Probability and Prodigality

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Abstract
I present a straightforward objection to the view that what we know has epistemic probability 1: when combined with Bayesian decision theory, the view seems to entail implausible conclusions concerning rational choice. I consider and reject three responses. The first holds that the fault is with decision theory, rather than the view that knowledge has probability 1. The second two try to reconcile the claim that knowledge has probability 1 with decision theory by appealing to contextualism and sensitive invariantism, respectively. I argue that each response fails, and that we can hold on to much of what was attractive in the responses while denying that what we know has probability 1.

1 Introduction
There seem to be systematic relations between knowledge and rational belief on the one hand, and rational decision on the other. If you know that it will rain, then if you are rational you will carry an umbrella. The more reason you have to believe that a bridge is unstable, the more reason you have not to walk across it. Insofar as we are interested in both epistemology and practical rationality, we will want some explanation of how they bear on one another.

One approach that seems to promise a powerful and unified explanation of these relations is that of Bayesian decision theory. According to Bayesian decision theory, agents ought to maximize expected utility. The relevant notion of expected utility, however, is defined relative to a probability function, and this seems like a promising place to try to forge a connection between decision theory and epistemology.

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A natural thought is that the probability functions relevant to rational choice are somehow sensitive to evidence. Suppose I am betting on the outcome of a coin toss, and the coin to be tossed is, unbeknownst to me, double-headed. If I have no reason to believe that it is double-headed, and in fact my evidence suggests that it is a fair coin, then I would plausibly be rationally required to accept a low-stakes bet at 1:2 odds on tails. This shows that the notion of probability relevant to rational choice is not objective physical probability; even though the objective physical probability of my winning the bet was zero, my decision was rationally required.

Orthodox subjective Bayesians take examples like the one above to show that the notion of probability relevant to rational choice is not objective chance, but instead subjective doxastic probability.\textsuperscript{1} While this suggestion is an improvement over the idea that objective probability is the sort of probability that is relevant to rational choice, it still faces serious obstacles. Suppose I am completely convinced that I am the target of a Martian conspiracy. Acting on this belief, I purchase large quantities of tin foil, so as to fashion hats that will render me invulnerable to Martian mind-scrambling rays. Even if my beliefs are probabilistically coherent, there is a perfectly natural, intuitive sense of “irrational” in which these beliefs and the actions based on them are irrational.\textsuperscript{2}

If we want to hold on to the idea that buying large quantities of tinfoil would be irrational in some sense, and we want to use decision theory to provide an account of rational choiceworthiness in this sense, then we need an evidence-sensitive notion of probability which is identical neither to objective physical probability, nor to subjective doxastic probability. Given such a notion of probability, we can then hold that agents are rational when they act so as to maximize expected utility relative to that probability function. Let us preliminarily assume that there is such a notion of probability, and let us call it “epistemic probability.”\textsuperscript{3}

In this paper, I will argue against a view about the connection between epis-
meric probability and knowledge. According to this view, what an agent knows has epistemic probability 1 for that agent. There are a number of ways one might be led to such a view. One way involves reflecting on the impropriety of “concessive knowledge attributions” such as the following:

(1) I know that I will always be too short to play in the NBA, but it is possible that I will have a dramatic mid-life growth spurt and be recruited by the Bulls.

(2) I know that I will never be able to afford a private jet, but there is some chance that I will win the $1,000,000,000 jackpot in the MegaBucks lottery.

There are many potential explanations for the impropriety of such knowledge attributions, but some explanations will involve the claim that what an agent knows has epistemic probability 1 for that agent. Suppose we take the impropriety of such attributions to indicate that they are false. If “chance” in (2) refers to epistemic probability, or if a proposition’s being possible in the sense of (1) requires that it have non-zero epistemic probability, then the claim that knowledge has epistemic probability 1 could explain why claims like (1) and (2) must be false.

Alternatively, we might accept that knowledge has epistemic probability 1 on other grounds. If we hold that an agent’s epistemic probabilities are obtained by conditionalizing some prior probability function on the conjunction of all the propositions the agent knows—as Williamson (2000) argues—then it is an immediate consequence that any proposition an agent knows must have epistemic probability 1 for that agent.

In the remainder of this paper I won’t be concerned with how we might motivate the claim that knowledge has epistemic probability 1, but instead with the consequences of this claim for our views about practical rationality. In §2, I will argue that the view that knowledge has probability 1 leads to unacceptable conclusions about rational choice. In §3, I will consider and reject a response from Timothy Williamson, according to which the blame for these conclusions should not be laid on the link between knowledge and probability 1, but instead on expected utility.

4 The phrase “concessive knowledge attribution” is from Rysiew (2001).

5 I will often drop “for an agent” and “epistemic,” and will just refer to this claim as the claim that knowledge has probability 1. Since epistemic probabilities are always agent-relative, and I won’t be talking about non-epistemic probability in the remainder of this paper except when I explicitly indicate otherwise, such omissions shouldn’t create confusion.

6 That such attributions are false is by no means uncontroversial. Rysiew (2001) holds that they are true, but inappropriate to assert on pragmatic grounds.

7 Williamson uses the term “evidential probability” rather than “epistemic probability,” but I have followed standard usage and gone with the latter. This may mark an important distinction, since as will become clear later in the paper, Williamson doesn’t think that his notion of evidential probability links up with decision theory in the way I have said epistemic probability should.
maximization; according to this response, we should reject decision theory as an all-purpose guide to rational choice, rather than rejecting the association between knowledge and probability 1. In §4, I will consider responses that try to reconcile decision theory with the claim that knowledge has probability 1 by appealing to contextualism or subject sensitive invariantism (SSI) about knowledge.\(^8\) I will argue that while such views may avoid the problem identified in §2, they do so only by generating other unacceptable conclusions about rational choice. Lastly, in §5, I will consider a response on behalf of the sensitive invariantist that will lead into more general issues about idealization in epistemology and decision theory.

2 The Prodigality Problem

If skepticism is false, then we know quite a lot. If skepticism is false and knowledge has epistemic probability 1, then quite a lot of propositions have epistemic probability 1. It is easy to see why we might worry about this—orthodox decision theory famously does some odd things when probability 1 is too easy to attain.\(^9\) The desire for a model of probabilistic updating in which updating doesn’t involve assigning any new propositions probability 1 was the main motivation behind Jeffrey’s (1965) generalization of strict conditionalization. Williamson (2000, chapter 10) stresses the problems associated with the fact that in orthodox decision theory, once a proposition has probability 1, it retains that probability for all time. This problem, along with many others associated with allowing propositions to have maximal probability, can be avoided by taking conditional probability as primitive and allowing for conditionalizing on probability zero events.\(^10\) However, the problem I will focus on in this paper cannot be solved by such methods.

The problem I will focus on is that, when combined with other plausible claims about knowledge, the claim that knowledge has probability 1 leads to implausible claims about rational choice. The following example will bring out the worry.

Suppose I read in the most recent edition of the *Encyclopedia Britannica* that the 2010 estimate of the population of Canada is just over thirty four million.\(^11\) Suppose that this is true, and that the actual population of Canada is quite close to the estimate. Given plausible, non-skeptical assumptions about knowledge,


\(^9\)Hájek (Draft) considers 10 apparent problems with liberal assignments of maximal probability, though he argues that they can each be circumvented. Easwaran (Draft) takes a similar line.

\(^10\)This is the strategy favored by Hájek (Draft) and Easwaran (Draft). Williamson (2000, chapter 10) opts for a different route.

\(^11\)Full disclosure: I actually read it in *Wikipedia*. 
in such a situation I know that the population of Canada is between thirty and forty million. Now suppose I am offered a bet by a credible bookie that will pay one cent if the population of Canada is between thirty and forty million, and which will bankrupt me if Canada has a population of fewer than thirty or greater than forty million. Given more controversial assumptions about knowledge to the effect that whether one knows that $P$ doesn’t depend on the practical significance of $P$, (assumptions which will be relaxed in later sections of this paper), I can still know that Canada’s population lies between thirty and forty million, even after being offered such a bet. Intuitively, however, it would be irrational for me to accept this bet—given the payoffs, I shouldn’t do so, even though it is extremely probable in light of my evidence that I will win if I take it. The view that knowledge has probability 1, however, cannot account for this fact about rational choiceworthiness, at least when combined with the assumptions about knowledge mentioned above.

The argument that I ought to accept the bet if knowledge has probability 1 and the aforementioned assumptions about knowledge hold is straightforward. While I spell it out in detail in a footnote, the basic idea is that if I know that the population of Canada is in the relevant bounds, and that I will win the bet if the population is in the relevant bounds, then the epistemic probability that I will win if I accept the bet is 1. And if this is the case, then the expected utility of accepting the bet is equal to the utility of winning the bet: the disutility of losing the bet doesn’t affect its expected utility at all.\footnote{Let $P$ represent my epistemic probability function, let $H$ be the proposition that the population of Canada is between thirty and forty million, let $A$ be the proposition that I accept the bet, let $W$ be the proposition that I win the bet, and let $L$ be the proposition that I lose the bet. Lastly, let $U$ be my utility function. I assume evidential decision theory for simplicity.

1. Expected utility of $(A) = P(W \mid A)U(W) + P(L \mid A)U(L)$ (by the definition of expected utility, together with assumptions that $P(W \lor L \mid A) = 1$, and $P(W \land L) = 0$.

2. $P(W \leftrightarrow H \mid A) = 1$ (Because I know that the bookie is telling the truth about the conditions of the bet)

3. $P(W \mid A) = P(H \mid A)$ (from 2)

4. $P(H) = 1$ (Because I know $H$)

5. $P(H \mid A) = 1$ (from 4)

6. $P(W \mid A) = 1$ (From 3 and 5)

7. $P(L \mid W) = 0$

8. $P(L \mid A) = 0$ (From 6 and 7)

9. Expected utility of $(A) = U(W)$ (From 1, 6, and 7)

Note that the expected utility of accepting the bet depends only on the utility of winning, and not at all on the disutility of losing.
Call this apparent result—that the defender of the view that knowledge has probability 1 is committed to the view that one is rationally required to accept high-stakes bets on propositions one knows no matter how unfavorable the odds—the prodigality problem. Perhaps it seems odd to call this a special problem for the view that knowledge has epistemic probability 1—orthodox subjective Bayesian agents could easily conditionalize on claims about Canada’s population and claims about the behavior of bookies in ways that would lead to their being committed to betting at any odds on claims about Canada’s population. But as mentioned earlier, the orthodox subjective Bayesian isn’t trying to explain constraints on rational choice that go beyond formal coherence constraints—she doesn’t think I am irrational if I blow the bank buying tin-foil to prevent Martian mind-scrambling, so long as such actions maximize expected utility relative to my subjective doxastic probability function. If we think there are further constraints, and we want to use decision theory to shed some light on them, then the orthodox subjective Bayesian isn’t a very helpful companion in guilt for the defender of the view that knowledge has epistemic probability 1.

Before considering responses to the prodigality problem, it is important to be clear on exactly what is alleged to be problematic. The worry isn’t that, if knowledge has probability 1 and some further assumptions hold, there are situations where we ought to take high-stakes bets at extremely unfavorable odds. Plausibly, there are at least possible cases where I should bet my life against a penny, if the proposition that I am betting on is probable enough. It is hard to imagine any attractive theory of rational choice that wouldn’t have this consequence—no matter how bad the potential downside of a bet, and how small the potential upside, if we hold these fixed, then we should be able to adjust the probabilities so that taking it is the rational thing to do, even without letting the proposition one is betting on have maximal probability.

Rather, the problem is that in everyday cases, once we hold fixed certain aspects of the situation—e.g., that I recently read the Encyclopedia Britannica, that I don’t have any defeating evidence for the claim that the population of Canada is in the relevant bounds, etc.—I ought to take certain high-stakes bets at extremely unfavorable odds. Plausibly, there are at least possible cases where I should bet my life against a penny, if the proposition that I am betting on is probable enough. It is hard to imagine any attractive theory of rational choice that wouldn’t have this consequence—no matter how bad the potential downside of a bet, and how small the potential upside, if we hold these fixed, then we should be able to adjust the probabilities so that taking it is the rational thing to do, even without letting the proposition one is betting on have maximal probability.

One potential line of response to the problem is a thoroughgoing skepticism about knowledge according to which we never know very much at all; for instance,

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13Unless, perhaps, infinite disutilities are at stake.
one might hold that when it comes to contingent propositions, the only ones we can know (if any at all) concern our present mental states. Such a position would avoid being committed to the view that we ought to accept bets at any odds, since while claims about one’s present mental states would have epistemic probability 1, claims about the population of Canada, or the betting behavior of bookies, will not. While this line of response would avoid the problem, it would do so at the considerable cost—it is a highly revisionary epistemological view. If the claim that knowledge has probability 1 is only plausible if we know very little, then it is less plausible than we might have hoped.

In the remainder of the paper, I will consider two lines of response to the prodigality problem that defenders of a link between knowledge and probability 1 might offer. The first response is due to Williamson (2000, 2005b, 2009). As I interpret him, he maintains that while we should hold onto the idea of epistemic probabilities as an epistemological notion—roughly, they represent the degree to which one’s evidence supports a hypothesis—we should reject the idea that they will combine in systematic ways with utilities to govern rational choice in the manner specified by decision theory. So we can agree that everyday propositions often have epistemic probability 1 in some sense without committing ourselves to any absurd conclusions about radical choice. Since this response gives up on decision theory as a guide to understanding choiceworthiness, it would be disappointing if we were forced to it. I will argue that we are not.

The second response involves allowing that while we may know propositions about, e.g., the population of Canada, before we are offered high-stakes bets on them, things change once we are offered such bets. So we can hold on to the link between probability 1 and knowledge, and the idea that decision theory is a guide to rational choiceworthiness, without committing ourselves to endorsing prodigal betting behavior. I will argue that while this response avoids the prodigality problem, it faces other serious objections.

14 Exactly how this response works will differ depending on whether we are discussing the sensitive invariantist, or contextualist version of this response.

15 While this response is inspired by Hawthorne (2004) and Stanley (2005), I don’t claim that they would endorse it. Hawthorne (2004, p.137) in particular seems to want to distance himself from broadly decision-theoretic approaches to rational choice, so I suspect that he would endorse the spirit of Williamson’s response to the prodigality problem.

16 There are other possible responses that I take seriously, but which I won’t consider here. See the discussion of “objection 7” in Hawthorne and Stanley (2008). Also, see “Unreasonable Knowledge,” by Maria Lasonen-Aarnio (2010). Lasonen-Aarnio might argue that while knowledge has probability 1 and acting so as to maximize expected utility is rational, such action may nevertheless be unreasonable, in a sense that she explicates.
3 Is Prodigality Everyone’s Problem?

It may seem as if no particular view about the connections between epistemic probability and knowledge is necessary to generate the prodigality problem. The assumption that rational agents maximize expected utility may seem to generate the problem all by itself. I take this to be the idea motivating the following comments of Williamson’s:

We should question the association between evidential probability 1 and absolute certainty. For subjective Bayesians, probability 1 is the highest possible degree of belief, which presumably is absolute certainty. If one’s credence in $P$ is 1, one should be willing to accept a bet on which one gains a penny if $P$ is true and is tortured horribly to death if $P$ is false. Few propositions pass that test. Surely complex logical truths do not, even though the probability axioms assign them probability 1. (2000, p. 213)

In replying to commentators on *Knowledge and its Limits*, Williamson (2005b, p.478) reiterates the point, claiming that “no decision theory based on expected utility, calculated according to the standard axioms and definitions of mathematical probability theory, will be everywhere consistent with what pretheoretic common sense predicts a sensible person would do.” His defense of this claim appeals to another example involving complex logical truths.

If Williamson really is right that the assumption that it is rational to maximize expected utility is enough to commit us to prodigality all on its own, then rejecting this assumption may be our best option. Still, a defender of decision theory who still wanted to press the prodigality problem against the view that knowledge has probability 1 might bite the bullet here, and hold that while betting the farm on contingent (but known) truths is irrational, betting at any odds on logical truths is rationally required.\textsuperscript{17}

After all, rejecting the link between expected utility maximization and choiceworthiness is a serious price to pay—once we do so, we lose out on a promising candidate explanation of the systematic connections between evidence and rational action. While Williamson’s remarks may suggest that it is just the identification between probability 1 and rational willingness to bet at any odds that we must reject, it is not clear whether we can maintain any general connection between expected utility and rational choiceworthiness once this link is severed. For instance, if we accept that 0.5 epistemic probability is associated with rational willingness to bet at \textit{even} odds, must we think that the relationship between choiceworthiness

\textsuperscript{17}As I understand Kaplan (2009, p. 136, note 22), this is his position.
and expected utility changes at some point between probability 0.5 and probability 1, or perhaps that there is a discontinuity at probability 1? Neither of these options is attractive.

The above objection doesn’t require that epistemic probabilities be identified with rational betting odds. While it is plausible that this link holds ceteris paribus, we should allow that it is severed when, e.g., agents have aversions to betting, or have other desires that bear on the attractiveness of accepting a bet independently of its potential payoff, as Ramsey (1931) notes. When ceteris is paribus, however, the link should hold, and this is all that is necessary to support the objection to the view that knowledge has probability 1.

Still, perhaps we should regard decision theory as applicable at best only in special cases, and should instead adopt a conception of practical rationality that takes something like Aristotelian practical syllogism as the paradigm of rational practical deliberation, and which shunts probabilistic notions to the sidelines; we might hold some version of the view that it would be rational for an agent to φ when she she knows the premises of a valid deductive argument whose conclusion is that she should φ.

For instance, we might follow Williamson (2005a) and hold that one often does know propositions when one is offered bets on them at extremely unfavorable odds, but avoid prodigality by holding that when one is making high stakes bets that depend on whether P, it is not enough to know that P in order to reasonably rely on P in one’s practical reasoning. Instead, as the stakes go up, one must have increasingly many iterations of knowledge that P for it to be reasonable to act as if P—one must know that one knows, or know that one knows that one knows, and so on. Since one will generally not have arbitrarily many iterations of knowledge, in cases of very high-stakes bets on a proposition P, one will typically not have enough iterations of knowledge in order to reasonably act on the assumption that P.

In the absence of some account of how the magnitude of the stakes in a situation fixes some level of iterations of knowledge that one must have in order for it to be reasonable to act as if some proposition is true, this approach is less elegant than Bayesian decision theory. There isn’t anything analogous to the notion of expected utility to give a systematic story about how epistemic and practical factors combine to generate facts about choiceworthiness. While I take this to be an unhappy position that gives up on much of the promise of decision theory, if decision theory commits us to prodigality, then we might reasonably retreat to something like the Williamsonian position sketched above.
3.1 Prodigality and Bridge Principles

Luckily, we can avoid prodigality without rejecting decision theory. Suppose we accept decision theory and the probabilistic framework associated with it—in particular, we accept that logical truths have epistemic probability 1.\textsuperscript{18} Nothing immediately follows about what bets we must make. Suppose I am offered a bet that is purported to pay off just in case it will rain tomorrow or it’s not the case that it will rain tomorrow. Call the proposition that it will rain tomorrow or it’s not the case that it will rain tomorrow—a logical truth if there ever was one—“Truth.” Let Accept be the proposition that I accept the bet on Truth. Let Win be the proposition that I win the bet. For it to be rational for me to accept the bet no matter the odds, it is not enough that Truth has epistemic probability 1.

What is also required is that the following probability be maximal: $P((\text{Win} \leftrightarrow \text{Truth}) \mid \text{Accept})$. Equivalently, it is required that $P(\text{Accept} \land \text{Truth} \land \lnot \text{Win}) = 0$. Put in the language of certainty, it isn’t enough that I be certain that it’s either going to rain or not in order for me to bet at any odds—I must also be certain that betting will have a favorable outcome if and only if it is either going to rain or not. But this claim—a claim about the result of handing some money to a bookie—is exactly the sort of contingent claim that a defender of decision theory can consistently be quite hesitant to assign probability 1.

Put more generally, logical truths on their own have no implications about which actions are more promising than others.\textsuperscript{19} Assigning probability 1 to logical claims only warrants prodigal betting behavior if bridge claims linking logical truths with empirical propositions also have probability 1. But nothing in the

\textsuperscript{18}This is a bit quick. There have been attempts to model logical ignorance in a Bayesian framework: see especially Garber (1982). However, while Garber’s approach does allow for logical equivalent empirical hypotheses to be assigned different probabilities, it doesn’t allow purely logical truths to be assigned sub-maximal probabilities. Generally, I am sympathetic to Williamson’s claim that “one could try to construct a non-standard probability calculus in which truth-functional tautologies can have probability less than 1, but such modifications tend to make huge sacrifices in mathematical power for tiny gains in psychological realism.” (Williamson, 2005b, p.478). However, later in this section I briefly discuss a different approach to the problem of logical omniscience, which doesn’t involve assigning logical truths sub-maximal probabilities.

\textsuperscript{19}I’m making some assumptions here about what counts as an action for the purposes of decision theory. In particular, I’m assuming that actions are the sorts of things one can in some sense “directly” perform, so that [accept a bet that will pay $1 if P $\lor$ $\lnot$ P] will not count as an action, but [utter “I’ll take the bet”], or perhaps [form an intention to utter “I’ll take the bet”] will. See Lewis (1981, p.7), Joyce (1999, p.57), and Hedden (Forthcoming) for views along these lines. Without this assumption, logical truths would have implications about which actions are more promising than others—[accept a bet that will pay $1 if P $\lor$ $\lnot$ P] would have to be more promising than [accept a bet that will pay $1 if P $\land$ $\lnot$ P]. I suspect my arguments could be reworked without this assumption, but matters would become more complicated.
probability calculus, and nothing in decision theory, requires that these bridge claims should have maximal probability. The view that knowledge has probability 1, on the other hand does require that bridge claims often have maximal probability, at least when it is married to views about knowledge according to which one can know everyday propositions even while being offered arbitrarily unfavorable bets. So while decision theory alone does not lead to the prodigality problem, decision theory, the claim that knowledge has probability 1, and some plausible assumptions about knowledge together do.

Before considering some ways of trying to reconcile the knowledge/probability 1 link with decision theory by adopting alternative theories about knowledge, I will consider two objections to my claim that decision theory alone doesn’t lead to prodigality.

3.1.1 Objection 1: Even Contingent Claims Must Have Probability 1

My defense of decision theory from prodigality relied on the claim that while decision theory requires logical truths to have probability 1, it doesn’t require bridge principles linking claims about logical truths to claims about betting outcomes to have probability 1, as such principles are not themselves logical truths. It might be objected, however, that this merely postpones the problem—in some cases, decision theory requires that even propositions that are contingent, non-logical truths (or even falsehoods), have probability 1; in particular, this is so in cases where the event space over which probabilities are divided is infinite. Williamson (2007) argues that if a coin is to be flipped infinitely many times, the probability of any particular outcome (e.g., the coin landing heads every time) is zero, even though that such an outcome should occur is logically possible. Assuming his argument is sound, must a decision theorist bet at arbitrarily unfavorable odds that a coin flipped infinitely many times will not land heads every time?

Perhaps, but not obviously. By the argument earlier in this section, this will not lead to prodigality, as long as crucial bridge claims do not have probability 1. And even if considerations about, e.g., zero measure sets, force us to grant that some contingent a posteriori claims have epistemic probability 1, such considerations do not force us to grant that the relevant bridge claims must have probability 1. For instance, in a case where someone is offered the chance to bet on the outcome of an infinite sequence of coin flips, nothing in decision theory forces the epistemic probabilities of bridge claims linking the bookie’s behavior with the outcome of the coin tosses to get probability 1. Still, I am willing to grant that one could design an infinitary example in which the epistemic probability of contingent claims about some bookie’s behavior are forced all the way to 1. In such a case, decision theory would tell us to bet on any odds against a possible (albeit probability zero) outcome. Is this an unwelcome result?
Not obviously. Betting at arbitrarily unfavorable odds in ordinary, humdrum cases involving topics like the population of Canada is clearly irrational. But how one ought to behave in complex cases crucially involving infinities is far from apparent—my intuitions don’t speak strongly about such cases, and if they did, I would regard them with skepticism, since we have independent evidence that intuitions aren’t very reliable when it comes to infinity. The defender of decision theory, then, may adopt the attitude recommended by David Lewis concerning certain putative infinitary counterexamples to analyses of causation:

I do not worry about... these far-fetched cases. They... go against what we take to be the ways of this world; they violate the presuppositions of our habits of thought; it would be no surprise if our common-sense judgments about them therefore went astray—spoils to the victor! (Lewis, 1986c)

The defender of decision theory can go on to add that if the war is fought on other ground, then decision theory is likely to win, given the powerful and systematic explanations it can provide of so many phenomena concerning rational choice.

3.1.2 Objection 2: Focusing on Bridge Principles Misidentifies the Problem

I have argued that even if logical truths have epistemic probability 1, decision theory doesn’t recommend that one bet on them at arbitrarily unfavorable odds. But it may seem that this addresses the letter of the prodigality problem without addressing the spirit. One might object that the reason we oughtn’t bet on logical truths at arbitrarily unfavorable odds isn’t that we can’t be sure whether the bookie will pay up, but is instead that we can’t be certain in the logical truths themselves.20 If the epistemic probability of a proposition is supposed to reflect something like the intuitive notion of how well supported that proposition is in light of our evidence, then according to this objection it simply isn’t true that logical truths have maximal epistemic probability.

While I grant that this objection has intuitive force, much of that force is dissipated, I think, once we realize just how wide a range of actions we can agree are irrational consistent with maintaining that logical truths have epistemic probability 1. For instance, in the face of disagreement, we needn’t hold that expected utility maximizing agents ought to confidently assert the sentences they take to express those truths. Claims about which logical propositions are expressed by which sentences are empirical claims that need not have epistemic probability 1.

Forceful disagreement from one’s logical superiors is, plausibly, exactly the sort of phenomenon that should lead one to revise one’s confidence that one hasn’t misunderstood the meaning of the sentence one initially took to be necessarily true. Once we have a clear picture of how an expected-utility maximizing agent will conduct herself if logical truths have epistemic probability 1, but sub-maximal epistemic probabilities are associated with biconditionals linking logical truths with propositions about the assertibility of sentences, the outcomes of bets, and so on, we may be inclined to think that such an agent’s conduct as a whole seems perfectly reasonable and undogmatic. And if that is our reaction, perhaps we needn’t insist that there is nevertheless some residual problem associated with holding that logical truths have maximal epistemic probability.

More generally, there are reasons completely independent of decision theory for thinking that uncertainty concerning logical truths should be treated differently from uncertainty concerning empirical propositions. If we follow Stalnaker (1984) and Lewis (1986a) in modelling beliefs with sets of possible worlds, then we will already want some special treatment of logical uncertainty, since logical truths are true in all possible worlds, and so on a crude version of the possible worlds account, they are automatically believed by all agents. To take one example already alluded to, we might treat some cases of ignorance about logical truths as metalinguistic ignorance; nothing in decision theory requires metalinguistic propositions to receive maximal probability.\(^{21}\)

Prodigality does not automatically threaten all decision theorists. We can avoid prodigality while retaining decision theory, so long as we don’t accept that knowledge has probability 1. In the next section, however, I will consider responses that try to reconcile the claim that knowledge has probability 1 with decision theory, while avoiding prodigality.

4 Contextualism, Sensitive Invariantism, and Rational Choice

In my presentation of the prodigality problem, I assumed a view about knowledge according to which anti-skeptical insensitive invariantism is correct. Anti-skeptical insensitive invariantism is called “anti-skeptical” because it says that it is possible to know everyday propositions about matters like the population of Canada, “in-

\(^{21}\)Whether such a strategy can be successfully carried out is a controversial matter. See Field (1986) for convincing arguments to the effect that solving the problem of logical omniscience requires more than just recognizing the possibility of metalinguistic ignorance. My own view is that if we understand logical ignorance as a combination of metalinguistic ignorance and fragmentation (in the sense discussed by Lewis (1982) and Stalnaker (1991, 1999)), the problems Field raises look more tractable.
sensitive” because it says that whether we know isn’t sensitive to practical stakes, and “invariantist” because it says that which propositions we express by uttering sentences containing “knows” doesn’t vary from context to context. In particular, I assumed that it is typically the case that when you know something, you will continue to know it when you are offered the chance to bet on it at very unfavorable odds. But many epistemologists reject this assumption, along with insensitive invariantism more generally. It might seem that views about knowledge other than insensitive invariantism would be well positioned to reconcile the claim that knowledge has probability 1 with decision theory. In this section I will discuss two such strategies for achieving a reconciliation—one contextualist, one sensitive invariantist. I will argue that neither strategy succeeds, though the bulk of my discussion will focus on sensitive invariantism.

4.1 Contextualism and Choiceworthiness

According to contextualists, which proposition one expresses by uttering a sentence of the form “S knows that P” depends on features of one’s context, often including how practically important the parties to one’s conversation take the truth of P to be. If we accept a link between knowledge and probability 1, then epistemic probability will be similarly context-sensitive. This might seem to give us the resources we need to handle the prodigality problem. In particular, contextualism seems to allow us to hold that within a context, what counts as “known” also counts as having “epistemic probability” 1, while also holding that the following two sorts of claims are both typically true:

1. Claims made in ordinary contexts to the effect that people know various things.

2. Claims made in contexts in which bets at extremely unfavorable odds have been offered to the effect that people oughtn’t accept such bets, even though they are bets on claims that we would ordinarily take ourselves to know.

The contextualist can accept the latter claims because she needn’t hold that propositions which we “know” in ordinary contexts can be truly described as having “epistemic probability” 1 in contexts in which extremely high stakes bets are under consideration.

Not all versions of contextualism are able to allow that epistemic probability inherits the context-sensitivity of knowledge. Some natural ways of developing contextualist theories of knowledge involve taking epistemic probability as insensitive

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22See Hawthorne (2004) and Stanley (2005) for some helpful taxonomizing with respect to these terminological issues.

23I won’t be questioning the anti-skeptical part of anti-skeptical insensitive invariantism.

24See e.g., DeRose’s (1992) discussion of what have come to be called “bank cases.”
to context, and holding that whether a proposition is known depends on how features of context interact with epistemic probabilities. For instance, Stewart Cohen (1988) defends a contextualist version of the relevant alternatives theory of knowledge; on his view epistemic probability is taken as fixed independently of context, and how probable an alternative has to be before it counts as relevant is determined by context. More simply, we might treat epistemic probability as fixed independently of context, and hold that propositions are known only if their probability exceeds some contextually determined threshold—in contexts in which skeptical scenarios are salient or high-stakes bets have been offered, the threshold might be quite high, while in normal contexts it would be lower. While this sort of strategy isn’t the only one available to the contextualist, it is a natural one, and it is incompatible with the claim that knowledge has probability 1.

Let us assume, however, that there are versions of contextualism that are consistent with the claim that knowledge has probability 1. If epistemic probability is context sensitive, then it is hard to see how rational choiceworthiness could fail to be context sensitive as well. While it is possible to come up with examples in which context insensitive notions can be analyzed in terms of context sensitive ones, the examples are unusual, and seem disanalogous to the present case.

But once we admit that rational choiceworthiness is context sensitive, we run into problems. Rational choiceworthiness is a concept that has roles to play in both third-person evaluation of agents’ choices, and first-person deliberation about which option to choose. Both of these roles are ill-suited to be played by a context-sensitive concept.

Let us first consider the third-person, evaluative role of the concept of rational choiceworthiness. If we are contextualists about rational choiceworthiness, then we can use the notion of rational choiceworthiness to assess an agent S’s action, but whether we assess S positively or negatively will depend on our context. If we are in a context in which “knowledge” is hard(easy) to have, we will treat S as “knowing” less(more), and will evaluate her actions relative to the associated epistemic probability function. This would seem to allow irrelevant factors to influence our evaluations, at least if we allow that features of context that determine the extension of “knows” typically appealed to by contextualists—e.g., which pos-
sibilities are salient in the conversation—in fact do determine the extension.\textsuperscript{27} If I am trying to decide whether some agent $S$ made a good decision, it shouldn’t matter what possibilities are salient in my conversational context; I am asking a question to which the only relevant factors concern $S$’s situation, not my own.

Perhaps the contextualist can find some way to finesse the above worries—they are sufficiently similar to standard objections to contextualism that we might expect that standard responses will serve.\textsuperscript{28} Where the contextualist has special difficulty is in accounting for the first-person, deliberative role of the concept of rational choiceworthiness. If we are contextualists about knowledge and rational choiceworthiness, we will recognize that there are in fact a family of context insensitive notions of rational choiceworthiness—there is one for each set of possible contextually fixed standards for knowledge, none of which seem to be privileged in any interesting sense. This makes it hard to see how to use the notion of choiceworthiness in deliberation. I could try to perform the action that is truly describable as “choiceworthy” in my context. But there is nothing special about my context—if my conversational partners and I were to start considering more (or fewer) possibilities, a different action might be truly describable as “choiceworthy,” even though nothing that is (intuitively) practically relevant would have changed. If there is nothing practically privileged about either context, or either extension of “choiceworthy,” then it is hard to see why I should prefer to act in the way that is truly describable as “choiceworthy” in my context. If all that was at stake was how to use language (e.g., whether to describe someone as “knowing”) then we might be perfectly happy to allow that considering more (or fewer) possibilities would change our (merely linguistic) behavior. But in cases where there is much of practical import at stake, letting our decisions be sensitive to features of our conversational context seems perverse.

The above considerations suggest that a contextualist view about knowledge attributions that emphasizes features of the conversational context of the person making the knowledge attributions—as traditional contextualist views have—won’t extend happily to a contextualist view about choiceworthiness. But accepting a link between knowledge and probability $1$, a decision theoretic account of choiceworthiness, and contextualism about knowledge requires accepting a contextualist view about choiceworthiness. Therefore, traditional contextualist views are ill-suited to reconcile decision theory with a link between knowledge and probability $1$.\textsuperscript{29}

\textsuperscript{27}See e.g., Lewis (1999).

\textsuperscript{28}See DeRose (1992, fn. 10).

\textsuperscript{29}I don’t rule out that non-traditional versions of contextualism—in particular, versions that reject David Lewis’ “Rule of Attention” (1999)—might be able to avoid the difficulties I’ve been discussing. See Williams (2001).
I believe this points to a more general tension in the theory of knowledge. One desideratum for a theory of knowledge is that it entail (or at least be consistent with) the claim that knowledge attributions are typically true. Developing a theory of knowledge on which knowledge attributions are typically true can seem to require holding that the truth of knowledge attributions is somehow sensitive to facts concerning which possibilities the parties in some conversation have mentioned, or are considering. Another desideratum for a theory of knowledge is that a theory of knowledge should explain the links between knowledge and rational action. But developing a theory of knowledge on which there is a close connection between knowledge and action—for instance, developing a view according to which knowledge has probability 1—seems to require holding that the truth of knowledge attributions not be sensitive to these facts about conversational contexts, since such facts are typically of no practical relevance; they don’t systematically bear on how it is rational to act.

Perhaps this isn’t a deep tension—there may be good reasons to reject one of the putative desiderata mentioned above, or the desiderata may ultimately be jointly satisfiable. In the case of contextualism, however, the tension is real; the features of contextualism that (at least seem to) allow it to accommodate the idea that knowledge attributions are typically true are the very same features that make trouble when we try to accommodate claims about links between knowledge and rational action.

We have already seen that contextualists have difficulty accommodating a knowledge/probability 1 link. Subject sensitive invariantism, however, emphasizes the respects in which the truth of knowledge attributions is (supposedly) sensitive not to the conversational context of the attributer, but to the practical situation of the subject who is said to know. Such a view might seem better placed than contextualism to accommodate links between knowledge and rational action, and it is to this view that we now turn.

4.2 SSI and Dutch Books

Unlike contextualists, sensitive invariantists hold that which proposition one expresses by uttering a sentence of the form “S knows that P” is the same across

\footnote{DeRose (1992) writes that “the obvious attraction of contextualism...is that it seems to have the result that very many of the knowledge attributions and denials uttered by speakers of English are true—more than any form of invariantism can allow for.”}

\footnote{Perhaps the conversation of the knowledge attributer, perhaps the conversation of the subject of the knowledge attribution, perhaps the conversation of the person assessing the truth of the knowledge attribution.}

\footnote{Williamson (2005a) argues that any theory of knowledge will have to attribute a great deal of metalinguistic error to speakers, and that it isn’t obvious that contextualism and sensitive invariantism attribute substantially fewer or less significant errors than insensitive invariantism.}
different contexts. However, they hold that the truth of the proposition expressed is sensitive to features not appealed to by traditional epistemological views. Sensitive invariantists hold that whether a subject knows some proposition depends not only on “truth-conducive factors,” but also on features of a subject’s practical situation. For instance, if I say “Susie knows that the coin is biased,” traditional views might have it that the truth of my utterance is sensitive to facts such as whether Susie has observed a distribution of heads and tails that would be unlikely if the coin were fair, whether Susie has received testimony to the effect that the coin is biased, whether Susie has subjected the coin to various tests, and so on. On traditional views, my utterance will not be sensitive to facts concerning how important it is for Susie whether the coin is biased. On sensitive invariantist views, both sorts of facts are relevant. Even if Susie has seen the coin land heads 600 out of 1000 times, and has received reliable testimony that it is biased, she might not know that it is biased if the proposition that the coin is biased is one on which a great deal of practical importance rests. For instance, a sensitive invariantist would hold that if Susie has been offered a bet at extremely unfavorable odds on the proposition that the coin is biased, it is much harder for her to know that the coin is biased than it is if there is less of practical importance at stake.

Sensitive invariantism might seem well-placed to reconcile decision theoretic accounts of choiceworthiness with the claim that knowledge has probability 1. Unlike insensitive invariantism, sensitive invariantism doesn’t seem to face the prodigality problem—a sensitive invariantist can agree that while we ordinarily know a great deal, we needn’t accept high-stakes bets at arbitrarily unfavorable odds, since being offered such bets destroys our knowledge; once we have been offered such bets, the claims we are betting on will not have probability 1. Unlike contextualism, sensitive invariantism seems to allow choiceworthiness to depend only on factors that are intuitively relevant—by emphasizing facts about a subject’s practical situation, rather than facts about conversational salience, sensitive invariantism can make sense of why choiceworthiness is a good standard for evaluating a third party’s actions, and a good thing to aim at in deliberation.

One obstacle to sensitive invariantist strategies for saving the claim that knowledge has probability 1 is similar to one mentioned earlier for contextualism—some

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33The phrase “truth-conducive factors” is from Stanley (2005).
34This may be a bit too charitable to the sensitive invariantist. The sensitive invariantist who holds that knowledge has probability 1 could hold that facts about conversational salience are relevant to the truth of knowledge attributions, and differ from the contextualist only in thinking that the relevant salience facts concern what is salient in the subject’s context, rather than the attributer’s context. If she takes this route, then she will have just as much trouble making sense of the deliberative role of the concept of choiceworthiness as will the contextualist. That is, the tension I pointed to at the end of the last section will be just as much of a difficulty for the sensitive invariantist as for the contextualist.
ways of implementing the sensitive invariantist proposal involve taking epistemic probability as fixed, and holding that whether one knows depends on the interaction of epistemic probability with practical factors. Fantl and McGrath (2009) employ a strategy along these lines, as does Stanley (2005, p.87).\footnote{Another example involves Weatherson (Draft), who argues that knowledge is sensitive not to the stakes of the bets one faces, but to the odds of those bets. As I understand his position, it requires that facts about subject’s epistemic probabilities are explanatorily prior to facts about what they know; his theory explains what you know in terms of facts about what your epistemic probabilities are, together with facts about what practical options are open to you. Like Fantl and McGrath’s view, Weatherson’s requires the failure of the knowledge-probability 1 link.} Because Stanley ultimately accepts the claim that knowledge has probability 1, however, he takes this presentation to be merely a convenient oversimplification, and not the official version of his view. In what follows, I won’t focus on problems like this one—I assume there are ways of developing sensitive invariantism that don’t require an insensitive notion of epistemic probability—and will instead argue that sensitive invariantism leads to implausible claims about rational choice when combined with the claim that knowledge has probability 1.

Suppose we have access to a device that randomly generates a natural number between 1 and 3, inclusive, with each number having an equal objective chance of being generated. We set the device working, and before looking to see what number is generated, we bet on its outcome. I offer you a bet at 1:2 odds that it generated the number 1. If we are non-skeptics, we should allow that you can know that the device really did generate a number between 1 and 3, and that each number had an equal chance of being generated. And if we accept that knowledge has probability 1, we should also agree that given that you know these things, they have probability 1, so evaluated relative to your epistemic probability function, the bet is fair. Suppose you take it, betting $1 against my $2. Now I make another offer with the following conditions. I will give you a penny if it generated a 1, 2, or 3, but I will take your life savings if it did not generate any of these numbers. You should not take this bet, and SSI has a say about why. Now it is much more important to you whether the machine did generate a 1, 2, or 3, and raising the stakes of betting on this proposition is exactly the sort of thing that can destroy one’s knowledge according to SSI. So the subject sensitive invariantist can grant that while earlier you knew that the machine had generated a 1, 2, or 3, now that this question has become one of great practical importance, you don’t (perhaps possibilities that previously had epistemic probability zero, e.g., that the device malfunctioned, now have positive probability).

However, now that my second offer has been made and your knowledge that the device will generate one of the first three natural numbers has been destroyed, your earlier bet doesn’t look quite as good as it did before.\footnote{One mechanism by which this could work is that my offering you the bet could straightfor-}
the device generated the number one is now less than \( \frac{1}{5} \), since there is a chance that it didn’t generate any number at all.\(^{37}\) I offer you the chance to cancel your earlier bet, for a small fee, and you take it, as cancelling the bet now has positive expected utility for you in light of your new probability function. All in all, without my having any information that you lack, I have extracted a small fee from you for nothing in return, by offering you a series of deals each of which you regarded as fair at the time.\(^{38}\)

The problem that the above example illustrates is this: if the probabilities that govern an agent’s choices are sensitive to the stakes of the bets she’s offered, then the agent is predictably manipulable by a bookie with the ability to offer high-stakes bets. Since this kind of predictable manipulability is irrational, SSI together with the claim that knowledge has probability 1 entails false conclusions about rational choice.\(^{39}\)

It shouldn’t be surprising that the defender of SSI is committed to recommending that agents accept Dutch books if she also accepts that knowledge has probability 1. According to SSI together with the claim that knowledge has probability 1, epistemic probabilities change when the stakes go up, but this change does not occur through conditionalization. But there are familiar arguments to the effect that agents who update the probabilities that guide their actions by rules other than conditionalization may be vulnerable to Dutch books; the argument I’ve given in this section can be seen as pointing to yet another example in which using an update rule other than conditionalization can get one into trouble.\(^{40}\) Should the defender of SSI and the knowledge/probability 1 link bite this bullet and endorse accepting Dutch books, or does she have a way out?

One potential option for the sensitive invariantist who wants to save the claim

\[^{37}\]I assume that the relative probabilities of the individual numbers being generated haven’t changed. While nothing I’ve said so far forces that assumption, and on some ways of fleshing out the case we might reject it, I assume that the example can be set up in such a way that it is a reasonable assumption to make.

\[^{38}\]A defender of SSI might object that while accepting the first bet is rationally permissible given your utility function, it is also permissible for you to reject it. If Dutch book worries only indicate irrationality when there is a series of bets each of which is rationally mandatory given one’s utility function, but which together guarantee a sure loss, then my initial example doesn’t threaten SSI. But we could have set up the example so that the bets were mandatory rather than merely permissible, at the cost of some elegance.

\[^{39}\]There is a vast literature on Dutch book arguments, and it is far from uncontroversial exactly what sort of rational shortcoming is indicated by susceptibility to a Dutch book. That susceptibility typically indicates some sort of rational shortcoming, however, is relatively less controversial. See Christensen (1996).

\[^{40}\]See Teller (1973), who credits David Lewis.
that knowledge has probability 1 is to employ a strategy similar to the one that I used in §3.1 to argue that decision theory alone isn’t enough to generate prodigality. The defender of SSI can claim that when the second bet is offered, it is not just one’s knowledge that the machine generated a 1, 2, or 3 that is destroyed, but also one’s knowledge of the bridge principles necessary to make canceling the initial bet rationally required. The defender of SSI could claim that after the second bet has been offered, one not only no longer knows that the machine generated a 1, 2, or 3, but one also fails to know that one will lose the first bet just in case the machine failed to generate a 1, 2, or 3. At the very least, nothing in the machinery of SSI seems to rule out this response. And this response allows for the possibility that canceling the first bet isn’t rationally required.

While the letter of the initial problem may have been avoided, a serious difficulty remains. Being offered the second bet should have no effect on one’s willingness to hold onto the first bet, and our theory of rational choice should explain why. While SSI may not be straightforwardly committed to endorsing the acceptance of Dutch books in cases like these, it lacks the resources to explain why the attractiveness of holding onto the first bet should have nothing to do with whether or not one has been offered the second. Rather than offering such an explanation, SSI seems to suggest that one’s willingness to stick to earlier bets should systematically depend on which bets one is offered later; according to decision theory together with claims linking knowledge and probability, one’s actions should depend on what one knows, and according to SSI, what one knows depends on which bets one has been offered. What we would like to say is that the epistemic probabilities of the device having generated various outcomes are unaffected by the fact that the second offer has been made, but SSI doesn’t seem to let us say this, at least once it is combined with claims linking knowledge and probability. This points to a more general problem involved in combining SSI with decision theory and the claim that knowledge has probability 1, which I discuss in the next section.

One possibility I have not considered is that the defender of SSI might modify her theory and stop treating knowledge as a relation between a subject and a proposition, but instead as a relation between a subject, a proposition, and an action. That is, the defender of SSI might allow that one can know that $P$ for the purpose of performing some actions, but not for the purpose of performing others. She could then hold that for the purposes of deciding whether or not to accept the first bet, one knows that the machine will generate a 1, 2, or 3, but not for the purpose of deciding whether or not to accept the second bet. She could also hold that being offered new bets typically doesn’t change whether or not you know something for the purpose of accepting bets you have already been offered. While this move would avoid the problems discussed above, it is relatively radical. More importantly, like SSI, it faces a problem which I discuss in §4.3. Thanks to Agustín Rayo (who argues for a view along similar lines in response to a different problem in his 2011 paper, “A Puzzle about Ineffable Propositions”) for suggesting this strategy on behalf of the sensitive invariantist.
4.3 SSI, Probabilities, and Utilities

Part of the appeal of the decision theoretic apparatus is that it lets us separate questions about the values of outcomes from questions about how likely it is that various outcomes will come about. Once we have separated considerations concerning utility from considerations concerning probability, we can isolate the contributions that each of these factors make to determining rational choiceworthiness. For example, a course of action might come to be more choiceworthy either because one of the outcomes it might lead to becomes more valuable, or because the probability that it will lead to a favorable outcome might increase (while the value of each possible outcome remains constant). When we marry the claim that knowledge has probability 1 to SSI, however, probabilities and utilities become interdependent in a way that makes it impossible for us to isolate the roles that they play in determining facts about rational choiceworthiness.

For instance, suppose, in a variant on the example from the previous section, I first offer you the chance to bet a dollar at even odds on the claim that the device generated either a 1, a 2, or a 3. Obviously, you ought to take the bet. Next, I make the knowledge-destroying offer—I offer you a bet on the same proposition, but with the potential payoff of a penny, and the potential downside of losing your life savings. You ought not take this new bet. How is this bet different from the earlier one? According to SSI, two things have changed—the disutility of one of the potential outcomes of the bet has increased (since there is now the potential downside of losing your life savings, rather than just $1), and the probability that the favorable potential outcome of the bet will be realized has decreased (since there is no longer a probability of 1 that the favorable outcome will be realized). But we can’t separate out these changes—they are inextricably linked.

I take it that this is a drawback of the combination of SSI with the claim that knowledge has probability 1. Intuitively, questions about probabilities and questions about utilities are distinct, and we should be able to separate factors responsible for changing probabilities from those responsible for changing utilities. Even if we could live with the phenomenon noted in the previous section, accepting SSI and the claim that knowledge has probability 1 would make decision theory much less well suited to offering an illuminating explanation of how facts about one’s evidence and facts about value come together to rationalize choices.\footnote{Hájek (2006) offers a similar criticism of a certain response to the Saint Petersburg paradox. According to this response, while the utilities a rational agent assigns to various outcomes can increase without bound, the expected utilities assigned to gambles must always be finite; on this account, one can never rationally take oneself to be playing a Saint Petersburg game. He argues that this creates an illegitimate dependence between probabilities and utilities—someone who changes her mind about the value of an outcome may be directly forced to revise her confidence that it will occur downwards. While in some cases indirect revision of this sort is rational, it is only rational when there is some sort of evidential link between probabilities and utilities, e.g.,}
5 Idealization

In this last section I will consider a potential response on behalf of the sensitive invariantist that will place the debate over whether knowledge has probability 1 in the context of a broader debate about the role of idealization in our theorizing about epistemology and practical rationality.

Keeping track of small probabilities is hard. If I am trying to decide whether to take the train or the bus, even if there is some tiny probability that the bus will be hijacked by terrorists, or that the train will derail, it is much easier (and more sensible) to ignore these probabilities in my deliberations than it is to factor each of them in. We might see sensitive invariantism combined with the claim that knowledge has probability 1 as a suggestion for how to ignore them—except in contexts where (prior to considering them) it seems likely that considering them might change our minds about what to do, we should treat ourselves as knowing that they won’t obtain, and should assign them probability 0 when we are calculating expected utilities. After all, considering such possibilities costs brainpower, and there are usually better things we could be doing with our time.

If we understand things this way, then it will look as if views on which propositions we ordinarily take ourselves to know have high but non-maximal epistemic probabilities are views that idealize away from computational costs, while views like sensitive invariantism (when combined with the claim that knowledge has probability 1) eschew this idealization. This would allow the sensitive invariantist to explain why—as it came out in §4.2—agents that act in accordance with her recommendations are predictably manipulable; they are predictably manipulable by agents who have more computational resources than them. It is not surprising that agents who have the time and computational resources to consider more possibilities than you can will be able to come up with strategies for offering you bets that will leave you predictably worse off—if they rely on the fact that you are ignoring certain possibilities, (while they are taking them into account) and then offer you bets that exploit your oversight, they can put you at a disadvantage.

Understood this way, arguments against the claim that knowledge has proba-

If you find out that a state lottery is offering a very large jackpot, you may reasonably revise your confidence that you will win downwards, given that you know that the state will only offer lotteries that raise revenues. What wouldn’t be rational would be to revise one’s probabilities as a result of revising one’s utilities directly, in the absence of such an evidential link. This is what the response to the Saint Petersburg paradox considered by Hájek amounts to, and it is also what seems to be recommended by SSI together with the claim that knowledge has probability 1.

Of course, there is another perspective from which it looks like it is the sensitive invariantist who is idealizing—we might say that she is the one who is idealizing by treating small probabilities as if they were zero. This looks a lot like standard ways of idealizing, e.g., treating small frictional forces as if they were nonexistent.
bility 1 are only attractive insofar as we are ignoring actual agents’ limitations—a realistic, useful theory of practical rationality will accept that knowledge has probability 1.

This take on the debate underestimates the resources available to the decision theorist who allows that known propositions have high but sub-maximal probability—such a theorist needn’t be understood as idealizing away from the computational costs of various courses of deliberation. In the remainder of this paper I will suggest a way for the opponent of the view that knowledge has probability 1 to find some room in her story for computational costs.

Just as we can use decision theory to evaluate actions like taking the train rather than the bus, we can use decision theory to evaluate long-term deliberative strategies such as taking into account a very wide range of possibilities when deliberating rather than only considering a narrower set. Even if we allow that propositions we ordinarily take ourselves to know don’t have maximal probability, we can hold that the long-term strategy of considering possibilities in which those propositions are false (and attempting to think about probabilities of those possibilities in deciding what to do) will have sub-maximal expected utility. This is because factoring such possibilities into our deliberations costs time and brain-power, and usually doesn’t have any benefit (since it usually won’t change what we do).

This allows for an interesting sort of case—it might be that an action \( \phi \) fails to maximize expected utility, even though engaging in a strategy for deliberating that leads to one’s \( \phi \)-ing does maximize expected utility. In those rare cases where which action maximizes expected utility does depend on what happens in far-out possibilities where propositions ordinarily taken to be known are false, it may be that the strategy of ignoring possibilities like those ones itself maximizes expected utility, even though the decision we go on to make about what to do (after having ignored those possibilities) will not maximize expected utility.

In fact, the example discussed in the previous section is plausibly such a case. The opponent of the claim that knowledge has probability 1 can hold that throughout the example, the proposition that the device generated a 1, 2, or 3 has high but sub-maximal probability for the subject facing the bets. When no high-stakes bets have been offered, however, it may be rational for the subject to ignore the possibility that this proposition is false in conducting her deliberations, and treat the proposition as if it had probability 1. After all, there is some cost associated

\[\text{We might argue that one can’t “directly” bring about that one pursues a long-term strategy, and conclude from this that we shouldn’t use decision theory to evaluate long-term strategies (perhaps because one accepts some of the arguments alluded to in footnote 19). If the reader is worried about this issue, she can instead take the present section to be pointing out that we can use decision theory to evaluate not only decisions about what bodily actions to perform, but also decisions about how to deliberate.}\]
with thinking about such possibilities, and in situations in which it is unlikely that considering them will provide any significant benefit, the expected utility of factoring such possibilities into one’s deliberations might be less than the expected utility of ignoring them and rounding high probabilities to 1.

Because of this, the decision to accept the first bet—the bet that pays off at 1:2 odds if the machine generated a 1—is a decision that is the upshot of a rational decision about how to deliberate, but which is not itself a rational decision; the expected utility of accepting the bet will be slightly negative if the probability that the machine generated a 1 is slightly less than $\frac{1}{3}$, as it plausibly is if the probability that it generated any number at all is slightly less than 1.45

Similar cases are familiar from the literature in game theory. It might be that binding oneself to a certain strategy maximizes expected utility, even though actually acting on that strategy in particular cases does not maximize expected utility. For instance, binding oneself to the strategy of always ignoring threats might be a good idea because it makes people less likely to threaten you, even though in some particular cases you would be better off complying with a threat—the costs of the threat’s being carried out might be quite high and the effects on one’s reputation as someone who gives into threats could be small or nonexistent.46

While such cases aren’t entirely uncontroversial, many philosophers hold that even if some strategy is generally a good one to follow (and even if it would make sense, ex ante, to bind oneself to acting in accordance with that strategy in all cases), if acting in accordance with it in some particular case would predictably have bad effects on net, it would be irrational to do so.47

Once we acknowledge the possibility of such cases, the view that propositions we ordinarily take ourselves to know have high but sub-maximal probability looks a good deal better. Such a view can give an appropriately nuanced treatment of cases in which agents who ignore far fetched scenarios in their deliberations make irrational decisions and are thereby predictably manipulable. The view can positively evaluate such agents by holding that they are rational to have a policy of ignoring such scenarios when engaging in deliberation. However, the view can also negatively evaluate such agents by holding that the actions they ultimately perform, having ignored such scenarios, are not rationally choiceworthy. This

45We can also use this framework to say plausible things about the decision to reconsider the first bet. Once the second bet has been offered, it clearly pays to consider the outlandish possibilities in which the machine generates no number at all. And as long as one is already devoting time and computational resources to considering those possibilities for the purpose of determining whether to accept the second bet, one may as well consider (at little extra cost) how their existence bears on the question of whether to cancel the first bet.


47See Parfit (1984) and Kelly (2002, 2004). For a contrary view, see Gauthier (1986). The way I have put things in the text presupposes that some version of consequentialism is correct, but this is inessential.
combination is more attractive than the verdicts of the view on which both SSI and the knowledge/probability 1 link hold; that view seems to entail that such agents are rationally unimpeachable.

Ultimately, denying that knowledge has probability 1 and holding that propositions we ordinarily take ourselves to know in fact have high but sub-maximal probability needn’t lead to an uninteresting, unrealistic view that is irrelevant to the concerns of computationally limited agents. To the contrary, such a theory can explain how it is rational to cope with our computational limitations (by positively evaluating the strategy of ignoring far-out possibilities), while also acknowledging that rational decisions sometimes beget irrational ones, as in cases where ignoring such possibilities makes us predictably manipulable.

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