ is the unit vector in z direction, Φ_k is the phase of the order parameter in superconductor k, $\phi_0 = \pi\hbar c/e$ is the flux quantum, $\mathbf{A}$ is the vector potential, $\varphi = \Phi_1 - \Phi_2 - (2\pi/\phi_0)\int_1^2 dz\, A_z$ is the gauge-invariant phase difference, d is the spacing between superconductors, and $\mathbf{j}_k^{(s)}$ is the current density at the surface. The magnetic field $\mathbf{H}$ inside the junction and the phase difference φ depend on coordinates x and y.

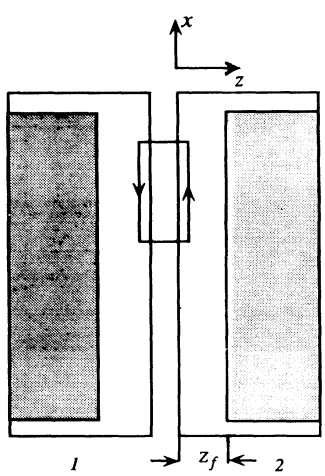

FIG. 2. Junction between two bricks showing the integration path used to derive Eq. (2). Shading represents the vortex-filled region. The vortex-free region is of thickness z_f.

In the mixed state, screening currents flow in the vortex-free region near the surface. The thickness of this region is z_f (Ref. 6) (see Fig. 2). Because of these currents, *in the equilibrium state,* the locally averaged magnetic field $\mathbf{h}(\mathbf{r})$ drops from its surface value $\mathbf{H}(x,y)$ to the induction $\mathbf{B}$ inside the superconductors as z increases from the surface ($z=0$) to z_f. Solving the London equation for the magnetic field, we obtain

$$\mathbf{h}(\mathbf{r}) = \left[\mathbf{H}(x,y)\sinh\frac{z_f - z}{\lambda_{ab}} + \mathbf{B}\sinh\frac{z}{\lambda_{ab}}\right] \Big/ \sinh\frac{z_f}{\lambda_{ab}}, \tag{2}$$

where $\mathbf{H}$ and $\mathbf{B}$ are parallel to the junction and $z_f = \lambda_{ab}\cosh^{-1}(H/B)$ is determined by the requirement that $j_x = j_y = 0$ at $z = z_f$. Inside the junction $\mathbf{H}$ coincides practically with the applied field $\mathbf{H}_a$, and thus H_a and B determine z_f. Using Maxwell's equations we find the surface currents $\mathbf{j}_k^{(s)}$:

$$\mathbf{j}_1^{(s)}(x,y) = -\mathbf{j}_2^{(s)}(x,y) = \left(\frac{c}{4\pi\lambda_{ab}^2}\right)\hat{\mathbf{z}}\times\left[\mathbf{H}(x,y)\coth\frac{z_f}{\lambda_{ab}} - \mathbf{B}\,\mathrm{csch}\frac{z_f}{\lambda_{ab}}\right]. \tag{3}$$

Using (1), (3), the Maxwell equation $(\nabla\times\mathbf{H})_z = (4\pi/c)j_z = (4\pi/c)j_0^\dagger\sin\varphi$, and assuming constant $\mathbf{B}$ we obtain the sine-Gordon equation:

$$\nabla^2\varphi - (1/\lambda_J^2)\sin\varphi = 0,\quad \lambda_J^2 = c\phi_0/8\pi^2\Lambda_1 j_0^\dagger, \tag{4}$$

where $\Lambda_1 = d + 2\lambda_{ab}\coth(z_f/\lambda_{ab})$. Equation (1) with the help of (3) gives the equation for φ for distances $z_f \ll r \ll \lambda_J$ counted from the edge of a junction (boundary conditions):

$$\nabla\varphi = -\frac{2\pi}{\phi_0}\Lambda_0\hat{\mathbf{z}}\times\mathbf{H}_a,\quad \Lambda_0 = d + 2\lambda_{ab}\tanh\frac{z_f}{\lambda_{ab}}. \tag{5}$$

Thus we find the Josephson penetration depth $\lambda_J(H)$ and the length $\Lambda_0(H)$ which determines the spatial variation of φ in magnetic field. The similar expressions for Λ_0 and Λ_1 with D instead of z_f were obtained for a junction made of thin superconductors in the absence of vortices ($H < H_{c1}$).[7]

For a short Josephson junction ($z_f \ll L \ll \lambda_J$) the critical current is

$$j_c(H) = j_0^\dagger\frac{\phi_0}{\pi HL\Lambda_0(H)}\left|\sin\frac{\pi HL\Lambda_0(H)}{\phi_0}\right|. \tag{6}$$

In the presence of vortices the period of oscillations depends on H via $\Lambda_0(H)$. The lengths Λ_0 and Λ_1 can be expressed via the averaged magnetization of superconductors $M(H)$. We note that $\Lambda_0 = \Lambda_1 = d + 2\lambda_{ab}$ for $H < H_{c1,g}$. When $H > H_{c1,g}$ and $-4\pi M \ll H$, we obtain $z_f = \lambda_{ab}[8\pi|M(H)/H|]^{1/2}$. As H increases, the magnetization and the length Λ_0 decrease; for $H \gg H_{c1,g}$, $\Lambda_0 \approx d + 2z_f$ and Λ_0 approaches d in the high-field limit. On the other hand, when $H \gg H_{c1,g}$, the length

$\Lambda_1(H) \approx 2\lambda_{ab}^2/z_f$ grows with H; correspondingly, λ_J diminishes and approaches $c\phi_0 z_f/8\pi^2\lambda_{ab}^2 j_0^\dagger$ (z_f should be larger than the interlayer spacing s in our phenomenological approach for a vortex structure near the surface). We note that inside the grains the internal Josephson penetration length λ_{0J} is $(c\phi_0 s/8\pi^2\lambda_{ab}^2 j_0)$, where s is the interlayer spacing. Thus $\lambda_J > \lambda_{0J}$ because $z_f > s$ and $j_0^\dagger < j_0$.

We see that in a perfect short junction for $H \gg H_{c1,g}$ the dependence of j_c upon H differs from the standard Fraunhofer dependence if $\phi_0/2\lambda_{ab}L \gg H_{c1,g}$, the decrease of j_c being slower and the period of oscillations larger.

Next we consider the combined effect on Λ_0 of anisotropy and small dimensions of the superconducting grains in the regime of weak fields ($H < H_{c1,g}$). In Bi 2:2:1:2 the London penetration depths λ_c and λ_{ab} for currents along the c axis and in the ab plane are quite different, with the anisotropy ratio $\lambda_c/\lambda_{ab} \approx 55$ according to torque measurements.[8] We assume a similar ratio for Bi 2:2:2:3. Moreover, the junctions in the tapes under study are not long; thus $D < \lambda_{ab}$ and $L < \lambda_c$. For superconductors with restricted dimensions L and D we find the magnetic field **h** using the London equation $\lambda_{ab}^2\nabla_z^2 h_y + \lambda_c^2\nabla_x^2 h_y = h_y$ with boundary conditions $h = H$ at $x = L/2$, $x = -L/2$, $z = D/2$, and $z = -D/2$. Then we obtain the currents at the surface using Maxwell's equation for **h**. To estimate the role of anisotropy we approximate the superconductors on both sides of the weak link by ellipses, neglecting the geometrical misfit. As a result we find $\Lambda_0 = d + D\alpha$ where $\alpha = [1 + (D\lambda_c/L\lambda_{ab})^2]^{-1}$. These expressions give the correct extrapolation to the limit $D\lambda_c/L\lambda_{ab} \gg 1$ (a more complete treatment of the problem with the help of a standard Fourier decomposition gives qualitatively the same result). We note that because of the coefficient α, which can be small, the effective penetration depth $D\alpha$ is smaller than the corresponding one in the isotropic case (D). We note that $D\alpha \approx (\lambda_{ab}^2/D)|M/H|$, where M is the magnetization of a grain. In the mixed state for high fields we again obtain a vortex-free region near the junction with thickness $z_f(H)$ obtained above. This behavior occurs for $z_f \ll D$ and $z_f\lambda_c/\lambda_{ab} \ll L$.

Thus for high fields of interest, Λ_0 coincides practically with d, and the period of the Fraunhofer oscillations is determined by the field $H_0 = \phi_0/Ld$.

Now we generalize to the case of a junction with disorder, where the value $j_0^\dagger$ depends on coordinates x and y. Yanson[9] first treated this problem for one-dimensional disorder; we treat the two-dimensional case. We assume that the random function $j_0^\dagger(x,y)$ has the correlation radius r_0; i.e., it does not change on a scale r_0 and on different scales correlations of the critical-current density are absent. Then the average critical current $\overline{j_c}$ drops at $H \approx H_0 = \phi_0/L\Lambda_0$, but now we find that it approaches the approximately constant value $j_{c1} = (r_0^2\overline{\delta j^2}/LW)^{1/2}$, where $\overline{\delta j^2}$ is the variance of the random value $j_0^\dagger(x,y)$. Above a second characteristic field $H_2 \approx \phi_0/r_0\Lambda_0$, the total critical current approaches zero as H increases due to development of the Fraunhofer dependence of the critical-current density on H with the scale r_0. Disorder in the vortex lattice in superconductors causes qualitatively the same effect as variations in the function $j_0^\dagger(x,y)$.[10]

With some variation in grain size along the length of the tape and distortions of the vortex lattice inside grains, we expect $J_c(H)$ initially to decrease slowly up to a characteristic field $H_0 = \phi_0/Ld$, with higher-order oscillations averaged out. Beyond this field we expect a plateau with the value $J_{c1} = j_{c1}L/D$.

III. COMPARISON TO EXPERIMENTAL DATA

Here we compare the above theoretical results to the Bi 2:2:2:3 experiments of Sato *et al.*,[2] who find a zero-applied-field current density $J_{c0} \approx 300000$ A/cm^2, a first characteristic drop off field $H_0 \approx 1$ T, and a high-field plateau $J_{c1} \approx 100000$ A/cm^2. There must be a second characteristic dropoff field H_1 above 23 T. The corresponding theoretical expressions in the high-field limit are $J_{c0} \approx j_0^\dagger L/D$, $H_0 \approx \phi_0/Ld$, $J_{c1} \approx (j_0^\dagger r_0/D)\sqrt{L/2W}$, and $H_1 \approx \phi_0/r_0 d$. The result for J_{c1} assumes that the variance $\overline{(\delta j)^2}$ is $(j_0^\dagger)^2/2$. If the grains are equiaxed in the ab plane, $2L \approx W$, and so $J_{c1} = j_0^\dagger r_0/2D$.

First we note that the theoretical ratios are $J_{c0}/J_{c1} = 2L/r_0$ and $H_1/H_0 = L/r_0$, while in experiments the corresponding ratios are 3 and > 23. We resolve this discrepancy by noting that the experimental result for the zero-field critical-current density J_{c0} can be reduced by the substantial self-field (of order 0.5 T) in these high-current density tapes. Further reduction can be caused by frozen-in flux,[9] for which there is evidence in these samples from transport-J_c hysteresis in increasing and decreasing fields.[2] Svistunov, D'yachenko, and Tarenkov[11] recently interpreted this hysteresis in the context of Yanson's model. Thus, for experimental comparison we focus only on J_{c1}, H_0, and H_1. Without more detailed TEM studies, the actual value for the grain dimensions D and L deduced from SEM micrographs[2] could lie anywhere in the ranges $0.01 < D < 1$ μm and $0.5 < L < 10$ μm. These experimental results, the formula for H_1, and the estimate $d \approx 10$ Å require us to consider an $L \approx 2$ μm. The ratio $H_1/H_0 = L/r_0 > 23$ then implies an inhomogeneity correlation length $r_0 < 1000$ Å, which seems reasonable. Finally, the experimental result for $J_{c1} = j_0^\dagger r_0/2D$, the inequality $j_0^\dagger \ll j_0$, with $j_0 \approx 2.5\times10^6$ A/cm^2 from $\lambda_c \approx 50\lambda_{ab} \approx 10$ μm, implies a reasonable estimation $D \ll 1$ μm. The condition $L \ll \lambda_J$, which is necessary for the small-junction limit to hold, is easily fulfilled.

We now discuss the effect of a perpendicular magnetic field. In a brick-wall structure this field penetrates into the sample in the form of vortices. Since the pinning sites inside the grains are located randomly, the vortices are misaligned in different grains and a component of the field parallel to the junction between grains again exists.[12] Thus the effect of perpendicular field is qualitatively the same as the effect of a parallel field, the strength of the effect depending on the transformation of the perpendicular field component into a parallel one. In the highly random system under consideration the transformation can be quite effective, causing a relatively small anisotropy in the dependence of J_c on the orientation of H.

Thus we conclude that our model provides a consistent interpretation of the Bi wire data, and is an alternative model to the strong-link models of Tenbrink, Heine, and Krauth[1] and Jin *et al.*[3] A conclusive discrimination be-

tween these models requires microscopic probes of the actual current paths. It is also interesting to ask what this model implies for optimizing the wire properties. The formula $H_0=\phi_0/Ld$ indicates that *short* grain size is favorable, and the formula $J_{c1}=j_0^{\dagger}r_0/D\sqrt{L/2W}$ indicates that *decreasing D* optimizes J_{c1}.

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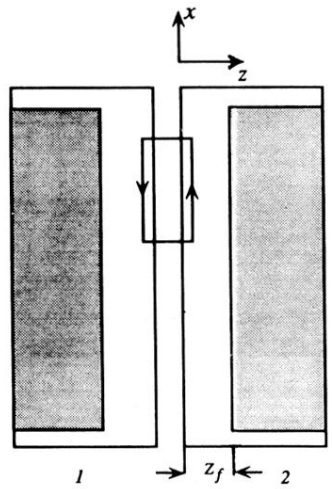

FIG. 2. Junction between two bricks showing the integration path used to derive Eq. (2). Shading represents the vortex-filled region. The vortex-free region is of thickness z_f.