Phil. 115 Review for the final exam

Our final is on Th., Dec. 15, at 2 PM (room TBA). You will have a total of 2-1/2 hours to complete your exam; at 4:30 PM you will have to turn in your work if you haven't done so already. (Fine print: Yale regulations state: "Final examinations normally last either two or three hours but, in either case, students are permitted to take an additional half-hour before being required to turn in their answers." Our exam, then, officially lasts for 2 hours. That, plus the half-hour of extra time, which is best used wrapping up your work, yields the total of 2-1/2 hours that you have before work must be turned in.)

Preparation for the exam:

1. Make sure you understand the homework questions and the questions we covered in class and in sections. Solving questions in the textbook is one of the best ways to review the material.
2. Open sentence files from Tarsk'si World Folder and try to understand them. Most probably, you need to build a world to see whether your understanding is correct or/and play a game with a computer.
3. Solve the questions in this review handout.
4. Take the sample final exam (attached here)

Review excerices

- Translate English sentences into FOL sentences:

$D(x)$: x is at the door
$H(x)$: x is honest
$I(x)$: x is an influence-peddler
$P(x)$: x is a politician
$R(x)$: x is a registered lobbyist
h: Harrington

1. All politicians are honest.
2. No politicians are honest.
3. Some politicians are honest.
4. Some politicians are not honest.
5. An honest politician is not an influence-peddler.
6. An honest politician is at the door.
7. Politicians and influence-peddlers are not all honest.
8. Honest influence-peddlers are nonexistent.
9. An influence-peddler is honest only if he or she is a registered lobbyist.
10. Some but not all registered lobbyists are honest.
11. If anyone is an influence-peddler, Harrington is.
12. If anyone is an influence-peddler, he or she is either a politician or a registered lobbyist.

13. If anyone is an influence-peddler, every registered lobbyist is.

- More translations

1. There are men who are unhappy unless everything is going well. (M_x, H_x, W_x: x is going well)
2. Bob likes anyone who pays attention to him. (L_{xy}: x likes y, A_{xy}: x pays attention to y, P_x: x is a person, b: Bob)
3. There's no time like the present. (T_x: x is a time, L_{xy}: x is like y, p: the present moment)
4. Tom is taller than anyone he knows. (P_x, T_{xy}: x is taller than y, K_{xy} x knows y, t: Tom)

- Translation into English. (Assume the domain of discourse is the set of people.)

1. $\forall x \exists y[Loves(x,y) \wedge \forall z(Loves(x,z) \rightarrow z = y)]$
2. $\exists y \forall x[Loves(x,y) \wedge \forall z(Loves(x,z) \rightarrow z = y)]$
3. $\exists y[\forall x Loves(x,y) \wedge \forall z(\forall x Loves(x,z) \rightarrow z = y)]$

- If everybody loves someone else, which of the following must be true? We assume that the domain of the discourse, i.e. the set of people, is not empty. (Choose as many as you think are true.)

1. Somebody loves somebody.
2. Someone is loved by everyone.
3. Somebody loves somebody else.
4. Everybody loves somebody.
5. Everyone is loved by someone.
6. Everyone is loved by someone else.

- Some of the following are valid. Soem of them are not. For a valid inference, give me a formal proof, and for an invalid inference, give me a world in which the premises are true but the conlcusion is false. (You may use Taut Con, Ana Con, and DeMorgan's Laws for quantifiers.)

1. $\forall x \exists y Likes(x,y)$ from the premise $\exists y \forall x Likes(x,y)$.
2. $\exists y \forall x Likes(x,y)$ from the premise $\forall x \exists y Likes(x,y)$.
3. $\exists x \exists y \exists z Between(x,y,z)$ from the premises
   $\forall x \forall y SameRow(x,y)$ and
   $\exists x \exists y \exists z(x \neq y \wedge y \neq z)$
4. $\exists x \forall y(\neg Likes(x,y) \wedge \neg Likes(y,x))$ from the premises
   $\exists x \neg Likes(x,x)$ and
   $\forall x[\exists y(Likes(y,x) \vee Likes(x,y)) \rightarrow Likes(x,x)]$

- Formal proofs:

1. $\forall x[\text{Person}(x) \to \forall z(\text{Person}(z) \to \exists y \text{GivesTo}(x, y, z))]$ from the premise
   $\forall x[\text{Person}(x) \to \exists y \forall z(\text{Person}(z) \to \text{GivesTo}(x, y, z))]$.
2. $\exists x(\neg B(x, x) \to \forall z M(a, z))$ from
   $B(c, c) \vee (K(c) \wedge L(b))$ and
   $\neg \forall z M(a, z) \to \neg K(c)$
3. $J(a, a)$ from
   $K(b, b) \wedge \neg G(b, c)$
   $\exists y \neg(K(b, y) \to G(y, c)) \to \forall x J(x, x)$
4. $\forall x[L(x) \to \exists y N(x, y)]$ from
   $\forall x[L(x) \to (P(x) \wedge \neg W(x) \wedge \neg F(x))]$
   $\forall x[(P(x) \wedge \neg \exists y N(x, y)) \to W(x)]$
5. $\exists z S(z)$ from
   $\exists x \forall y N(x, x, y)$,
   $\exists y \exists z(B(z, y) \wedge \neg P(y))$ and
   $\forall z \forall w[(N(z, z, w) \wedge \neg P(w)) \to S(z)]$
6. $\exists z M(z)$ from
   $\forall x[\exists y L(x, y) \to \exists y \neg J(y)]$,
   $\exists y \exists z L(y, z)$, and
   $\forall x[\neg J(x) \leftrightarrow M(x)]$

- Show that it would be impossible to construct a reference book that lists all and only those reference books that do not list themselves.

1. Using the abbreviation suggested, translate each of the following statements into **FOL**. [If a sentence is ambigous, give us more than one reading.]

(5 pts. each)

   (a) Every cube is in front of some tetrahedron.
   (b) There are at most three cubes.
   (c) The same things are left of **b** are left of **c**.
   (d) If anyone is perfect, Ted is. ($P(x)$, t: Ted)
   (e) We are happy only when we love someone else. ($H(x)$, $Love(x, y)$, and assume that the domain of discourse is the set of people.)
   (f) Some professors bore all other professors. ($P(x)$, $B(x, y)$: x bores y)

2. Each of the following translations into FOL is wrong. (i) Present a situation where the two sentences (one in English and one in FOL) get different truth values, and (ii) give me a correct translation. (5 pts. each)

   (a) There are museums in Chicago and New York.
   $\exists x(\mathbf{Museum}(x) \wedge \mathbf{Chicago}(x) \wedge \mathbf{NY}(x))$.
   (b) Every cube is behind the dodecahedron. (according to Russell's analysis of "the")
   $\forall x(\mathbf{Cube}(x) \rightarrow \exists y[\mathbf{Dod}(y) \wedge \forall z(\mathbf{Dod}(z) \rightarrow y = z) \wedge \mathbf{BackOf}(x, y)])$

3. The following pairs are not equivalent to each other. (i) Translate the following pairs into first-order logic. (ii) Explain a difference by presenting a situation where two sentences have different truth values.

(5 pts. each)

   (a) 1. The large cube is left of a dodecahedron.
   2. The cube is large and left of a dodecahedron.
   (b) 1. At most **a** and **b** are small tetrahedra.
   2. The only small tetrahedra are **a** and **b**.

4. Is the following inference valid? If so, formalize this inference using FOL and prove it. If not, point out where and why the inference went wrong, by using FOL. (5 pts.)

   In English, the following two sentences mean the same thing:

   Mary is smart and Mary is nice. (1)
   Mary is smart and nice. (2)

   We know the following is true in reality:

   Some one is rich and someone is poor. (3)

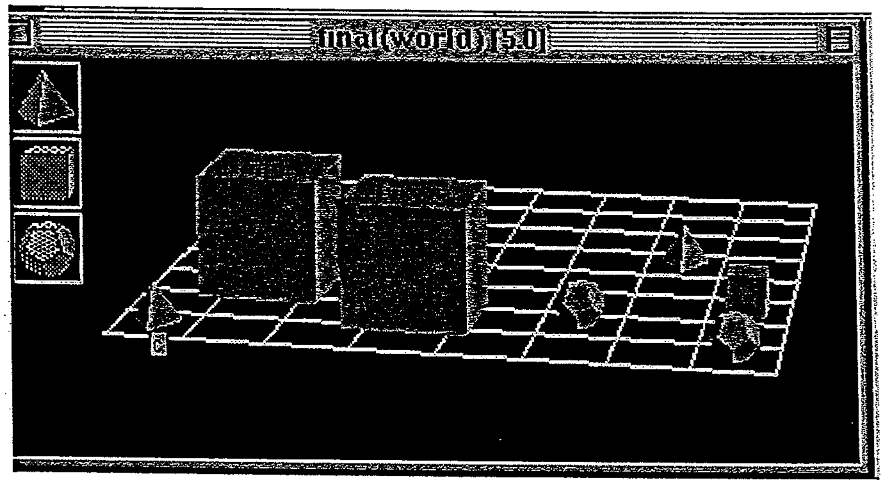
   By using the same reasoning as we saw in (1) and (2), the following sentence must mean the same thing as (3):

   Someone is rich and poor. (4)

   Who could this rich-and-poor person be ?

5. For the following twelve sentences, give me a truth-value of each sentence in the given world.

(3 pts. each)



(a) $\exists x \exists y[\textbf{Cube}(x) \land \textbf{Cube}(y) \land x \neq y]$

(b) $\forall x\ \textbf{Large}(x) \lor \forall x\ \textbf{Medium}(x) \lor \forall x\ \textbf{Small}(x)$

(c) $\forall x[\textbf{Tet}(x) \rightarrow \forall y(\textbf{Dodec}(y) \rightarrow \textbf{FrontOf}(x, y))]$

(d) $\exists x \forall y[(x \neq y \rightarrow \textbf{BackOf}(x, y)) \land \forall z(\textbf{BackOf}(z, y) \rightarrow z = x)]$

(e) $\forall x[(\textbf{Cube}(x) \land \neg\textbf{Large}(x)) \leftrightarrow x = \textbf{c}]$

(f) $\forall x \forall y\ (\textbf{Larger}(x, y) \rightarrow \textbf{Cube}(x))$

(g) $\forall x(\forall y\ \textbf{Larger}(x, y) \rightarrow \textbf{Cube}(x))$

(h) $\forall x(\exists y\ \textbf{Larger}(x, y) \rightarrow \textbf{Cube}(x))$

(i) $\forall x \forall y[(\textbf{Tet}(x) \land \textbf{Tet}(y)) \rightarrow x = y]$

(j) $\exists x \forall y[\ \textbf{Tet}(y) \rightarrow x = y]$

(k) $\exists x(\textbf{Tet}(x) \rightarrow \forall y\ \textbf{Tet}(y))$

(l) $\exists x \exists y[\textbf{Dod}(x) \land \textbf{Dod}(y) \land x \neq y \land \forall z(\ \textbf{Dod}(z) \rightarrow (z = x \lor z = y))]$

6. Assume the following premises: (5 pts.)

(a) $\forall x \forall y[\ \textbf{LeftOf}(x, y) \rightarrow \textbf{Larger}(x, y)]$

(b) $\forall x \forall y[\ \textbf{Smaller}(x, y) \rightarrow (\textbf{Cube}(y) \land \textbf{Dodec}(x))]$

Does it follow that $\neg\exists x \exists y[\textbf{LeftOf}(x, y) \land \textbf{Cube}(y)]$? If so, give a proof (either formal or informal). (You may use Taut Con, Ana Con, and DeMorgan's laws for quantifers if needed.) If not, present a situation in which the premises are true and the conclusion is false.

7. Give a formal proof of the following: (10 pts. each)

(a) Prove $\exists x$ **Cube**$(x) \land \exists y$ **Small**(y)
    from $\exists x$ (**Cube**$(x)\land$ **Small**(x))

(b) Prove $\exists x \neg$**Criticize**(x, x) from the following premises:
    $\forall z$ [**Criticize**$(z, z) \rightarrow$ (**Wise**$(z)\land$ **Cool**(z))]
    $\exists y \neg$**Cool**(y)

(c) Prove $\forall x[L(x) \rightarrow \exists y N(x, y)]$ from the following premises
    $\forall x[L(x) \rightarrow (P(x) \land \neg W(x) \land \neg F(x))]$
    $\forall x[(P(x) \land \neg\exists y N(x, y)) \rightarrow W(x)]$

(d) Prove $\exists x C(x)$ from the following premises
    $\forall x \forall y[G(x, y) \leftrightarrow (K(y) \rightarrow C(x))]$
    $\forall w G(b, w)$
    $\exists z K(z)$.

(e) $\exists z$[**Woman**$(z) \land \forall y$(**Woman**$(y) \rightarrow$ **Admires**(z, y))] from the following premises:
    $\forall x \forall y$[(**Admires**$(x$,m$)\land$**Admires**(m$, y$)) $\rightarrow$ **Admires**(x, y)]
    $\forall x$[**Woman**$(x) \rightarrow$ **Admires**(m$, x$)]
    $\exists x$[**Woman**$(x)\land$**Admires**$(x$,m)]

Have a wonderful break!