Comment

John Geanakoplos
YALE UNIVERSITY

The rapidly growing field called 'general equilibrium with incomplete markets' (GEI) has already made fundamental contributions in three different areas of economic theory. First, it has thrown up mathematical problems so delicate that they have forced economists for the first time to abandon that old war-horse – Brouwer's fixed point theorem – and to invent (or borrow from mathematics) a new methodology for constructing existence proofs for general equilibrium. Second, it has demonstrated a significant difference between real and financial assets. Third, it has, to my mind, greatly increased the presumption against Pareto efficiency of the market process. The model appears to be flexible and rich enough to include many of the fundamental paradigms of finance and macroeconomics.

David Cass has been a pioneer in developing the GEI theory, and indeed he has already made fundamental contributions in at least two of the areas named above. Cass (1984) reformulated the definition of GEI equilibrium, suppressing the portfolio notation, and bringing the accounting in GEI much closer to that in conventional general equilibrium (GE) models. This reformulation led to a quick and elegant proof of the existence of equilibrium for purely financial assets. (It could also have been used to give an alternative proof of the existence of *numéraire* asset equilibria.) The Cass accounting was an essential ingredient in the concept of a pseudo-equilibrium, which is the object of analysis by all the new mathematical techniques that have been brought to bear on the subject.

Cass (1985) was also the first to present an example of an economy, with two 'states of nature', but only one asset, promising one 'dollar' in both states, in which there is a continuum of equilibria. This example led to the general theory of real indeterminacy with financial assets.

I do not have time to give more examples of Cass's contributions to GEI, but let me say that we all eagerly await further discoveries from him.

In the next few pages I sketch four of the many open problems in GEI analysis. To give some concrete idea of the potential power and future evolution of the GEI model, I will try to indicate a specific approach to coping with the last problem. Before proceeding to future developments, let me briefly consider the meaning of the established inefficiency results for GEI equilibrium.

It is a very old idea – which, for example, my adviser Kenneth Arrow often repeated – that when markets are missing, competitive equilibrium should not be expected to be Pareto optimal. There may be a useful role for the government or other institutions to act in place of the missing markets. The GEI model strengthens the role for intervention in a non-trivial way. In the Arrow–Debreu world, if a market is missing, a central planner can improve the final allocation, but only in effect by *replacing* the missing market. An illuminating real world example is the so-called 'operation bubble' that is in place in some mid-western American states, where polluters must buy the right to pollute the air on an artificial government-run market. In the Arrow–Debreu model, if the planner has no ability to affect the allocation of goods that would have been traded on the missing market, then it should not intervene at all. If, for example, apples cannot be traded on the market, there is no reason for the government to induce firms to change the mix of pears and oranges they are selling. In the GEI model this is not the case. If some kinds of 'risk' cannot be traded, then the government indeed ought to intervene and induce consumers and producers to alter their trades of the existing assets.

UNSOLVED PROBLEMS

The incomplete markets approach is still comparatively young, and a host of open problems remain. Let me mention four of them. First, there is no doubt that the financial assets posited in GEI models leave money very sketchily modelled. Several authors have suggested that if money were properly modelled, then the indeterminacy would disappear. By properly modelled, these authors presumably mean that money should be given an explicit transactions role, as in the cash-in-advance constraints of Clower (1967), Lucas (1980), and so on. The same qualification presumably applies also to the indeterminacy results with several currencies ('exchange-rate' indeterminacy).

Indeed, in several of the cash-in-advance models formulated by Lucas, Svensson (1985) and others, there is no indeterminacy.

An important feature of many of these cash-in-advance models is the hypothesis of a representative agent. Moreover, the way the models are typically rigged, the velocity of money is fixed at one, so a crude quantity theory of money must prevail. The question is, to what extent do these two special features bias the model in favour of determinacy, or even optimality? I propose embedding the cash-in-advance constraints inside the GEI model. I conjecture that if a bank is added to the model, and if loans from the bank can be either short-term or long-term, then a representative agent solution will not give an accurate portrayal of the multi-agent general equilibrium. In any case, further work in demystifying money in the GEI model is certainly desirable.

Second, the new proofs that have been given for the existence of GEI equilibrium are very attractive, but they do not rule out the possibility that a more conventional fixed-point approach might ultimately also generate the existence of GEI equilibrium. In particular, I believe the potential for using liquidity constraints (as in problem one) in constructing existence proofs has so far been underestimated. Even the smallest liquidity 'tax' guarantees the existence of equilibrium through a standard, elementary fixed-point argument. It would be very informative to understand the structure of the set of allocations that are the limits of liquidity tax equilibria as the tax grows arbitrarily small.

Third, the proposition that a central planner could make welfare improvements without introducing new assets, and without knowing anything about the state of nature, presumes that the planner does know the preferences of the agents. One would like a practical guide to how much, and precisely what, the planner must know about preferences in order to effect the improvements.

Which assets should be assumed to exist and which not? The failure so far of the GEI approach to provide a rigorous model yielding an explanation is regarded by some as a death-blow to the whole enterprise. That seems to me an entirely unjustified conclusion. In the first place, to the extent that the GEI theorems hold *no matter what* the asset structure (provided that it is incomplete, that there are many individuals with different preferences, and that the endowment allocation is not Pareto optimal), the objection loses force. Second, a theory can be useful long before all its foundational

questions are settled. For example, the standard theory of risk aversion based on taking expectations of concave utilities still cannot account for the widespread presence of gambling and lotteries. Third, and most importantly, the time is just now becoming right to develop such a model.

An important first step to explaining which assets are present would be to enlarge the GEI model to account for the possibility that agents default on their promises to deliver. Default and bankruptcy of course are widespread in the market. There is a natural tendency to regard them as disequilibrium phenomena, but I believe that a simple extension of the GEI model can bring them within the reach of equilibrium theory.

Default is a quintessential general equilibrium phenomenon. In a world in which promises can exceed physical endowments, each default can begin a chain reaction. A creditor in one market where payment does not occur is deprived of the means of delivery in another market where he is the debtor, thereby causing a further default in some other market, etc. The indirect effects of default might be as important as the direct effects, but they are missed in partial equilibrium models.

The GEI model, suitably extended, seems to me the ideal vehicle for analysing the system-wide consequences of default. Of course, allowing for strategic default creates a host of modelling questions. What informational assumptions should be made? Is perfect competition consistent with default? How should agents who default be punished? Consider the following bare-bones approach (following research I am doing with P. Dubey and M. Shubik, 1989).

Time lasts two periods. In the last period there are S states of nature. As in the conventional Arrow–Debreu model, agents $h = 1, \ldots, H$ are characterised by endowments $e^h \in \mathbb{R}^L_+ \times \mathbb{R}^{SL}_+$ and utilities $u^h \colon \mathbb{R}^L_+ \times \mathbb{R}^{SL}_+ \to \mathbb{R}$. But in addition there are J assets exogenously specified. Each asset j is represented by an SL-dimensional vector A^j denoting the 'promised' delivery of commodities sl, $s = 1, \ldots, S, l = 1, \ldots, L$. Agent h can purchase any amount θ^h_j or sell any amount φ^h_j of asset j at the price π_j. By selling φ^h_j units of the asset, the agent obliges himself to deliver $A_{slj}\varphi^h_j$ of commodity sl. He may choose to deliver only D^h_{slj}, but then he must pay the penalty $\mu^h_{slj} [A_{slj}\varphi^h_j - D^h_{slj}]^+$, where the μ are exogenously given, and $[x]^+$ denotes max $(x, 0)$. The penalty is extra economic. It can be thought of as days in jail, as opposed to an economic penalty like being barred from using asset markets in the future. An agent h who purchases θ^h_j

units of an asset cannot expect that he will receive payment in full. Let K_{slj} be the proportion of promised deliveries of good sl that he expects to receive from asset j. We can write the conditions for a GEI_μ equilibrium as follows.

We call $(p, \pi, (x^h, \theta^h, \varphi^h, D^h)_{h=1}^H, K)$ a GEI_μ for the usual economy $((u^h, e^h)_{h=1}^H, A)$ where $A \geq 0$, $\mu = (\mu^h)_{h=1}^H$, $\mu^h \in \mathbb{R}_{++}^{SLJ}$, $K \in \mathbb{R}_+^{SLJ}$, $0 \leq K_{slj} \leq 1$, $D^h \in \mathbb{R}_+^{SLJ}$, if

(A_μ) $\sum\limits_{h=1}^{H} (x^h - e^h) = 0$

(B_μ) $\sum\limits_{h=1}^{H} (\theta^h - \varphi^h) = 0$

(C_μ) $(x^h, \theta^h, \varphi^h, D) \in B^h(p, \pi, K) = \{(x, \theta, \varphi, D) \in \mathbb{R}_+^L \times \mathbb{R}_+^{SL} \times \mathbb{R}_+^J \times \mathbb{R}_+^J \times \mathbb{R}_+^{SLJ} \mid p_0(x_0 - e_0^h) + \pi(\theta - \varphi) \leq 0$ and $\forall s = 1, \ldots, S, \quad p_s(x_s - e_s^h) + p_s D_s \leq p_s(K_s \times A_s)\theta\}$ for $h = 1, \ldots, H$, where $K \times A$ means the coordinate by coordinate multiplication $(K \times A)_{slj} = K_{slj} \cdot A_{slj}$

(D_μ) If $(x, \theta, \varphi, D) \in B^h(p, \pi, K)$ then

$$u^h(x^h) - \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{l=1}^{L} \mu_{slj}^h [A_{slj}\varphi_j^h - D_{slj}^j]^+$$

$$\geq u^h(x) - \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{l=1}^{L} \mu_{slj}^h [A_{slj}\varphi_j - D_{slj}]^+, \text{ for } h = 1, \ldots, H$$

(E_μ) $K_{slj} = \dfrac{\sum\limits_{h=1}^{H} D_{slj}^h}{A_{sjl}\sum\limits_{h=1}^{H} \varphi_j^h}$ if denominator > 0

Note that in (C_μ) agents expect to receive only the percentage K_{slj} of the promises to deliver good sl by asset j. By condition (E_μ) these expectations are rational, and perfect competition prevails despite the defaults. We must think of all the default risk on each asset being pooled among the holders of the asset. This will tend to be true if there is a great deal of financial intermediation.

Note finally that agents choose what to deliver, and how much to default. There is no agency that can garnish commodities, or force agents to deliver. The penalties μ are the only incentives. Despite all these extreme hypotheses, I believe that such a model can shed a great deal of light on bankruptcy/default law, as well as being a prelude to modelling the endogenous formation of assets.

Consider the question why some societies are less severe in punishing defaulters than others. Laxness in punishing defaulters results in more default. There are non-economic explanations for this gentleness (the punishment should fit the crime, etc.). But is there an economic rationale for encouraging some default?

There are three drawbacks to intermediate or low values of μ. First, default results in penalties, which are a deadweight loss. Second, lenders (that is, buyers of assets), rationally anticipating the increased chances of default, will be less willing to lend. Third, borrowers may choose not to deliver even when they are able. The economic advantage of allowing default is that is improves the allocation of risk. Agents who default alter the effective assets. The span of the new assets is different from the original span, and may be better.

There is a second sense in which the span may be better. In GEI, there is no point in an agent going long and short in the same asset, for by doing so he effectively undoes with one hand what he has done with the other. But when default is allowed, going long and short in the same asset can sometimes double the asset span. Old Polonius's advice, 'neither borrower nor lender be' should be stood on its head: 'both borrower and lender be', if default is possible.

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