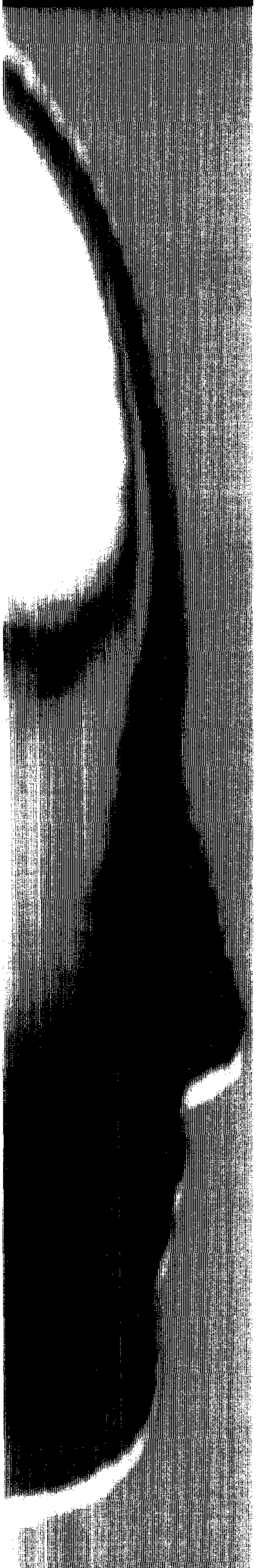
# Psychological foundations of number: numerical competence in human infants

Karen Wynn

**An enduring question in philosophy and psychology is that of how we come to possess knowledge of number. Here I review research suggesting that the capacity to represent and reason about number is part of the inherent structure of the human mind. In the first few months of life, human infants can enumerate sets of entities and perform numerical computations. One proposal is that these abilities arise from general cognitive capacities not specific to number. I argue that the body of data supports a very different proposal: humans possess a specialized mental mechanism for number, one which we share with other species and which has evolved through natural selection. This mechanism is inherently restricted in the kinds of numerical knowledge it can support, leading to some striking limitations to early competence.**



*K. Wynn is at the Department of Psychology, University of Arizona, Tucson, AZ 85721, USA.*

tel: +1 520 621 7447
fax: +1 520 621 9306
e-mail: wynn@u.arizona.edu

Over the past two decades, many studies have been conducted showing that infants can discriminate between different small numbers of entities. When repeatedly presented with displays of a given number of visual objects (say, two), infants will become bored and their looking time to the displays will decrease, but they will regain interest if they are then presented with displays containing a new number (say, three) of objects[1–3]. Infants respond this way even when the configuration of the objects in the displays, overall area taken up by the objects, and density of spacing are all controlled for[4], and even when the objects in the displays are in continuous motion[5]. These findings suggest that infants possess a sensitivity to number *per se*.

Infants can also enumerate entities other than material objects. They can enumerate physical actions: when habituated to a puppet jumping either two times or three times, six-month-old infants looked significantly longer when the puppet jumped a new number of times than when it again jumped the old, habituated number (see Fig. 1A), even when duration and tempo of the sequences was controlled for[6]. This held even when the jumps were embedded within a sequence of continuous motion, in which the puppet wagged its head back and forth in an exaggerated fashion between jumps, thereby preventing infants from simply picking out portions of motion from non-motion (Fig. 1B). Rather, infants had to parse the continuous sequence into discrete 'action units', and then enumerate those units. Further studies[7] showed that six-month-old infants are also able to enumerate heterogeneous, as well as homogeneous, action sequences.

Finally, there is some evidence that infants can enumerate sounds, and, moreover, can match a number of sounds to that same number of material objects: when habituated to pictures of either two or three household objects, infants subsequently looked longer at a black disk when it emitted the same number of drumbeats as the objects they had been habituated to, than when it emitted the other number. That is, infants habituated to pictures of two objects looked longer at the disk when it gave two drumbeats; those habituated to pictures of three objects looked longer when it gave three drumbeats[4]. However, attempts to replicate these findings[8,9] have produced mixed results, so they must be regarded as tentative at present.

## Infants can compute over their representations of number

Infants are not only sensitive to number; they can also engage in numerical computation. This was first shown in a series of experiments in which five-month-old infants were shown simple addition and subtraction operations on small sets of objects[10]. The numerical outcome was in some cases consistent with the operations, in some cases inconsistent; infants' looking time to the different outcomes was measured. Because of infants' well-known tendency to look longer at unexpected events, this provides a measure of infants' expectations as to the results of the operations they witnessed. In one experiment, one group of infants saw a '1 + 1' operation (Fig. 2A): an object (a Mickey Mouse doll) was seen placed in a display stage; a screen then rotated upwards to temporarily hide it from view; a hand then entered the display stage with another, identical object, placed that



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object behind the screen along with the first, and exited the stage, empty; the screen then came down to reveal either the correct number of objects (two dolls), or an incorrect number (one doll). Another group of infants saw a '2 − 1' operation (Fig. 2B): two objects were placed in the display, the screen rotated up to occlude them, an empty hand then entered the display, reached behind the screen, and removed one of the objects from the display, and the screen then dropped, again to reveal either one doll (now the correct outcome) or two dolls (now the incorrect outcome). Both groups of infants saw three pairs of the two kinds of test trials.

Infants in the two groups showed significant differences in their looking patterns: those shown the addition operation looked longer at an outcome of one doll than two dolls, while those shown the subtraction operation showed the reverse pattern (Fig. 3A,B).

Another experiment addressed the question of whether infants were computing the actual numerical result of the '1 + 1' operation, or whether they were simply expecting that the display should have 'more' objects than it had before, or be otherwise changed from its initial appearance before the screen rose. Here, five-month-old infants were again presented with a '1 + 1' addition situation, but in this case, the screen dropped to reveal either two or three objects – both different from the initial number of one object. If infants were simply expecting the display to be altered in some way as a result of the operation, this expectation should be equally met by both outcomes. In fact, infants looked significantly longer at the result of three items than of two (Fig. 3C). Another experiment, reported separately, found that five-month-olds also appreciate that one object removed from three should not result in three objects[11].

The finding that infants can anticipate the correct outcome of such addition and subtraction operations is quite robust, and has been replicated and extended in several independent laboratories (Refs 12–14; D.S. Moore, unpublished; M.C. Uller *et al.*, unpublished). Of particular interest, Koechlin *et al.*[13] asked whether infants' performance is due to a sensitivity to the precise spatial locations of objects in the stage, rather than to the number of objects. Infants might have been recording the specific location of the objects involved, and their longer looking times to the impossible outcomes revealing surprise at the existence of an object in a location where none should be, or at the absence of any object in a location where there should be one. In their experiment, they again presented infants with '1 + 1' and '2 − 1' situations that resulted in either one or two items, as in Wynn[10]. However, the items were all placed on a large rotating plate located on the stage area behind the screen; thus, no single unique location could be attributed to any of the objects. Nonetheless, the same results obtained: infants looked longer at the numerically incorrect outcomes, showing that they were computing over number of objects, not over filled versus empty spatial locations.

These findings suggest a picture of the nature of numerical knowledge very different from some proposals, such as that of Kitcher[15], who proposes that humans acquire an appreciation of some fundamental numerical facts, such as that one and one make two, through our own earliest physical manipulations of objects – collecting and segregating small

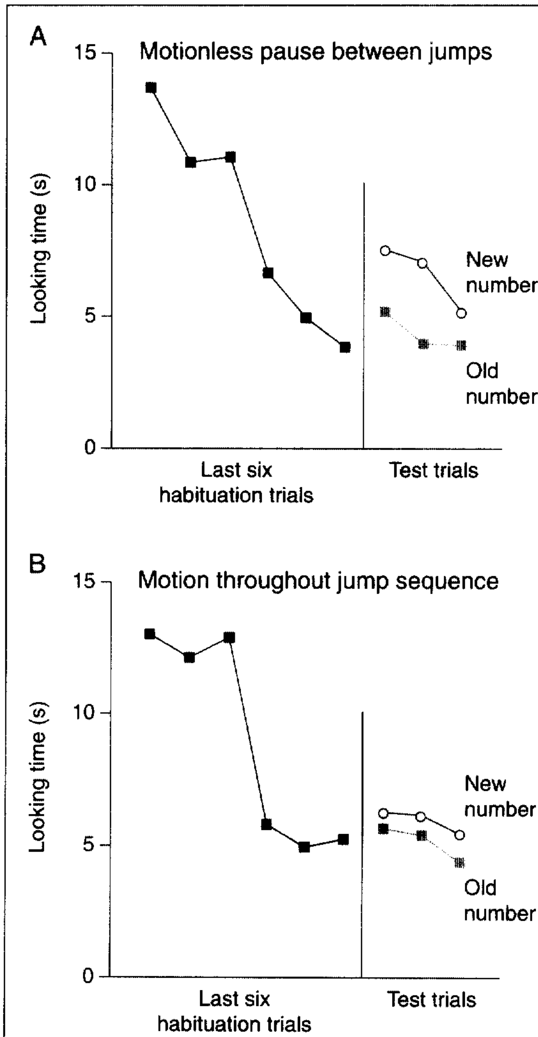


**Fig. 1 Six-month-olds' looking times on last six habituation trials and on old- and novel-number test trial jump sequences. (A)** sequences with motionless inter-jump intervals; **(B)** sequences in which the puppet was in continuous motion. (Adapted from Ref. 6, courtesy of *Psychological Science*.)

groups of objects, and observing the results. This empiricist account of the attainment of such basic numerical relationships cannot be true – already, by five months of age, well before infants are able to engage in systematic physical manipulations of objects, they appreciate the precise numerical relationships that hold between small numbers.

### Two alternative accounts of infants' numerical competence

I propose that there exists a mental mechanism, dedicated to representing and reasoning about number, that comprises part of the inherent structure of the human mind[11,16]. A range of warmblood vertebrate species, both avian and mammalian, have been found to exhibit numerical discrimination and reasoning abilities similar to those documented in human infants[17,18]. Because of its adaptive function[18–20], this mechanism quite likely evolved through natural selection, either at a point in evolutionary history prior to the branching off of these different species, or separately but analogously within several branches of these species[18].

The 'accumulator model' is a specific proposal of just such a mechanism. It was initially proposed by Meck and

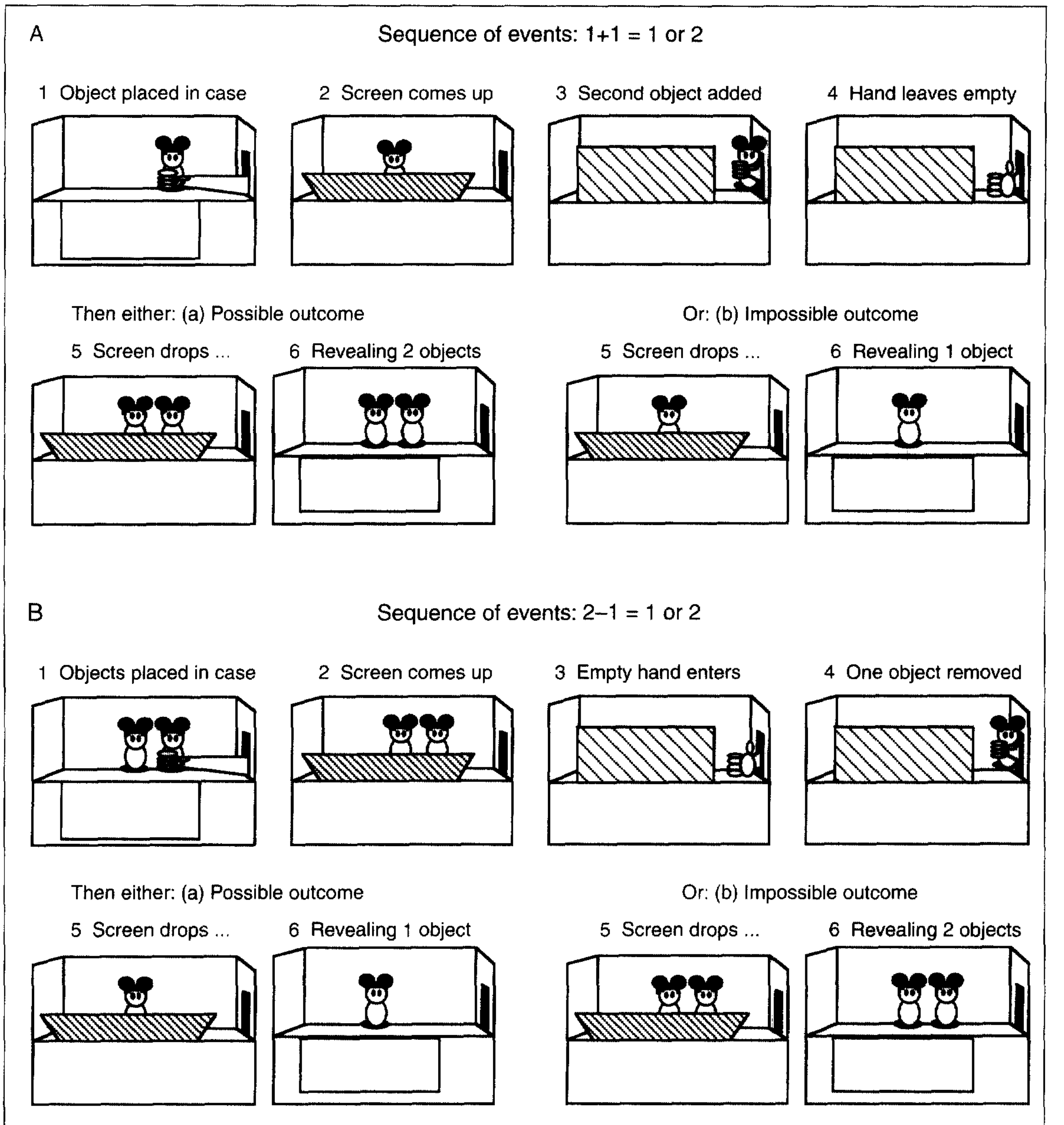


**Fig. 2 Sequence of events shown to five-month-olds.** The sequence was either an addition operation in which a second Mickey Mouse doll was added to the display **(A)**, or a subtraction operation, in which one of two dolls was removed **(B)**. Both sequences ended with either a possible or an impossible outcome, each occurring with equal probability. (Adapted from Ref. 10, courtesy of *Nature*.)

Church[21] to account for numerical abilities in rats, adapted from a model for measurement of duration by Gibbon[22]. Thus, the accumulator can both measure duration, and count. On this model, a pulse generator puts out 'pulses' at a constant rate, which can be passed into an accumulator by the closing of a switch. To count an entity, the switch closes for a brief, fixed amount of time, passing pulses into the accumulator (see Fig. 4). Because the generator always produces pulses at a constant rate, roughly the same amount is passed into the accumulator for each item counted. The final accumulation of pulses at the end of the count is the output of the mechanism, and represents the total number of items in that count. When the accumulator is in its timing mode, the switch closes at the beginning of the event to be timed, and stays closed until the conclusion of the event, so it fills up continuously, rather than incrementally as it does when counting. The final fullness of the accumulator is proportional to the duration of the timed event.

The existence of numerous separate accumulators allows for timing and counting to occur simultaneously, and enables parallel enumeration of distinct sets of entities; multiple accumulators also permit one magnitude to be retrieved from memory, or stored, for purposes of comparison. Different accumulators can be compared and operated upon to make evaluations as to whether one value is more or less than another, to compute the sum or difference of two values, and so on. For example, consider a '2 – 1' situation as in Wynn[10]. The accumulator is first incremented by two increments to represent the two items in the display; when one item is then removed, the contents of the accumulator are decremented by one increment to yield an accumulator filled to the level of a single increment – a representation of the value *one*.

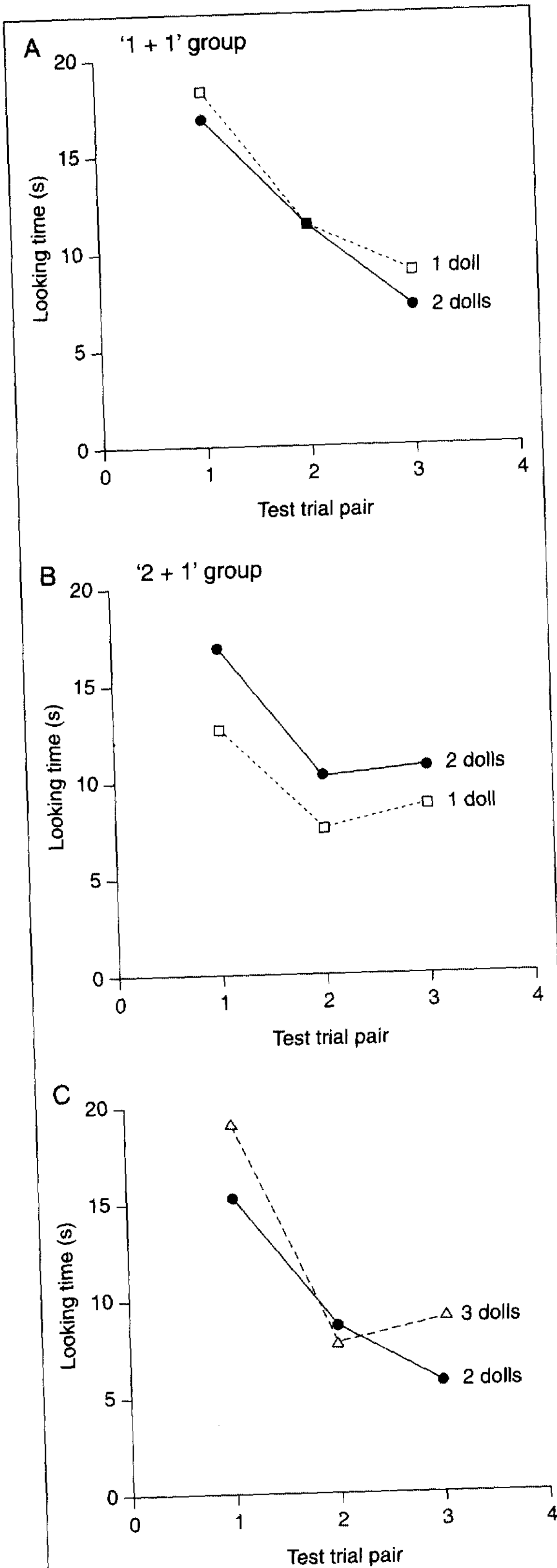


**Fig. 3 Five-month-olds' looking times to outcomes of the experiments shown in Fig. 2. (A)** following '1+1' event sequence. **(B)** following '2–1' event sequence. **(C)** Five-month-olds' looking times to two versus three objects, following a '1+1' sequence of events. (Data from Ref. 10.)

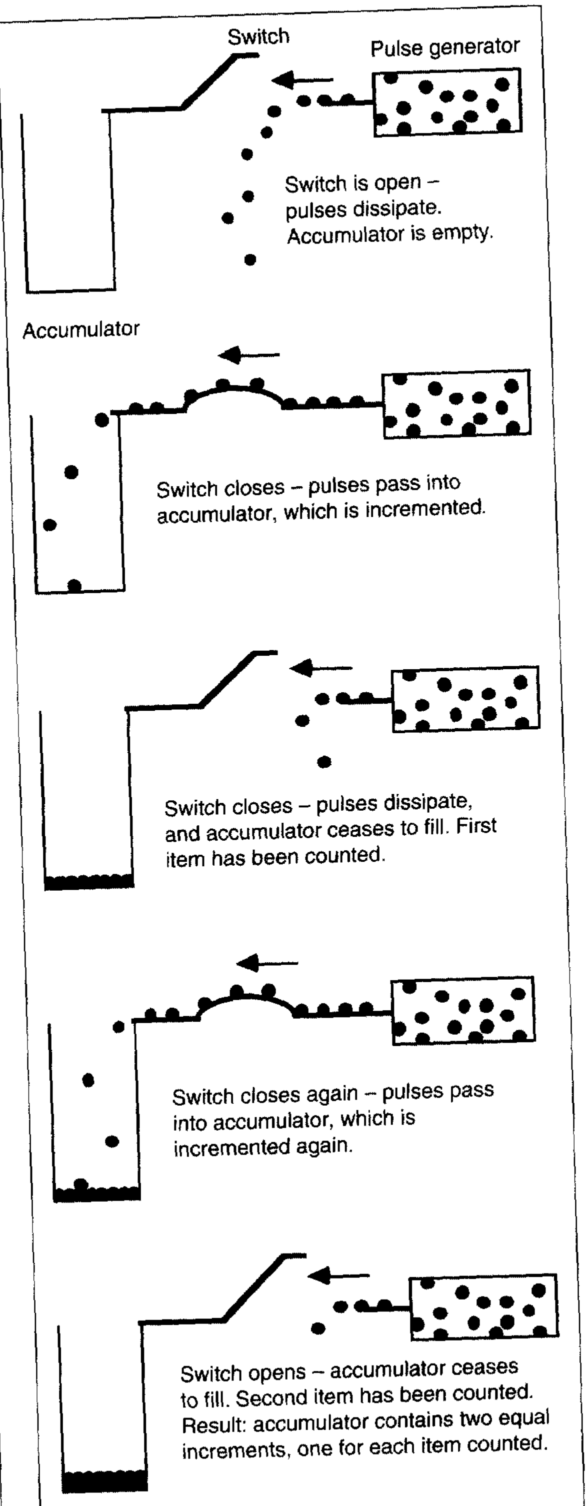


**Fig. 4 Schematic diagram of the states of the Meck and Church[21] accumulator mechanism as it enumerates two items.** The resulting fullness level of the accumulator is the mental representation for 'two'. (Adapted, with permission, from Ref. 21.)

An alternative possibility is that cognitive capacities not themselves specific to number provide the basis for infants' competence. On theories of object-tracking[23,24], the visual system constructs temporary representations ('object files' or 'tokens') of objects in a scene. The system places (or attempts to place) these tokens in 1–1 correspondence with tokens from immediately preceding scenes. In this way, the system is responsible for the perception of continuity of object identity over time, maintaining object identity as objects move about and change, and from saccade to saccade. In an account by Simon of infants' numerical *discrimination* abilities[25], infants are simply responding to mismatches between sets of tokens representing currently perceived items, and sets of tokens constructed from previous scenes (such as displays presented during habituation trials). Failure to achieve a 1 – 1 correspondence matching, as would occur on test trials in which a new number of objects was presented, results in longer looking.

This account has also been proposed to explain infants' numerical *reasoning* abilities (Refs 23,24 and M.C. Uller *et al.*, unpublished). On this view, infants are simply tracking the specific objects involved, and expecting them to maintain spatiotemporal continuity. This expectation is violated when a tracked individual magically disappears 'into thin air', or when a new, untracked object appears 'from out of nowhere'. That is, infants are not computing over mental representations of number; rather, they are operating over representations of specific individual objects. For example, consider a '2 – 1' situation. When the screen drops following the operation to reveal an impossible outcome of two objects, it is the sudden appearance of an additional, untracked object with no corresponding mental token, not a mismatch between expected and observed number of objects, that causes infants' longer looking.

### The two alternatives as accounts of infants' numerical discrimination abilities

There are strong similarities between non-human animals' and human infants' numerical abilities which suggest that the same mechanism is responsible for both; and extensive empirical findings on numerical competence in rats support the accumulator mechanism specifically.

Similarities between animals and infants' numerical abilities are numerous: animals as well as infants enumerate a wide range of entities, including objects and events, entities that move and entities that are stationary, entities presented simultaneously and entities presented sequentially[17–19]. And, the pattern of discriminability observed is similar: animals[19] as well as infants (F. Xu and E.S. Spelke, unpublished) show a greater precision in distinguishing between smaller numbers than in distinguishing between larger numbers. But both animals and infants can discriminate larger numbers if those numbers are sufficiently far away from each other. These similarities strongly suggest a common mechanism as the basis for numerical abilities in both infants and animals.

Moreover, the evidence favoring the accumulator as the basis of numerical competence in animals is compelling. Meck and Church[21] found strong similarities between animals' counting and timing processes, which led them to conclude that the same mechanism is responsible both for animals' measurement of duration, and their measurement of number. First, both numerical and duration discriminations transfer to novel stimuli to exactly the same extent. Second, if rats are administered methamphetamine, it causes them to overestimate duration. It also causes them to overestimate number, by exactly the same factor. Finally, they obtained transfer effects between the two: animals respond to a trained fullness of the accumulator regardless of whether that fullness level results from a counting or a timing process: that is, a count calculated to fill up the accumulator to the same level as a previously rewarded duration, was responded to as if it *were* that duration, and vice versa. The accumulator model accounts for these similarities; no other existing model of numerical discrimination can explain them.

There are several reasons to doubt the utility of the object-tracking theory as an adequate account of infants' number-discrimination abilities. First, the proposed object-tracking processes[23,24,26] create mental tokens specifically of *objects* in the visual scene; they do not create tokens of other kinds of individuals. But it is clear that infants can enumerate not only material objects, but other kinds of entities as well, such as physical actions[6,7].

Second, on object-tracking theories, there is assumed to be a strict limit on the number of objects that can be simultaneously tracked, to three or at most four objects[23,24,26]. This assumption is based upon empirical evidence on the limits of humans' capacity for parallel individuation of objects[24]. However, recent experiments by F. Xu and E.S. Spelke (unpublished) have found that infants' enumeration capacities significantly exceed these limits: infants can discriminate 8 from 16, significantly exceeding the limits of object-tracking processes, even with factors such as density and overall size of objects controlled for. Interestingly, infants did not discriminate 8 from 12 in a similar design. Such a decrease in discriminability with increasing number is a necessary consequence of the accumulator model. As all physical processes entail some amount of variability, variance in the exact size of each increment of pulses transferred into the accumulator during a count provide a source of variation which is additive. Thus the variation in the total fullness of the accumulator will be greater for counts of larger numbers than for counts of smaller numbers, meaning that consecutive numbers will become more difficult to discriminate the larger they are, following a Weber's Law discrimination function[19].

### The two alternatives as accounts of infants' numerical reasoning abilities

Even if infants possess a mechanism such as the accumulator for enumerating, infants' ability to determine the results of numerical computations may not rest on this mechanism. It is possible that the accumulator supports infants' numerical discriminations, but that their numerical reasoning abilities result from more general object-tracking capacities (Ref. 26 and M.C. Uller *et al.*, unpublished).

One finding claimed to favor the object-tracking view is that in numerical reasoning experiments, infants' performance is affected by factors such as physical properties of the arrays, the relative locations of the objects in the array, and the temporal sequences of events (M.C. Uller *et al.*, unpublished). Uller and colleagues reason that, if infants are creating a mental model of the situation complete with mental tokens for the objects, as entailed on the object-tracking alternative, then factors that influence their ability to accurately construct mental models of a scene should affect their success in these tasks. However, these findings do not decide the issue: there is evidence the accumulator attends to such factors as the locations of objects in deciding when to set up distinct counts[19], and so the same considerations that affect the accuracy of a mental model of a scene could also influence the accumulator. In other words, if the accumulator applies over a mental representation of a scene, with information such as spatial relationships between entities being relevant to a count, then any factors that negatively affect the construction of a mental model of a scene will, by necessity, also affect the enumeration process.

A different way to evaluate the two alternatives is to examine certain idiosyncratic properties of the accumulator mechanism. Because of the structure of the accumulator mechanism, it is limited in terms of the mathematical concepts, computations, and values it can represent[11,18,27]. It cannot represent abstract mathematical concepts, such as infinity. Because of the complexity of the operations that would be required, it probably cannot support many kinds of computations, such as the determination of a cube root. And, notably, it cannot represent numerical values other than positive integers.

This last limitation of the accumulator suggests a means of distinguishing between it and the 'object-tracking' alternative as accounts of infants' performance in the numerical reasoning experiments. By virtue of its inherent architecture, the accumulator cannot represent the value 'zero'; this limitation makes sense as, by hypothesis, the accumulator evolved to enumerate collections of real-world entities, and all such collections will have some positive number of members. No natural collection of entities will have zero members – and no collection that has zero members will activate an enumeration process. Why isn't an empty accumulator a representation of 'zero items'? It is the accumulation of pulses in the accumulator that serve as the output of this mechanism; therefore, an empty accumulator is a silent accumulator, one which is giving no output at all. This becomes clear when we consider the fact that, before it has begun counting in any given situation, the accumulator is empty – emptiness is simply the state that the accumulator is in when it has no expectations at all about a given situation. This limitation results in a somewhat counter-intuitive consequence: infants will have no numerical expectations about an operation that yields a result of zero items.

Consider a hypothetical '$1 - 1 = 1$' situation. An item is seen placed into the display; the screen comes up to hide it from view; a hand then enters the display, reaches behind the screen, and removes the object from the display; down comes the screen to reveal an object behind it. On the accumulator model, in this situation the infant first represents one object behind the screen: the switch is closed once, passing one increment of pulses into the accumulator. When the infant then sees an object removed from behind the screen, the contents of the accumulator are decremented by one increment, yielding … an empty accumulator. The subsequent presence of an object behind the screen will therefore not conflict with any *numerical* representation, because there *is* none for it to conflict with. The 'object-tracking' account, however, makes a different prediction. When the item is placed in the display, the infant creates a token for that object. When the infant sees the object removed from the display, the object-tracking processes update the mental model of the situation; there is no longer a mental token for an object behind the screen. Down comes the screen to, impossibly, reveal an object still there. The 'magical appearance' of this new, untracked object with no corresponding token will trigger the same surprise as it does in, for example, a '$2 - 1 = 2$' situation.

Wynn and Chiang presented a '$1 - 1 = 1$' situation to eight-month-old infants[28]. The infants were shown 'magical' versus 'expected' appearances. In the magical appearances, an object was placed in the display; a screen rotated upward to occlude it; a hand then reached in and removed the object from the display; and subsequently the screen dropped to reveal an object behind it. In the expected appearances, an object was placed in the display; the screen rotated upward to occlude an empty area of the stage beside the object; a hand then reached in, pushed the object behind the screen, and exited the display; and the screen then dropped to reveal an object behind it (Fig. 5).

Infants looked equally at the magical and expected appearances (Fig. 6). Infants are not surprised when an object 'magically' appears in a location where there should be no objects. This finding was reliably obtained under several different conditions involving different display characteristics and using different stimuli, and obtained even though, with similar stimuli and procedures, infants clearly distinguished magical from expected *dis*appearances (in which a screen dropped to reveal no object behind it), showing that they were able to follow our event sequences and could discriminate 'pushing-object-behind-screen' trials from 'removing-object-from-display' trials. This result suggests that infants' object-tracking capacities are not sufficient to explain infants' performance in the numerical reasoning studies. The impossible appearance of a new, untracked object does not by itself elicit longer looking. As the accumulator model predicts, infants show no numerical expectation about the outcome of an operation that yields a result of zero items (see Box 1 for further discussion of this finding).

### Other evidence of difficulty with 'zero'

This finding of a difficulty in representing the value 'zero' fits quite well with some other facts about the nature and development of numerical knowledge, in particular the historical and ontological development of zero. The introduction of the numerical value zero was a surprisingly late achievement in the historical development of mathematics[29]. Zero was not part of early formal number systems; the Babylonian place-value numeral notation system had no symbol for zero for over 1500 years, and when a symbol was finally introduced, it was not as a *conceptual* advance – it was merely a place holder symbol representing an absence of any value in that column in place value numeral notation. It was much later that this symbol slowly developed meaning in its own right, as a numerical value with its own place relative to other numerical values.

Within individual development, a similar story holds. An understanding of zero as a number does not follow the same pattern of development as children's understanding of positive integer values. Even once they have learned what the word 'zero' and its corresponding Arabic numeral symbol stand for, young children take a considerable time to appreciate that zero is a numerical value, one which can be

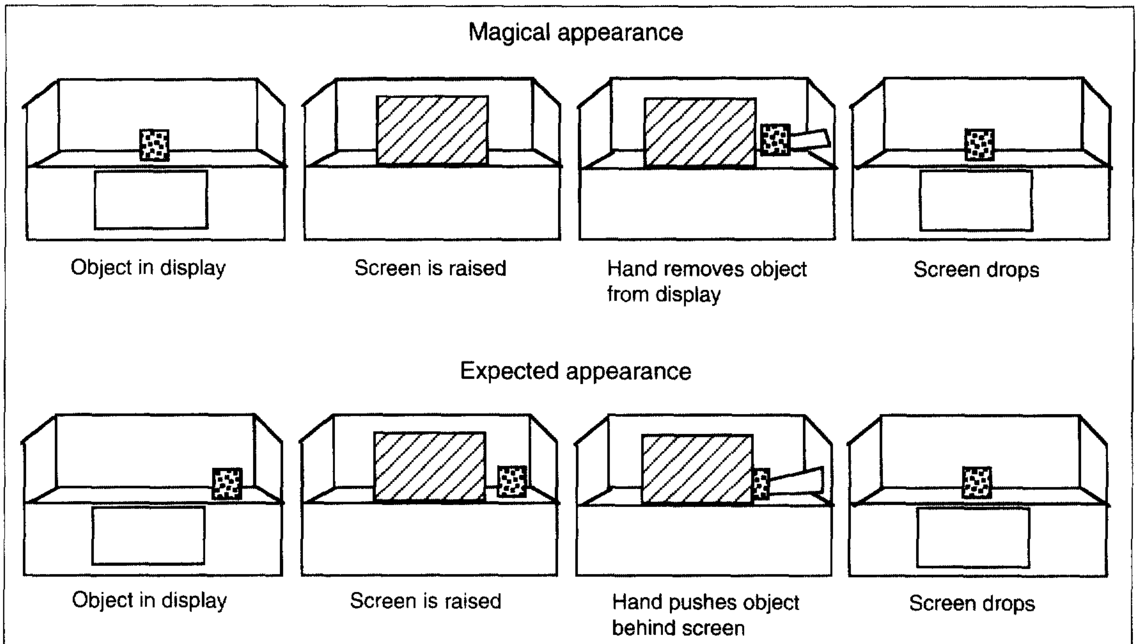


**Fig. 5 'Magical appearance' and 'expected appearance' events presented to eight-month-old infants.** In the first case, the appearance of an object when the screen drops is unexpected or'magical'. In the second, the appearance of the object is expected from the sequence of previous events. (Adapted from Ref. 28, courtesy of *Psychological Science*.)

ordered relative to other integer values. For example, even once they know that the word 'zero' applies to 'no items', preschoolers will name 'one' as the smallest number. Even more interestingly, when asked to identify the smaller of two numbers, if one of the choices is zero, preschoolers perform at chance – they are as likely to say that three is smaller than zero as that zero is smaller than three[30]! That is, zero is simply not part of their own internal number system: one is the smallest *number*, zero is something completely different, a synonym for 'none' or 'nothing'.

Other non-positive integer numbers, such as negative numbers, fractions, and irrational numbers, also emerged only gradually and with much controversy in the history of mathematics, and pose considerable developmental challenges in children's acquisition (see, for example, Gelman *et al.*[31]). The earliest numbers to appear in mathematical thought, both historically and within individual development, are the positive integers. The positive integers appear to be psychologically privileged numerical values. These are precisely the values that the accumulator mechanism is capable of representing.

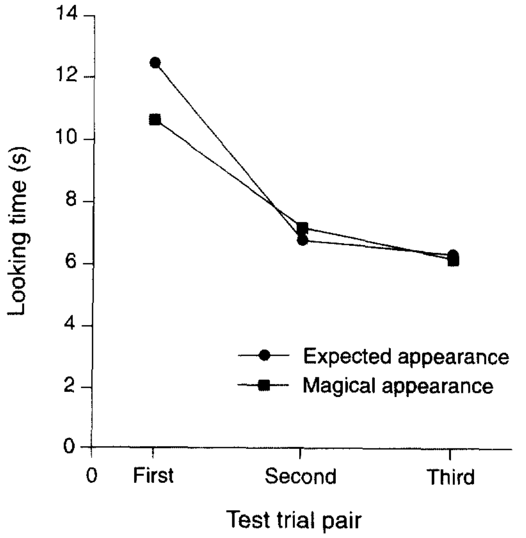


**Fig. 6 Eight-month-olds' looking times to the magical versus expected appearances illustrated in Fig. 5.** (Data from Ref. 28, courtesy of *Psychological Science*.)

## Conclusions

I have reviewed evidence showing that infants can enumerate different kinds of entities, and can compute the numerical outcomes of operations over small numbers of entities. These abilities cannot be fully explained in terms of infants' cognitive capacities for tracking and reasoning about objects. Rather, findings support the existence of a dedicated mental mechanism specific to number, one which may have evolved through natural selection and which we may share with other warm-blooded vertebrate species. This mechanism serves as a foundational core of numerical knowledge, providing us with a toe-hold upon which to enter the realm of mathematical thought. But it is strictly limited in the numerical values it can represent and in the kinds of numerical knowledge it supports. I began this article by asking how we come to have knowledge of number. I hope to have provided the beginnings of an answer – we are built specifically to do so, probably because there was adaptive benefit to having a degree of numerical competence. But to develop a full answer to this question will require a better understanding of how it is that the human mind goes beyond the limits of this initial foundation. The system of formal mathematics, a uniquely human achievement, goes far beyond this core competence. It is in accounting for this broader achievement that an appeal to more general cognitive capacities may prove essential.

## Box 1. Magical appearance and knowledge of objects

As discussed in the main article, it is not surprising that infants' number mechanism failed to detect a 'magical appearance' in a '$1-1=1$' situation[a]; the accumulator does not represent the value zero, so would be unable to represent the result of the operation. But the experimental results indicate that it is not only the *number* mechanism that failed to detect a discrepancy; no other cognitive system detected one either. This is surprising, as there are many non-numerical ways infants might have succeeded at the task: for example, they could have represented, not that there should be 'zero items' behind the screen, but that the location behind the screen was now empty where once it was filled.

It is particularly interesting that infants' system of object knowledge did not detect the incongruity of the outcome. There is much evidence that infants possess a sophisticated understanding of many of the properties of physical objects. For example, numerous experiments show that infants expect an object to continue to exist once out of sight (Refs a–c; W-C. Chiang and K. Wynn, unpublished) and expect an object to follow a continuous pathway through space[d]. These expectations have been characterized as an understanding of the principle of spatiotemporal continuity: objects have continuous existence through space and time; they cannot 'blip' discontinuously into and out of existence, or from one location to another[e]. But a magical appearance is as much a violation of continuity as a magical disappearance.

We suggest[a] that there are several ways that infants' failure to detect a magical appearance can be reconciled with findings showing infants' appreciation of other aspects of object continuity. One possibility is that infants possess only partial knowledge of the continuity principle: they appreciate that existing objects continue to endure, but have no expectations about the conditions under which objects come into existence. Another possibility is that infants possess the full principle of continuity, but simply have greater difficulty in detecting when an appearance is discontinuous than they do in detecting when a disappearance is discontinuous. Different reasoning processes might be required to detect a discontinuous appearance than are required to detect a discontinuous disappearance, and the former might develop later than the latter. Yet a third possibility is that object-tracking processes themselves highlight the occurrence of an illicit disappearance: when a tracked object disappears from a specified location, this conflicts with representations produced by object-tracking processes. However, because these processes produce information only about tracked objects, the appearance of a new, untracked object cannot conflict with the outputs of these processes, since none of the outputs pertains to the new object. (For further discussion, see Ref. a.)

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