than the first three as a necessary condition for understanding natural language. Haugeland says, "Only a being that cares about who it is, as some sort of enduring whole, can care about guilt or folly, self-respect or achievement, life or death. And only such a being can read" (631). However, it certainly seems that somebody, such as a Buddhist monk, can eschew self-identity and the cares of life and still read. So I assume that Haugeland has something deeper in mind. Perhaps the condition is that in order to understand a passage about certain cares or feelings one must have experienced those cares and feelings. But this claim is too strong, for one can understand discussions of death, for example, without ever having had cares about death. To insist that one does not really, really understand discussions of subjects one does not care about or has not experienced is to exclude much of what we usually take to be natural-language understanding. Although knowledge is essential for natural-language understanding, caring is not.

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THEORIES OF PROPERTIES, RELATIONS, AND PROPOSITIONS *

I. THE NEED FOR A NEW FORMULATION

THE theory of properties, relations, and propositions promises to be an important tool in logic, philosophy, psychology, and linguistics. Indeed, talk about properties, relations, and propositions (PRP's for short) is commonplace in informal discussions in these disciplines. However, no formal theory of PRP's has ever been completely and adequately formulated. To be convinced of this, consider two representative arguments:

Whatever x believes y believes.
x believes that A.
∴ y believes that A.

Being a bachelor is the same thing as being an unmarried man.
∴ It is necessary that all and only bachelors are unmarried men.

* To be presented in an APA symposium of the same title, December 29, 1979. Edmund Gettier will comment; his paper is not available at this time.

Earlier versions were presented at the Northwest Conference of Philosophy (fall 1978) and at a Reed College Philosophy-Mathematics Colloquium (spring 1979). For support, thanks go to the National Endowment for the Humanities, the Mellon Foundation, and Reed College; for improvements, to Mark Bedau.

0022-362X/79/7611/0634$01.50

Neither of these intuitively valid arguments is even expressible in standard first-order predicate logic even when epistemic and modal operators are adjoined. And though it is true that both of these arguments can be expressed in certain higher-order intensional logics, such higher-order logics are essentially incomplete, to mention just one of their shortcomings. But things are better than they might seem. When an intensional abstraction operation is adjoined to first-order logic, the result is a logic for PRP's which is equipped to represent the above arguments—and indeed, nearly all problematic arguments in intensional logic. At the same time, unlike higher-order intensional logics, this first-order quantified intensional logic is, surprising as it might seem, provably complete.

In what follows I will show how to construct such a first-order theory of PRP's. The construction requires the development of both a new formal language and a new semantic method. The new semantic method does not appeal to possible worlds, even as a heuristic. The heuristic used is simply that of properties, relations, and propositions taken at face value. And, unlike the various possible-worlds approaches to intensional logic, the approach developed here is adequate for treating not just the logical modalities but intentional matters as well. I will bring the paper to a close by speculating briefly on the intriguing philosophical question of why a complete logic for PRP's can be achieved in the setting of first-order logic but not in the setting of higher-order logic. The answer to this question suggests an account of the origin of incompleteness in logic generally.

II. TWO TRADITIONAL CONCEPTIONS

Historically, there have been two fundamentally different conceptions of properties, relations, and propositions.[1] On the first conception, intensional entities are considered to be identical if and only if they are *necessarily equivalent*. A corollary is that, beyond the requirement of necessary equivalence, there are no constraints on what is to count as a correct definition. For example, both of the

[1] There are intermediate conceptions between the two that I isolate. No treatment of PRP's would be satisfactory unless it addressed this and related topics including: the relation between PRP's and sets, the logic for the predication relation and the status of logicism, the logical and intentional paradoxes, the paradox of analysis, the relation between extensional and intensional logic, and the relation between theories of reference and theories of meaning. These topics are explored in my forthcoming book *Quality and Concept*, where the two traditional conceptions of PRP's are synthesized into a single theory of qualities and concepts. This synthesis leads to noncircular purely logical definitions of concepts ranging from truth and analyticity to necessary equivalence $\approx_N$, necessary connection, and intentionality. This synthesis also leads to a solution to Goodman's new problem of induction.

following sentences taken from contemporary philosophy:

(a) x is grue iff x is green if examined before t and blue otherwise.

(b) x is green iff x is grue if examined before t and bleen otherwise.

qualify as correct definitions on this conception.

On the second conception, by contrast, each definable intensional entity is such that, when it is defined completely, it has a *unique, noncircular definition.* (The possibility that such complete definitions might in some or even all cases be infinite need not be ruled out.) Hence, on this conception, there are severe constraints on what is to count as a correct definition. For example, in view of its stipulative character, the original definition of grue in terms of green (and blue) is certainly correct, even if green should itself be definable. Therefore, on the assumption that there is a *unique* way of completely spelling out the correct definition of grue, green must show up in that definition either as a defined or as an undefined term. Consequently, on the assumption that correct definitions cannot be *circular*, green cannot in turn be defined in terms of grue. Thus, although (a) and (b) above both express necessary truths, on the second conception of intensional entities (a) alone is a correct definition. Although necessary equivalence is a necessary condition for identity, it is not a sufficient condition.

The first conception of intensional entities is that which underlies Alonzo Church's "alternative 2" formulation of Frege's theory of senses.[2] This conception is also built into the possible-worlds treatment of PRP's. Indeed, this conception is commonly attributed to Leibniz. Whether Leibniz actually subscribed to it, however, is open to doubt.

The second conception of intensional entities has a far livelier history. Perhaps the clearest instance of it is to be found in Russell's doctrine of logical atomism. (On this doctrine it is required that all complete definitions be *finite* as well as unique and noncircular.) Traces of this conception are also clearly evident in Leibniz's remarks on the distinction between simple and complex properties. Moreover, if concepts (ideas, thoughts) are identified with properties, relations, and propositions, evidence of this conception can be

[2] "A Formulation of the Logic of Sense and Denotation," in Paul Henle, Horace Kallen, and Susanne K. Langer, eds., *Structure, Method, and Meaning: Essays in Honor of Henry M. Sheffer* (New York: Liberal Arts Press, 1951), pp. 3–24; and "Outline of a Revised Formulation of the Logic of Sense and Denotation," in two parts, *Noûs*, VII, 1 (March 1973): 24–33, and VIII, 2 (May 1974): 135–156. For Church's theory of synonymy, see his "Intensional Isomorphism and Identity of Belief," *Philosophical Studies*, V, 5 (October 1954): 65–73, and his "Outline . . . ," *op. cit.*

found in the writings of philosophers from Descartes and Locke, through Kant, and on to even Frege. In spite of its lively history, this conception has never been invoked as the intuitive motivation for a formal theory of PRP's. Although Russell's informal doctrine of logical atomism provides us with perhaps the clearest instance of this conception, *Principia Mathematica*, ironically, is neutral with regard to the two conceptions. And despite what one might expect, Alonzo Church does not intuitively motivate his "alternative 0" formulation of Frege's theory of senses with this conception of PRP's; instead, the intuitive motivation that Church explicitly invokes is a problematic conception of synonymy based on the notion of synonymous isomorphism. However, scrutiny of Church's axioms (axioms 63–65 and 66–68, *ibid.*) reveals that the second conception does in fact implicitly underlie this formulation of Frege's theory.

The first conception of intensional entities is ideally suited for treating the logical modalities—logical necessity, logical possibility, etc. It has proved to be of little value, however, in the treatment of intentional matters—belief, desire, perception, decision, assertion, etc. Indeed, it has led its major contemporary proponents to construct theories that provide strikingly inadequate treatments of them. The second conception, on the other hand, though ideally suited for the treatment of intentional matters, has only complicated the treatment of logical modalities.

The value of each conception of intensional entities is evident. Therefore, I propose to develop the two conceptions side by side. Once this is done, a natural synthesis will suggest itself.

III. A NEW FORMAL LANGUAGE

I will now specify the syntax for a first-order language with intensional abstraction. This language will be called L_ω. Primitive symbols:

Logical operators: $\&$, $\neg$, $\exists$
Predicate letters: $F_1^1, F_2^1, \ldots, F_m^n$
Variables: $x, y, z, \ldots$
Punctuation: (,), [,]

Simultaneous inductive definition of *term* and *formula* of L_ω:

(1) All variables are terms.
(2) If $t_1, \ldots, t_j$ are terms, then $F_i^j(t_1, \ldots, t_j)$ is a formula.
(3) If A and B are formulas, then $(A \& B)$, $\neg A$, and $(\exists v_k)A$ are formulas.
(4) If A is a formula and $v_1, \ldots, v_m$, $0 \leq m$, are *distinct* variables, then $[A]_{v_1 \cdots v_m}$ is a term.[3]

[3] In the limiting case where $m = 0$, $[A]$ is a term. All and only formulas and terms are *well-formed expressions*. An *occurrence* of a variable v_i in a well-formed

L_ω is just like a standard first-order language except for its singular terms $[A]_{v_1 \ldots v_m}$. On the intended informal interpretation of L_ω, the singular term $[A]_{v_1 \ldots v_m}$ denotes a proposition if $m = 0$, a property if $m = 1$, and m-ary relation-in-intension if $m \geq 2$. From a syntactic point of view the intuitively valid arguments mentioned at the outset of the paper can be perspicuously represented in L_ω:

$$\frac{(\forall z)(B(x, z) \supset B(y, z)) \quad B(x, [A])}{\therefore B(y, [A])} \qquad \frac{[B(x)]_x = [(U(x) \;\&\; M(x))]_x}{\therefore N([(\forall x)(B(x) \equiv (U(x) \;\&\; M(x)))])}$$

Of course, in order to guarantee that these and other intuitively valid arguments come out valid in L_ω, we must first specify the semantics for L_ω.

IV. A NEW SEMANTIC METHOD

By what means should we characterize the semantics for L_ω? Since the aim is simply to characterize the logically valid formulas of L_ω, it will suffice to construct a Tarski-style definition of logical validity for L_ω. Such definition will be built on Tarski-style definitions of truth for L_ω. These definitions will in turn depend in part on specifications of the *denotations* of the singular terms in L_ω. As already indicated, every formula of L_ω is just like a formula in a standard first-order extensional language except perhaps for the singular terms occurring in it. Therefore, once we have found a method for specifying the denotations of the singular terms of L_ω, the Tarski-style definitions of truth and validity for L_ω may be given in the customary way. What we are looking for specifically is a method for characterizing the denotations of the singular terms of L_ω in such a way that a given singular term $[A]_{v_1 \ldots v_m}$ will denote an appropriate property, relation, or proposition, depending on the value of m.

Since L_ω has infinitely many singular terms $[A]_\alpha$, what is called for is a recursive specification of the denotation relation for L_ω. To do this we will arrange these singular terms into an order according to their syntactic kind and complexity. So, for example, just as the complex formula $((\exists x)Fx \;\&\; (\exists y)Gy)$ is the conjunction of the simpler

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expression is *bound* (*free*) if and only if it lies (does not lie) within a formula of the form $(\exists v_i)A$ or a term of the form $[A]_{v_1 \ldots v_i \ldots v_m}$. A *variable* is *free* (*bound*) in a well-formed expression if and only if it has (does not have) a free occurrence in that well-formed expression. A *sentence* is a formula having no free variables. The predicate letter F_1^2 is singled out as a *distinguished logical predicate* and formulas of the form $F_1^2(t_1, t_2)$ are to be rewritten in the form $t_1 = t_2$. $\forall$, $\supset$, $\equiv$, $\equiv_{v_i \ldots v_j}$ are to be defined in terms of $\exists$, &, and $\neg$ in the usual way. If v_i occurs free in A and is not one of the variables in the sequence of variables α, then v_i is an *externally quantifiable free variable* in the term $[A]_\alpha$. Let δ be the sequence of externally quantifiable free variables in $[A]_\alpha$. For readability $[A]_\alpha$ will sometimes be rewritten $[A]_\alpha^\delta$.

formulas $(\exists x)Fx$ and $(\exists x)Gx$, we will say that the complex term $[((\exists x)Fx \,\&\, (\exists y)Gy)]$ is the *conjunction* of the simpler terms $[(\exists x)Fx]$ and $[(\exists y)Gy]$. Similarly, just as the complex formula $\neg(\exists x)Fx$ is the negation of the simpler formula $(\exists x)Fx$, we will say that the complex term $[\neg(\exists x)Fx]$ is the *negation* of the simpler term $[(\exists x)Fx]$. The following are other examples: $[Rxy]_{yx}$ is the *conversion* of $[Rxy]_{xy}$; $[Sxyz]_{zxy}$ is the *inversion* of $[Sxyz]_{xyz}$; $[Rxx]_x$ is the *reflexivization* of $[Rxy]_{xy}$; $[Fx]_{xy}$ is the *expansion* of $[Fx]_x$; $[(\exists x)Fx]$ is the *existential generalization* of $[Fx]_x$; $[Fy]^y$ is the *predication* of $[Fx]_x$ of y; $[F[Gy]_y]$ is the *predication* of $[Fx]_x$ of $[Gy]_y$; $[F[Gy]^y]_y$ is the *relativized prediction* of $[Fx]_x$ and $[Gy]_y$. Thus, nine syntactic operations are isolated in this way: conjunction, negation, conversion, inversion, reflexivization, expansion, existential generalization, predication, and relativized predication.[4]

The complex singular terms of L_ω that are syntactically simpler than all other complex singular terms are those whose form is $[F_h{}^m(v_1, \ldots, v_m)]_{v_1 \ldots v_m}$. These will be called *elementary*. The denotation of such an elementary complex term is just the property or relation expressed by the primitive predicate $F_h{}^m$. The denotation of a more complex term $[A]_\alpha$ is defined in terms of the denotation(s) of the relevant syntactically simpler term(s). However, to state this definition, we must have a general technique for modeling PRP's.

Suppose that we were to use one of the previous approaches to this subject—namely, the approach of Russell, of Church, or of the possible-worlds theorists Montague, Kaplan, D. Lewis, *et al.* In that case we would be led to identify properties and relations with certain *functions*. I find such identification highly unintuitive. But this is not all. The identification of properties and relations with functions leads naturally—and perhaps inevitably—to a hierarchy of artificially restricted *logical types*. Since the thesis that properties and relations are functions is linked in this way to type theory, it proves to be more compatible with the higher-order approach to the logic of PRP's than it is with the first-order approach. In a first-order setting—such as that provided by L_ω—the identification of properties and relations with functions merely generates a jungle of unwanted and unnecessary complications and restrictions. The alternative is to take properties and relations—as well as propositions—at face value, i.e., as real, irreducible entities. This is what I will do.

[4] These nine syntactic operations are precisely defined in my *Quality and Concept*, *op. cit.*

The identification of intensional entities with functions lies at the very heart of the possible-worlds semantic method. If, as I have proposed, intensional entities are taken at face value and not as covert functions, then the possible-worlds semantic method will be of no use to us. But how, then, is the denotation of a given complex term $[A]_\alpha$ determined from the denotation(s) of the relevant syntactically simpler terms? My answer is that the new denotation is determined *algebraically*. That is, the new denotation is determined by the application of the relevant *fundamental logical operation* to the denotation(s) of the relevant syntactically simpler term(s). Let me explain.

Consider the following three propositions: $[(\exists x)Fx]$, $[(\exists y)Gy]$, $[(\exists x)Gx \,\&\, (\exists y)Gy]$. (Note: in this paragraph I will be *using*—not mentioning—terms from L_ω.) What is the most obvious relation holding among these propositions? Answer: the third proposition is the *conjunction* of the first two. Similarly, what is the most obvious relation among the properties $[Fx]_x$, $[Gx]_x$, and $[(Fx \,\&\, Gx)]_x$? As before, the third is the *conjunction* of the first two. And what is the most obvious relation holding between the propositions $[(\exists x)Fx]$ and $[\neg(\exists x)Fx]$? Answer: the second is the *negation* of the first. Similarly, what is the most obvious relation holding between the properties $[Fx]_x$ and $[\neg Fx]_x$? As before, the second is the *negation* of the first. In a like manner we arrive at the following fundamental relationships: $[Rxy]_{yx}$ is the *converse* of $[Rxy]_{xy}$; $[Sxyz]_{zxy}$ is the *inverse* of $[Sxyz]_{xyz}$; $[Rxx]_x$ is the *reflexivization* of $[Rxy]_{xy}$; $[Fx]_{xy}$ is the *expansion* of $[Fx]_x$; $[(\exists x)Fx]$ is the *existential generalization* of $[Fx]_x$; $[Fy]^y$ is the result of *predicating* $[Fx]_x$ of y; $[F[Gy]_y]$ is the result of *predicating* $[Fx]_x$ of $[Gy]_y$, and $[F[Gy]^y]_y$ is the result of a *relativized predication* of $[Fx]_x$ of $[Gy]_y$. The above examples serve to isolate nine fundamental logical operations on intensional entities. These nine fundamental logical operations, of course, correspond to the nine syntactic operations listed earlier.

The first two fundamental logical operations are intensional analogues of the two operations from Boolean algebra. A Boolean algebra having two elements (T and F) is an extensional model of first-order sentential logic. The next four operations are intensional analogues of operations from the algebra of relations, whose origins are found in the work of Pierce and Shröder. The algebra of relations, or transformation algebra, as it is called, is the algebra for extensional relations. A transformation algebra is an extensional model of first-order predicate logic *without* quantifiers. The next operation, existential generalization, is an intensional analogue of the special new operation found in polyadic algebra. Polyadic algebra is just the algebra for extensional relations *with* quantification. A polyadic algebra is an extensional model of first-order predicate logic with quantifiers. Finally, predication and relativized predication are two further operations that I have isolated for the purpose of modeling first-order quantifier logic with distinguished singular terms, including in particular intensional abstracts.

Taken together, these nine fundamental logical operations have the following the following property. Choose any complex term $[A]_\alpha$ in L_ω that is not elementary. If $[A]_\alpha$ is obtained from $[B]_\beta$ via the syntactic operation of negation (conversion, inversion, reflexivization, expansion, existential generalization), then the denotation of $[A]_\alpha$ is the result of applying the logical operation of negation (conversion, inversion, reflexivization, expansion, existential generalization) to the denotation of $[B]_\beta$. The same thing holds *mutatis mutandis* for complex terms that, syntactically, are conjunctions, predications, or relativized predications. In this way, therefore, these nine fundamental logical operation make it possible to define recursively the denotation relation for all of the complex intensional terms $[A]_\alpha$ in L_ω.[5]

The algebraic semantics for L_ω is thus to be specified in stages. (1) An algebra of properties, relations and propositions—or an *algebraic model structure*, as I will call it—is posited. (2) Relative to this an intensional *interpretation* of the primitive predicates is given. (3) Relative to this, the *denotation* relation for the terms of L_ω is recursively defined. (4) Relative to this, the notion of truth for formulas is defined. (5) In the customary Tarski fashion, the notion of *logical validity* for formulas of L_ω is defined in terms of the notion of truth.

Now a structure β is a Boolean algebra if and only if (i) β is an ordered set consisting of a universe or domain $\mathfrak{D}$ and two operations on $\mathfrak{D} \times \mathfrak{D}$ and $\mathfrak{D}$, respectively, and (ii) the elements of β satisfy certain finitely specifiable conditions. By analogy, $\mathfrak{M}$ is an algebraic model structure if and only if (i) $\mathfrak{M}$ is an ordered set consisting of a universe or domain $\mathfrak{D}$ and nine fundamental logical operations on $\mathfrak{D} \times \mathfrak{D}$, $\mathfrak{D}$, . . ., respectively (plus certain supplementary elements), and (ii) the elements of $\mathfrak{M}$ satisfy certain finitely specifiable conditions. In section II, I mentioned that, historically, there have

[5] Notice that the *meaning relation* for L_ω may then be defined simply as follows:

$$\text{The meaning of } A =_{df} \text{the denotation of } [A]_{v_1 \ldots v_m}$$

(in order of their first free occurrences, $v_1, \ldots v_m$ are the free variables in A).

been two competing conceptions of intensional entities. According to conception 1, intensional entities are identical if and only if they are necessarily equivalent. According to conception 2, each definable intensional entity is such that, when it is defined completely, it has a unique, noncircular definition. By suitably adjusting the conditions imposed on the elements of a given algebraic model structure $\mathfrak{M}$, we can fix the exact character of the intensional entities that $\mathfrak{M}$ is designed to model. In particular, by suitably formulating the conditions imposed on the elements of $\mathfrak{M}$, we can make precise what it takes for the intensional entities modeled by $\mathfrak{M}$ to conform to conception 1 or conception 2.

In this way we actually arrive at two distinct types of algebraic model structures—type 1 and type 2. In turn, we arrive at two distinct notions of logical validity for L_ω—validity$_1$ and validity$_2$, i.e., truth-in-all-type-1-model-structures and truth-in-all-type-2-model-structures. (Specifications of both types of model structure and both notions of logical validity are laid out in greater detail in the Appendix.)

V. THE LOGIC FOR PRP'S ON CONCEPTION 1

On conception 1 intensional entities are identical if and only if they are necessarily equivalent. Thus, on conception 1, the following definition captures the properties usually attributed to the modal operator $\Box$:

$$\Box A \equiv_{df} [A] = [[A] = [A]]$$ [6]

For the purpose of formulating the logic for L_ω on conception 1, this definition will be adopted as a notational convenience. The modal operator $\Diamond$ is then defined in terms of $\Box$ in the usual way: $\Diamond A \equiv_{df} \neg \Box \neg A$. The logic T1 for L_ω on conception 1 consists of (a) the axiom schemes and rules for the modal logic S5 with quantifiers and identity and (b) three additional axiom schemas for intensional abstracts.

Axiom Schemas and Rules of T1

A1: Truth-functional tautologies

A2: $(\forall v_i)A(v_i) \supset A(t)$ (where t is free for v_i in A)[7]

[6] That is, necessarily A iff the proposition that A is identical with a trivial necessary truth. Since on conception 1 there is only one necessary truth, this definition is adequate.

[7] A term *t is free for v_i* in A iff$_{df}$ for all v_k, if v_k is free in t, then no free occurrence of v_i in A occurs either in a subcontext of the form $(\exists v_k)(\ldots)$ or in a subcontext of the form $[\ldots]_{\alpha v_k \beta}$. Thus, if t is free for v_i in A, the result of substituting t for the free occurrences of v_i in A produces no "collision of variables." [Recall that $(\forall v_k)(\ldots v_k \ldots)$ is an abbreviation for $\neg(\exists v_k)\neg(\ldots v_k \ldots)$.] Let $A(v_1, \ldots, v_k)$ be any formula; $v_1, \ldots, v_k$ may or may not occur free in A. Then, we write $A(t_1, \ldots, t_k)$ to indicate the formula that results when, for each j, $1 \leq j \leq k$, the

A3: $(\forall v_i)(A \supset B) \supset (A \supset (\forall v_i)B)$ (where v_i is not free in A)

A4: $v_i = v_i$

A5: $v_i = v_j \supset (A(v_i, v_i) \equiv A(v_i, v_j))$
[where $A(v_i, v_j)$ is a formula that arises from $A(v_i, v_i)$ by replacing some, but not necessarily all, free occurrences of v_i by v_j, and v_j is free for the occurrences of v_i that it replaces]

A6: $[A]_{v_1 \dots v_p} \neq [B]_{u_1 \dots u_q}$ (where $p \neq q$)

A7: $[A(v_1, \dots, v_p)]_{v_1 \dots v_p} = [A(u_1, \dots, u_p)]_{u_1 \dots u_p}$
(where the externally quantifiable free variables in these two complex terms are the same and, for each k, $1 \leq k \leq p$, v_k is free in A for u_k, and conversely)

A8: $[A]_\alpha = [B]_\alpha \equiv \Box(A \equiv_\alpha B)$

A9: $\Box(A) \supset A$

A10: $\Box(A \supset B) \supset (\Box A \supset \Box B)$

A11: $\Diamond A \supset \Box \Diamond A$

R1: if $\vdash A$ and $\vdash A \supset B$, then $\vdash B$

R2: if $\vdash A$, then $\vdash (\forall v_i)A$

R3: if $\vdash A$, then $\vdash \Box A$

A1 is of course concerned with the truth-functional sentential connectives & and $\neg$. A2 and A3 are familiar axioms for first-order quantifiers. A4 asserts the reflexivity of identity. A5 is Leibniz's law. A6 asserts the distinctness, respectively, of propositions, properties and relations. A7 asserts the validity$_1$ of a change of bound variables within intensional abstracts. A8 asserts the necessary equivalence of identicals and the identity of necessary equivalents. This principle is, of course, the hallmark of conception 1. A9–A11 are the standard S5 axioms for $\Box$ and $\Diamond$. R1 is *modus ponens*. R2 is universal generalization. R3 is the necessitation rule from S5.[8]

Given the definition of $\Box$ and $\Diamond$ in terms of identity and intensional abstraction, modal logic may be viewed as just the identity theory for intensional abstracts. In this connection, notice that, whereas the principle of necessary identity:

$$x = y \supset \Box\, x = y$$

is an immediate consequence of Leibniz's law A5 (given the reflexivity of identity A4), the S5 axiom A11 is just an instance of the principle of necessary distinctness:

$$x \neq y \supset \Box\, x \neq y$$

term t_j replaces each free occurrence of v_j in A. Example: if $A(v)$ is $F[Gv]$ and t is $[Hw]$, then $A(t)$ is $F[G[Hw]]$. In this example t is free for v in $A(v)$; v is an externally quantifiable free variable in $A(v)$, and w is an externally quantifiable free variable in $A(t)$.

[8] T1 is the simplest formulation of conception 1. In it the Barcan formula and its converse are derivable. This feature can be removed by slightly complicating the axioms and rules. Corresponding adjustments would then be made in the semantics.

In fact, the S5 axiom and the principle of necessary distinctness are actually equivalent. For, given A1–A10 and R1–R3, not only is A11 derivable from the principle of necessary distinctness, but also the principle of necessary distinctness is derivable from A11.

Now I will state the primary result for T1:

THEOREM (Soundness and Completeness):

For all formulas A in L_ω, A is valid$_1$ if and only if A is a theorem of T1 (i.e., $\models_1 A$ iff $\vdash_{T1} A$).[9]

VI. THE LOGIC FOR PRP'S ON CONCEPTION 2

On conception 2, each definable intensional entity is such that when it is defined completely, it has a unique, noncircular definition. The logic T2 for L_ω on conception 2 consists of (a) axioms A1–A7 and rules R1–R2 from T1, (b) five additional axiom schemas for intensional abstracts, and (c) one additional rule. In stating the additional principles, I will write $t(F_p^q)$ to indicate that t is a complex term of L_ω in which primitive predicate F_p^q occurs.

Additional Axiom Schemas and Rules for T2

$\mathcal{A}$8: $[A]_\alpha = [B]_\alpha \supset (A \equiv B)$

$\mathcal{A}$9: $t \neq r$
(where t and r are non-elementary complex terms of different syntactic kinds)

$\mathcal{A}$10: $t = r \equiv t' = r'$
(where t and r are the negations (existential generalizations, expansions, inversions, conversions, reflexivizations) of t' and r', respectively)

$\mathcal{A}$11: $t = r \equiv (t' = r' \,\&\, t'' = r'')$
(where *either* t is the conjunction of t' and t'' and r is the conjunction of r' and r'' *or* t is the predication of t' of t'' and r is the predication of r' of r'' *or*, for some term t^*, t is the relativized predication of t, t^*, t'' and, for some term r^*, r is the relativized predication of r', r^*, r'')[10]

$\mathcal{A}$12: $t(F_i^j) = r(F_h^k) \supset q(F_i^j) \neq s(F_h^k)$
[where $t(F_i^j)$ and $s(F_h^k)$ are elementary, and $r(F_h^k)$ or $q(F_i^j)$ is non-elementary]

$\mathcal{R}$3: Let F_k^p be a nonlogical predicate that does not occur in $A(v_i)$; let $t(F_k^p)$ be an elementary complex term, and let t' be any complex term of degree p that is free for v_i in $A(v_i)$. If $\vdash A(t)$, then $\vdash A(t')$.

[9] The proof of this theorem is given in *Quality and Concept*. An important corollary of this theorem is that first-order logic with identity and extensional abstraction (i.e., class abstraction) is complete.

[10] For the explanation of the clause concerning relativized predication, see *Quality and Concept*. For the present suffice it to say that the clause is stated this way in order to handle relativized predication generally, not just the simple example given earlier.

$\mathcal{A}8$ affirms the equivalence of identical intensional entities. Schemas $\mathcal{A}9$–11 capture the principle that a complete definition of an intensional entity is unique. And schema $\mathcal{A}12$ captures the principle that a definition of an intensional entity must be noncircular. $\mathcal{R}3$ says roughly that, if $A(t)$ is valid$_2$ for an arbitrary elementary p-ary term t, then $A(t')$ is valid$_2$ for any p-ary term t'.

The following is the primary result for T2:

THEOREM (Soundness and completeness):

> For all formulas A in L_ω, A is valid$_2$ if and only if A is a theorem of T2 (i.e., $\models_2 A$ iff $\vdash_{T2} A$).

Now recall the two intuitively valid arguments mentioned at the outset of the paper. Symbolized in L_ω, these arguments are both valid$_1$ and valid$_2$, and, relatedly, in both T1 and T2 the conclusion of each argument is derivable from its premise(s).

To bring out the difference between T1 and T2 (and between validity$_1$ and validity$_2$), an example will be helpful. Consider the following *invalid* argument involving the intentional predicate 'wonders':

> x wonders whether there is a trilateral that is not a triangle.
> Necessarily, all and only trilaterals are triangles.
> ∴ x wonders whether there is a triangle that is not a triangle.

In L_ω this argument is symbolized as follows:

$$x W^2[(\exists y)(\text{Trilateral } (y) \,\&\, \neg\text{Triangle } (y))]$$
$$\Box(\forall y)(\text{Trilateral } (y) \equiv \text{Triangle } (y))$$
$$\therefore\ x W^2[(\exists y)(\text{Triangle } (y) \,\&\, \neg\text{Triangle } (y))]$$

In T1—but not in T2—the conclusion of this argument is derivable from the two premises. And, relatedly, the argument is valid$_1$ but not valid$_2$. So, clearly, the formal logic—and semantics—that is based on conception 2 is that which is appropriate for the treatment of intentional matters. The fact that Church's "alternative 2" and the various possible-worlds constructions of intensional logic (including Carnap's original construction in *Meaning and Necessity*) are all based on conception 1 is what lies at the root of their failure to provide adequate treatments of intentional matters.

Now for the synthesis of the two approaches. In relevant type 2 algebraic model structures, single out a new distinguished logical relation *Necessary Equivalence*. Adjoin an associated primitive 2-place logical predicate $\approx_N$ to L_ω. Add to T2 axioms and rules for $\approx_N$ fashioned after the T1 axioms and rules for $=$. (That is, add $\approx_N$ analogues of A4, A6–A11, R3, plus two $\approx_N$ analogues of A5, viz., $\approx_N$-symmetry and $\approx_N$-transitivity.) The resulting system T2′ is a unified logic for both modal and intentional matters. It appears that T2′ can also be proved sound and complete.

VII. THE ORIGIN OF INCOMPLETENESS

Why is it that complete logics for PRP's can be achieved in the setting of first-order logic but not in the setting of standard higher-order logic?

Consider the following intuitively valid argument:

$$\frac{x \text{ is red and } y \text{ is red}}{\therefore \text{ There is something that } x \text{ is and that } y \text{ is.}}$$

There are two approaches to the representation of this argument: the first-order approach and the higher-order approach. On the higher-order approach the argument is represented as an instance of second-order existential generalization:

$$\frac{Rx \mathbin{\&} Ry}{\therefore\ (\exists f)(fx \mathbin{\&} fy)}$$

where R is a name of the color red and f is a predicate variable for which R is a substituend. On the first-order approach the argument is represented as an instance of first-order existential generalization:

$$\frac{x\Delta r \mathbin{\&} y\Delta r}{\therefore\ (\exists z)(x\Delta z \mathbin{\&} y\Delta z)}$$

where r is a name of the color red and Δ is a distinguished 2-place logical predicate that expresses the *predication relation*, i.e., a relation expressed by the *copula* in natural language.

As I have said, the logics T1 and T2 for L_ω are provably complete relative to the standard notions of validity—i.e., validity$_1$ or validity$_2$. But suppose that a 2-place predicate (e.g., Δ) is singled out as a distinguished logical predicate and that the interpretations of L_ω are restricted in such a way that this 2-place predicate always expresses the predication relation. In this event, the logic for L_ω would be rendered essentially incomplete relative to the resulting special notion of validity. In this sense, then, it is not the apparatus of intensional abstraction—nor is it the associated infinite abstract ontology of intensional entities—that is responsible for incompleteness in the logic for PRP's. Rather, it is a fundamental logical relation on that ontology, namely, the predication relation. The reason that higher-order logics are essentially incomplete relative to their standard notions of validity is that the notation for the predication relation is built into the very syntax of higher-order languages, and consequently, the semantic import of the notation for the predication relation is never permitted to vary from one standard interpretation to another.

Completeness is possible in the first-order setting because the standard interpretations of L_ω are not restricted in such a way that a distinguished logical predicate (e.g., Δ) must always express the predication relation. Such a restriction on the interpretations of L_ω would be relevant if the theory of PRP's were being used in a construction of classical mathematics. It is not relevant, however, if the theory is being used merely to treat modal and intentional matters. Modal and intentional matters require a theory of properties, relations, and propositions of decidedly less logical power.

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Appendix

An *algebraic model structure* is any structure $\langle \mathfrak{D}, \mathcal{P}, \mathcal{K}, \mathcal{G}, \text{Conj}, \text{Neg}, \text{Exist}, \text{Exp}, \text{Inv}, \text{Conv}, \text{Ref}, \text{Pred}, \text{RelPred}, \text{Id}\rangle$ whose elements simultaneously satisfy the conditions set forth below. $\mathfrak{D}$ is the domain of discourse and is nonempty. $\mathcal{P}$ is an equivalence relation on $\mathfrak{D}$ that serves to partition $\mathfrak{D}$ into a denumerable number of disjoint subdomains: $\mathfrak{D}_{-1}, \mathfrak{D}_0, \mathfrak{D}_1, \mathfrak{D}_2, \mathfrak{D}_3, \ldots$ Although $\mathfrak{D}_i, i \geq 0$ may not be empty, we do permit $\mathfrak{D}_{-1}$ to be empty. The elements of $\mathfrak{D}_{-1}$ are to be thought of as particulars; the elements of $\mathfrak{D}_0$, as propositions; the elements of $\mathfrak{D}_1$, as properties, and the elements of $\mathfrak{D}_i$, for $i \geq 2$, as i-ary relations. $\mathcal{K}$ is a set of functions on $\mathfrak{D}$. These functions are to be thought of as determining alternate or possible extensions of the elements of $\mathfrak{D}$. $\mathcal{G}$ is a distinguished element of $\mathcal{K}$ and is to be thought of as that function which determines the actual extensions of the elements of $\mathfrak{D}$. The next nine elements of an algebraic model structure are operations each of which must satisfy an associated defining condition. For example, for all $H \in \mathcal{K}$ and for all $x_1, \ldots, x_i \in \mathfrak{D}$:

1.a. $\langle x_1, \ldots, x_i\rangle \in H(\text{Conj}(u, v)) \equiv (\langle x_1, \ldots, x_i\rangle \in H(u) \;\&\; \langle x_1, \ldots, x_i\rangle \in H(v))$ (for $u, v \in \mathfrak{D}_i, i \geq 1$)

b. $H(\text{Conj}(u, v)) = \text{T} \equiv (H(u) = \text{T} \;\&\; H(v) = \text{T})$ (for $u, v \in \mathfrak{D}_0$)

2.a. $\langle x_1, \ldots, x_{i-1}\rangle \in H(\text{Exist}(u)) \equiv (\exists x_i)(\langle x_1, \ldots, x_{i-1}, x_i\rangle \in H(u))$ (for $u \in \mathfrak{D}_i, i \geq 2$)

b. $H(\text{Exist}(u)) = \text{T} \equiv (\exists x_1)(x_1 \in H(u))$ (for $u \in \mathfrak{D}_1$)

c. $H(\text{Exist}(u)) = \text{T} \equiv H(u) = \text{T}$ (for $u \in \mathfrak{D}_0$)

3.a. $\langle x_1, \ldots, x_{i-1}\rangle \in H(\text{Pred}(u, x_i)) \equiv \langle x_1, \ldots, x_{i-1}, x_i\rangle \in H(u)$ (for $u \in \mathfrak{D}_i, i \leq 2$)

b. $H(\text{Pred}(u, x_1)) = \text{T} \equiv x_1 \in H(u)$ (for $u \in \mathfrak{D}_1$)

The last element of a model structure is Id. Id is a distinguished element of $\mathfrak{D}_2$ and is thought of as the fundamental logical relation-in-intension, *Identity*. Id must, of course, satisfy the following condition:

$$(\forall H \in \mathcal{K})(H(\text{Id}) = \{xy \in \mathfrak{D}: x = y\})$$

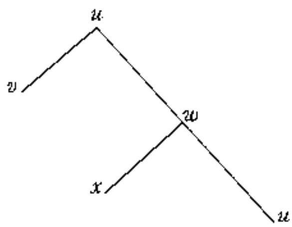
That is, every $H \in \mathcal{K}$ singles out the extensional identity relation on $\mathfrak{D}$ to be the extension of the intensional identity relation Id.

An algebraic model structure is *type 1* iff$_{df}$ it satisfies the following further condition:

$$(\forall x, y \in \mathfrak{D}_i)((\forall H \in \mathcal{K})(H(x) = H(y)) \rightarrow x = y) \quad \text{for all } i \geq -1$$

This condition provides us with precise statement of conception 1. Specifically, this condition rules out the possibility of there being two (or more) elements of any given subdomain $\mathfrak{D}_i$ which are necessarily equivalent in extension.

An algebraic model structure is *type 2* iff$_{df}$ its nine operations Conj, Neg, Exist, Exp, Inv, Conv, Ref, Pred, RelPred are (i) one-one, (ii) disjoint in their ranges, and (iii) noncycling. Conditions (i)–(iii) provide us with a precise formulation of conception 2. For, taken together, (i) and (ii) guarantee that the action of the inverses of the nine fundamental logical operations in a given type 2 model structure $\mathfrak{M}$ is to *decompose* the elements of $\mathfrak{D}$ into *unique* (possibly infinite) trees. And condition (iii) ensures that, for each item u in such a decomposition tree, u cannot occur on any path descending from u. So the following is the sort of situation ruled out by condition (iii):



Hence, whereas conditions (i) and (ii) ensure that the elements of $\mathfrak{D}$ have at most one complete definition in terms of the elements of $\mathfrak{D}$ plus the nine fundamental logical operations, condition (iii) ensures that such definitions are never circular.

Notice that in the formal characterizations of what it is to be a type 1 or type 2 algebraic model structure no use is made of any of the following intuitive notions: particular, property, relation, proposition, alternative or possible extension, actual extension, complete definition. For what it is worth, type 1 and type 2 model structures are characterized formally in exclusively set-theoretic terms.

An interpretation $\mathcal{I}$ for L_ω *relative to model structure* $\mathfrak{M}$ is any function that assigns to the predicate letter F_1^2 (i.e., =) the distinguished element Id $\in \mathfrak{M}$ and, for each remaining predicate letter F_i^j in L_ω, assigns to F_i^j some element of the sub-domain $\mathfrak{D}_j \subset \mathfrak{D} \in \mathfrak{M}$. Given the above notions, *denotation* and, in turn, *truth* for L_ω relative interpretation $\mathcal{I}$ and algebraic model structure $\mathfrak{M}$ are definable in a relatively straightforward manner. And then Tarski-style definitions of *type 1* and *type 2 validity* are immediate.

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