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Uniqueness and Stability of Equilibrium in Economies with Two Goods*

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Abstract

We offer new sufficient conditions ensuring demand is downward sloping local to equilibrium. It follows that equilibrium is unique and stable in the sense that rising supply implies falling prices. In our setting, there are two goods, which we interpret as consumption in different time periods, and many impatience types. Agents have the same Bernoulli utility function, but the types differ arbitrarily in time preference. Our main result is that if endowments are identical and utility displays nonincreasing absolute risk aversion, then market demand is strictly downward sloping local to equilibrium. We discuss implications for the Diamond-Dybvig literature.

Keywords: uniqueness of equilibrium, absolute risk aversion, excess demand functions, stability of equilibrium, Diamond-Dybvig models

JEL Classification: C62, D51, D58

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1 Introduction and Related Literature

When the aggregate endowment temporarily increases, will interest rates fall? Will discoveries of oil push down gas prices? The intuitive answer to these questions is yes. The logic is that since individual demand is normally downward sloping, if supply of a good increases, its price must fall to clear markets and maintain equilibrium. But this reasoning requires that aggregate, market demand inherit the properties of individual demand. If, for example, markets are complete and agents have identical homothetic utility functions, equilibrium prices are as if there were a representative agent with homothetic preferences. In this case, micro intuition extends to the macroeconomy. But what if we place less restrictive assumptions on individual preferences? Will market demand still look like the demand curve of a rational person?

The Sonnenschein-Mantel-Debreu results give a negative answer to this question (see Shafer and Sonnenschein (1982) for a survey). They say that arbitrary continuous market excess demand functions can be generated by individuals with positive endowments and continuous, increasing, concave utility functions. In the case of Mantel (1976), the utility functions can even be restricted to be homothetic. The striking implication is that strong assumptions about individuals (such as homotheticity) may yield wild market excess demand functions exhibiting, for example, multiple equilibria and thus equilibria with upward sloping demand. In this case, equilibrium may be unstable in the sense that increasing supply may lead to *higher* prices. In short, the Sonnenschein-Mantel-Debreu results show that concavity, continuity, and homotheticity are not sufficient for aggregate demand to behave like individual demand. See Toda and Walsh (2016) for examples of and sufficient conditions for unstable equilibria in Edgeworth box economies with identical homothetic or quadratic Bernoulli utility functions.

Yet theorists have uncovered many cases where competitive equilibrium is unique and thus stable, meaning that aggregate demand is downward sloping at least local to equilibrium. See Kehoe (1998) and Mas-Colell (1991) for surveys of the uniqueness literature.[1] For example, suppose agent $i \in I$ has differentiable, increasing, concave utility $u_i(x) = \sum_{j=1}^{J} u_{i,j}(x_j)$ over J goods and a positive endowment of each good. If for all $i \in I$ relative risk aversion is everywhere less than 1, $-x_j u''_{i,j}(x_j) / u'_{i,j}(x_j) < 1$, then all excess demands functions are downward sloping, and the resulting equilibrium is unique and stable (see Mas-Colell, Whinston, and Green (1995)). By assuming collinear endowments, the result of Mitiushin and Polterovich (1978) weakens this condition to $-xu''/u' < 4$. However, as Kehoe (1998) observes, "useful conditions that guarantee uniqueness of equilibrium are very restrictive," involving, say, *quantitative* bounds on relative risk aversion, as in these two instances.[2] Generally, as Kehoe

[1] See Negishi (1962), Arrow and Hurwicz (1958), and Walras (1954) for early treatments of the topic of stability of competitive equilibrium.

[2] Also, 4 is not a large value for relative risk aversion in the sense that many theoretical and empirical studies assume or estimate relative risk aversion to be well in excess of 4. See, for example, the meta-analysis of Havranek, Horvath, Irsova, and Rusnak (2015). Note that while their study is about the elasticity of intertemporal substitution (EIS), most of the papers they reference restrict

(1998) continues, conditions sufficient for uniqueness have been difficult to translate into economic intuition without losing necessity. Furthermore, while there are many applied general equilibrium models for which we do not have uniqueness proofs (as in the infinite horizon macroeconomics literature), non-uniqueness examples are equally rare in some settings. Therefore, as Kehoe (1998) writes, "It may be the case that most applied models have unique equilibria." Indeed, it may be that relatively weak, easily verified sufficient conditions are simply unknown.

In this paper, we offer new sufficient conditions ensuring aggregate demand is downward sloping local to equilibrium. It follows that equilibrium is unique and locally stable. In our setting, there are two goods, which we interpret as consumption in different time periods, and $I < \infty$ impatience types. Agents have the same Bernoulli utility function u, but the types differ arbitrarily in time preference. That is, for arbitrary agent i utility is of the form $u(c_1^i) + \beta^i u(c_2^i)$, where β^i captures agent i's patience. Our main result (Proposition 1) is that when agents have identical, strictly positive endowments, if u displays nonincreasing absolute risk aversion then market demand is strictly downward sloping local to equilibrium. That is, when the interest rate rises, aggregate demand for $t = 2$ goods increases. It follows that (i) equilibrium is unique and (ii) increasing the supply of $t = 1$ ($t = 2$) goods leads to a fall (rise) in the equilibrium interest rate, in line with supply and demand intuition.

The assumption of nonincreasing absolute risk aversion is weak: at least since Arrow (1965) economists have almost universally held that increasing absolute risk aversion is an undesirable property because it implies investors spend less on risky assets as they become richer. Assuming identical endowments and two goods is more restrictive. However, this case is important because many papers have used it to study maturity mismatch and the role of government in managing liquidity. For example, the second and third periods of Diamond-Dybvig models consist of different patience types with identical endowments. These papers, which include Jacklin (1987), Bhattacharya and Gale (1987), Farhi, Golosov, and Tsyvinski (2009), Yared (2013), and Geanakoplos and Walsh (2015), show that the government can improve welfare by encouraging more short-term, liquid investment. Doing so increases welfare by pushing down the interest rate, which redistributes from patient to impatient types and hence mitigates a pecuniary externality. The key mechanism at play is rising liquidity leading to a fall in interest rates. In other words, proving inefficient liquidity provision in Diamond-Dybvig models requires stability of equilibrium. Geanakoplos and Walsh (2015) apply our Proposition 1 to show that in their generalized version of the Diamond-Dybvig model agents will always overinvest in high yielding but illiquid long-term assets and underinvest in short-term liquid assets.

Intuitively, why does nonincreasing absolute risk aversion rule out unstable equilibria and thus ensure uniqueness? Unstable equilibria arise when income effects are strong and positive in the aggregate, leading to upward sloping excess demand crossing 0. Absolute risk aversion $-u''/u'$ is inversely proportional to the change in demand from increasing wealth. Thus, when absolute risk aversion is declining, the agent consuming the most is the most sensitive to changes in wealth. This implies

risk aversion to be the reciprocal of the EIS.

that the aggregate income effect from a price rise is not positive because buyers face an income loss when the price increases, meaning the income effect reinforces the price effect for the most wealth sensitive consumers. Technically, we show that, given any price, nonincreasing absolute risk aversion implies that the derivative of consumption with respect to wealth is positively correlated with excess demand across agents, and thus that the average total income effect of a price increase is less than the total income effect of the "average" agent.[3] In equilibrium, market clearing implies the "average" agent must have zero excess demand, meaning his total income effect is zero. Therefore, equilibrium is essential for our result. Indeed, demand could be upward sloping away from equilibrium.

In Section 2, we exposit the model and prove our propositions.

2 Model and Results

Consider an economy consisting of two time periods, $t \in \{1, 2\}$, and a unit mass of agents. There is a single consumption good in each time period. We normalize to 1 the price of $t = 1$ goods and define q to be the price of $t = 2$ goods.[4] There are I impatience types indexed by i. Type i is distinguished by the parameter $\beta^i > 0$. The $t = 1$ present value utility of impatience type i is

$$U^i\left(c_1^i, c_2^i\right) = u\left(c_1^i\right) + \beta^i u\left(c_2^i\right),$$

where c_t^i is the consumption of type i at $t \in \{1, 2\}$. π_i is the fraction of i -types, and $\sum_{i=1}^{I} \pi_i = 1$. That is, while the agents have identical Bernoulli utility functions, the economy exhibits arbitrary time preference heterogeneity. Agent i has endowment $(e_1^i, e_2^i) \gg 0$, and the agents are ordered by patience:

$$\beta^1 < ... < \beta^I.$$

Stars denote equilibrium quantities. We assume that u is twice continuously differentiable, $u' > 0$, $u'' < 0$, and $\lim_{x \downarrow 0} u'(x) = \infty$.

Define $a(\cdot) \equiv -u''(\cdot)/u'(\cdot)$ to be the Arrow-Pratt measure of absolute risk aversion. The budget set of an agent is

$$\mathcal{B}^i\left(q, e_1^i, e_2^i\right) = \left\{\left(c_1^i, c_2^i\right) \in \mathbb{R}_+^2 \left| c_1^i + q c_2^i \leq e_1^i + q e_2^i \right.\right\}.$$

Definition 1 (Competitive Equilibrium) *Competitive Equilibrium consists of a price $q^* > 0$ and consumption bundles $\{(c_1^{i*}, c_2^{i*})\}_{i \in I}$ such that*

[3] As we have an endowment economy, there are two income effects from a price change: the indirect effect on purchasing power, which is the income effect in the standard textbook Slutsky equation, and the direct endowment wealth effect. We refer to the sum as the "total" income effect.

[4] For exposition we call the two goods consumption at $t = 1$ and $t = 2$, which means the relative price is the interest rate, but our set up and analysis are identical to a static setting with two different goods.

1. *For all* $i \in I$*, given* q^**, if* $(c_1^i, c_2^i) \in \mathcal{B}^i(q, e_1^i, e_2^i)$ *then* $U^i(c_1^i, c_2^i) \leq U^i(c_1^{i*}, c_2^{i*})$.

2. $\sum_{i=1}^{I} \pi_i (c_1^{i*}, c_2^{i*}) = \sum_{i=1}^{I} \pi_i (e_1^i, e_2^i)$.

Define $\omega^i(q) \equiv e_1^i + qe_2^i$ to be an agent's wealth at price q. The budget constraint is $c_1^i + qc_2^i \leq \omega^i(q)$. Define $c_t^i(q, \omega^i)$ to be type i's time t demand at price q and wealth ω^i. $u' > 0$, $u'' < 0$, and $\lim_{x \downarrow 0} u'(x) = \infty$ imply that the first order condition $\beta^i u'(c_2^i) = qu'(\omega^i(q) - qc_2^i)$ characterizes the demand function for any $q > 0$. Since u is twice continuously differentiable, the implicit function theorem and the first order condition give us that $c_2^i(q, \omega^i)$ is continuously differentiable. Define $D(q) \equiv \sum_{i=1}^{I} \pi_i c_2^i(q, \omega^i(q))$ to be market demand for $t = 2$ goods. $D(q)$ is continuously differentiable since $c_2^i(q, \omega^i)$ is, and, by standard arguments, $\lim_{q \to 0} D(q) = \infty$ (see Mas-Colell, Whinston, and Green (1995)).

The goal is to show that when agents have identical endowments decreasing absolute risk aversion ensures market demand for $t = 2$ goods is downward sloping, local to any equilibrium: $D'(q^*) < 0$. We break most of the proof down into four lemmas. Lemma 1 shows that for any price $q > 0$ demand is ordered according to patience. This is the first step in ordering total income effects.

Lemma 1 (Consumption Lemma) *If* $(e_1^i, e_2^i) = (e_1, e_2)$ $\forall i$*, then*

$$c_2^1(q, \omega^1(q)) < ... < c_2^I(q, \omega^I(q))$$
$$c_1^1(q, \omega^1(q)) > ... > c_1^I(q, \omega^I(q))$$

for any $q > 0$.

Proof. By the assumption on u, $\lim_{x \downarrow 0} u'(x) = \infty$ in particular, the solution to each agent's optimization problem is interior, so the first order conditions characterize the demand functions:

$$q = \beta^i \frac{u'(c_2^i)}{u'(c_1^i)} = \beta^j \frac{u'(c_2^j)}{u'(c_1^j)}, \ \forall i, j \in I.$$

Since endowments are identical, $\omega^i(q) = \omega(q) = e_1 + qe_2$ for all $i \in I$. Using the budget constraint and $\beta^i < \beta^j$ for $i < j$, it follows that if $i < j$ then

$$\frac{u'(c_2^i)}{u'(\omega(q) - qc_2^i)} > \frac{u'(c_2^j)}{u'(\omega(q) - qc_2^j)}.$$

$c_2^j > c_2^i$ follows immediately from $u'' < 0$, and the budget constraint then gives $c_1^j < c_1^i$. ■

The second lemma, which invokes Lemma 1, says that if absolute risk aversion is nonincreasing then the derivative of consumption with respect to wealth is ordered according to patience.

Lemma 2 (Income Effect Lemma) *If* $(e_1^i, e_2^i) = (e_1, e_2)\ \forall i$ *and* $a(x) \geq a(y)$ *whenever* $x \leq y$*, then*

$$\frac{\partial c_2^1(q, \omega(q))}{\partial \omega} < ... < \frac{\partial c_2^I(q, \omega(q))}{\partial \omega}$$

for any $q > 0$.

Proof. Suppressing superscripts for now and implicitly differentiating with respect to ω the first order condition of an arbitrary agent, $\beta u'(c_2) = qu'(\omega - qc_2)$, we get

$$\frac{\partial c_2}{\partial \omega}\beta u''(c_2) = \left(1 - q\frac{\partial c_2}{\partial \omega}\right) qu''(c_1)$$
$$\Longrightarrow$$
$$\frac{\partial c_2}{\partial \omega} = \frac{qu''(c_1)}{\beta u''(c_2) + q^2 u''(c_1)}.$$

Using the first order condition $q = \beta u'(c_2)/u'(c_1)$ and the definition of $a(x)$, we get

$$\frac{\partial c_2}{\partial \omega} = \frac{1}{\frac{a(c_2)}{a(c_1)} + q}.$$

Since a is a decreasing function, the lemma follows immediately from the ordering of consumption (Lemma 1). ■

The upshot of Lemmas 1 and 2 is that the wealth effect term $\partial c_2^i/\partial \omega$ is monotonically increasing in c_2^i. Choosing agent i with probability π_i, we can write market excess demand as expected excess demand: $\sum_{i=1}^{I} \pi_i (c_2^i(q, \omega(q)) - e_2^i) = E_\pi [c_2^i(q, \omega(q)) - e_2^i]$. Moreover, the monotonic relationship gives us the following covariance result.

Lemma 3 (Covariance Lemma) *If* $(e_1^i, e_2^i) = (e_1, e_2)\ \forall i$ *and* $a(x) \geq a(y)$ *whenever* $x \leq y$*, then for any* $q > 0$

$$cov_\pi\left(\frac{\partial c_2^i(q, \omega(q))}{\partial \omega}, c_2^i(q, \omega(q))\right) =$$
$$\sum_{i=1}^{I} \pi_i \frac{\partial c_2^i(q, \omega(q))}{\partial \omega} c_2^i(q, \omega(q)) - \left(\sum_{i=1}^{I} \pi_i \frac{\partial c_2^i(q, \omega(q))}{\partial \omega}\right)\left(\sum_{i=1}^{I} \pi_i c_2^i(q, \omega(q))\right) \geq 0.$$

Intuitively, why does nonincreasing absolute risk aversion yield this positive relationship between consumption and the strength of the income effect? Since the first order condition is

$$\frac{u'(c_2)}{u'(c_1)} = \frac{q}{\beta},$$

when an agent receives more wealth, optimization entails maintaining the ratio between $t = 2$ and $t = 1$ marginal utility. That is, the agent must adjust consumption so

that the percentage increase in marginal utility is the same in both periods. Absolute risk aversion $a = -u''/u'$ measures the percentage change in marginal utility per unit change in consumption. With declining absolute risk aversion, the most patient agents, who by Lemma 1 have high c_2 and low c_1, must make a large adjustment to c_2 (and a small adjustment to c_1) to maintain the first order condition after receiving new wealth. Thus the covariance (across agents according to the probabilities π_i) between income effects and consumption is nonnegative.

What is the relevance of these lemmas for the slope of demand? As we will see in the proof of Proposition 1, $-(c_2 - e_2)(\partial c_2/\partial\omega)$ is the total income effect in the Slutsky equation for agent i. Therefore, the Covariance Lemma tells us, loosely, that the average income effect is bounded above by the income effect of the "average" agent:

$$\sum_{i=1}^{I} \pi_i \frac{-\partial c_2^i(q,\omega(q))}{\partial\omega}\left(c_2^i(q,\omega(q)) - e_2\right)$$
$$\leq \left(\sum_{i=1}^{I} \pi_i \frac{-\partial c_2^i(q,\omega(q))}{\partial\omega}\right)\left(\sum_{i=1}^{I} \pi_i \left(c_2^i(q,\omega(q)) - e_2\right)\right).$$

When we impose the market clearing price, q^*, average excess demand is 0, implying the market total income effect is negative:

Lemma 4 (Market Income Effect Lemma) *If* $(e_1^i, e_2^i) = (e_1, e_2)\ \forall i$ *and* $a(x) \geq a(y)$ *whenever* $x \leq y$, *then*

$$\sum_{i=1}^{I} \pi_i \frac{-\partial c_2^i(q^*,\omega(q^*))}{\partial\omega}\left(c_2^i(q^*,\omega(q^*)) - e_2\right) \leq 0.$$

Proof. Since the agents have the same endowment, the Covariance Lemma implies $cov_\pi(\partial c_2^i/\partial\omega, c_2^i - e_2) \geq 0$. The lemma then immediately follows from the definition of covariance and market clearing at $q = q^*$, $\sum_{i=1}^{I} \pi_i (c_2^i(q^*,\omega(q^*)) - e_2) = 0$. ■

It follows that demand is downward sloping local to any equilibrium q^*:

Proposition 1 (Downward Sloping Demand) *If* $(e_1^i, e_2^i) = (e_1, e_2)\ \forall i$ *and* $a(x) \geq a(y)$ *whenever* $x \leq y$, *then* $D'(q^*) < 0$.

Proof. For any agent i, the endowment economy Slutsky equation is

$$\frac{dc_2^i}{dq} = \frac{\partial h^i}{\partial q} + \frac{-\partial c_2^i}{\partial\omega}\left(c_2^i - e_2\right),$$

where h^i is Hicksian demand. The first term is the substitution effect. It is strictly negative for all $q > 0$ by the compensated law of demand. The second term, the total income effect, may be positive or negative. However, at an equilibrium q^* the Market

Income Effect Lemma says that the average income effect is negative. Therefore, $D'(q^*) < 0$ follows immediately from averaging dc_2^i/dq across agents and imposing market clearing.[5] ■

In summary, demand is downward sloping local to equilibrium. Separability and $u' > 0$, $u'' < 0$ ensure the market substitution effect is negative. Thus, whether intuitive supply and demand statics apply depends on the market's total income effect. Declining absolute risk aversion puts an upper bound on this total market income effect: the buyer income effect, which has the "correct" sign, has the greatest weight in aggregation. This is because declining absolute risk aversion means buyers have insensitive marginal utility and must make large adjustments to equate marginal utility across goods when optimizing. Market clearing means the upper bound is zero.

Proposition 1 implies a unique equilibrium exists.

Proposition 2 (Uniqueness) *If $(e_1^i, e_2^i) = (e_1, e_2)\ \forall i$ and $a(x) \geq a(y)$ whenever $x \leq y$, then there is a unique equilibrium.*

Proof. Since $\lim_{q\to 0} D(q) = \infty$, we can find $\underline{q} > 0$ such that $z(q) \equiv D(q) - e_2 > 0$ if $q \leq \underline{q}$. Since the budget constraint is $q(c_2 - e_2) = e_1 - c_1$, by applying the same argument to c_1 and $1/q$ we can find $\overline{q}$ such that $z(q) < 0$ if $q \geq \overline{q}$. Therefore, by the intermediate value theorem, we can find $q^* \in [\underline{q}, \overline{q}]$ such that $z(q^*) = 0$. That is, an equilibrium exists. Let Q^* denote the set of equilibrium prices, and let $\underline{q}^* = \inf Q^*$. $\underline{q}^* > \underline{q}$ exists and is an equilibrium because demand is continuous and $z(q) > 0$ if $q \leq \underline{q}$. Thus, there is no equilibrium to the left of $\underline{q}^*$ by definition. By Proposition 1, $D'(\underline{q}^*) < 0$, so if there is an equilibrium to the right of $\underline{q}^*$, it must satisfy $D' \geq 0$ by the continuity of D. This would contradict Proposition 1, so $\underline{q}^*$ constitutes the unique equilibrium. See Figure 1. ■

Finally, let $z(q; e_1, e_2)$ denote market excess demand for $t = 2$ goods at price q and the common endowment (e_1, e_2). Also, let $Q(e_1, e_2)$ denote the equilibrium price corresponding to endowment (e_1, e_2). Q is continuously differentiable by the implicit function theorem, as demand is continuously differentiable. Our final result shows that increasing the supply of $t = 1$ ($t = 2$) goods leads to a fall (rise) in the equilibrium interest rate, in line with supply and demand intuition.

Proposition 3 (Stability) *If $(e_1^i, e_2^i) = (e_1, e_2)\ \forall i$ and $a(x) \geq a(y)$ whenever $x \leq y$, then (i) $\frac{\partial}{\partial e_1} Q(e_1, e_2) > 0$ and (ii) $\frac{\partial}{\partial e_2} Q(e_1, e_2) < 0$.*

[5] Formally, implicitly differentiating the first order condition of an agent we get

$$\frac{dc_2}{dq} = \underbrace{\frac{u'(c_1)}{\beta u''(c_2) + u''(c_1) q^2}}_{SE(q)} + \underbrace{\frac{-\partial c_2}{\partial \omega}(c_2 - e_2)}_{TI(q)}.$$

Thus,

$$D'(q) = \sum_{i\in I} \pi_i SE^i(q) + \sum_{i\in I} \pi_i TI^i(q).$$

By Lemma 4, we have $\sum_{i\in I} \pi_i TI^i(q^*) \leq 0$. Since $SE^i(q) < 0$ by $u' > 0$ and $u'' < 0$, it follows that $D'(q^*) < 0$.

Proof. First, note that for any q, if $e_1' > e_1$ then $z(q; e_1', e_2) > z(q; e_1, e_2)$, so $\partial z/\partial e_1 > 0$. This follows from $\partial c_2^i/\partial \omega = (a(c_2^i)/a(\omega(q) - qc_2^i) + q)^{-1} > 0$ for all i, which we derived in the proof of Lemma 2. Also, we know that

$$\frac{\partial c_2^i}{\partial e_2} = q\frac{\partial c_2^i}{\partial \omega} = \frac{q}{\frac{a(c_2^i)}{a(\omega(q) - qc_2^i)} + q} < 1.$$

Therefore,

$$\frac{\partial}{\partial e_2}\left(c_2^i - e_2\right) = \frac{\partial c_2^i}{\partial e_2} - 1 < 0,$$

so $\partial z/\partial e_2 < 0$. Since $z_q(Q(e_1, e_2); e_1, e_2) < 0$ by Proposition 1, it follows that $\frac{\partial}{\partial e_1}Q(e_1, e_2) > 0$ and $\frac{\partial}{\partial e_2}Q(e_1, e_2) < 0$, as we see graphically in Figure 1. ■

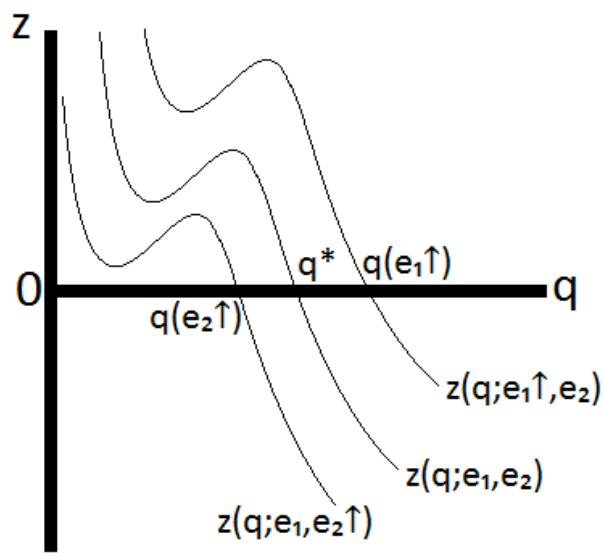

Figure 1: The figure shows market excess $t = 2$ demand, $z(q; e_1, e_2)$, as a function of the $t = 2$ goods price, q. Increasing e_1 (e_2) shifts z up (down) and leads to an increase (decrease) in equilibrium q.

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