9

Logic

45. Truth

The ancient problem of truth was to say informally what truth is. That problem was solved by Plato and Aristotle by means of their germinal versions of the correspondence theory. The modern problem of truth is not to seek a novel definition; rather, it is to find a logically precise and clear expression of the ancient Greek definition. This at any rate is how Alfred Tarski sees the modern problem:

> We should like our definition to do justice to the intuitions which adhere to the *classical Aristotelian conception of truth*—intuitions which find their expression in the well-known words of Aristotle's *Metaphysics*:
>
> > To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true.
>
> However, all these formulations [i.e., the original Aristotelian formulation and the sundry modern attempts to capture it] can lead to various misunderstandings, for none of them is sufficiently precise and clear . . .; at any rate none of them can be considered a satisfactory definition of truth. It is up to us to look for a more precise expression of our intuitions. (§3, 'The Semantic Conception of Truth and the Foundations of Semantics')

Tarski's semantic conception of truth is the backbone of the Tarskian technique for characterizing validity for formal languages. I will show in a while (§47) that there are significant obstacles to a general account of validity along Tarski's semantic lines. But first let me examine the semantic conception of truth, not as a formal device used in the model-theoretic characterization of validity, but rather as a serious account of truth itself. This exercise will help to motivate my own solution to the modern problem of truth.

I gave evidence in §6 and §8 for the thesis that 'that'-clauses are singular terms whose semantical correlates are propositions, rather than linguistic entities such as sentences. Given this thesis, there is good evidence that truth is a property of propositions, as is suggested by the following intuitive validity containing a 'that'-clause:

It is true *that snow is white* if and only if snow is white.

Given the theory of qualities and concepts advanced in the previous chapter, the type of propositions that can have the property of truth are the ones known as *thoughts*. Therefore, I conclude that, as a minimum requirement, an adequate theory of truth must provide an account of what it takes for a thought to be true.

Now despite the fact that truth is a property of thoughts, we also commonly say of *sentences* that they are true relative to some language or other. For example, we say that English sentences are true (i.e., true-in-English), that French sentences are true (i.e., true-in-French), that Polish sentences are true (i.e., true-in-Polish), and so on for all languages. The semantic conception is predicated on this elementary linguistic fact.

The word 'true' thus has a multiplicity of uses—one use for propositions and an indefinitely large constellation of uses for sentences, one distinct use for each of the various languages. How are we to explain this multiplicity of uses?

One explanation is that it is merely ambiguity, merely *chance homonymy*. Tarski's polemical remarks about the 'meaninglessness' of 'those endless, often violent discussions on the subject: "What is the right conception of truth?"' and his advice that we should 'reconcile ourselves with the fact that we are confronted, not with one concept, but with several different concepts which are denoted by the same word' (§14, 'The Semantic Conception') suggest that he is sympathetic with this chance homonymy explanation. This suggestion is borne out by Tarski's strategy for giving semantic truth definitions. For he holds that,

> ...we must always relate the notion of truth, like that of a sentence, to a specific language; for it is obvious that the same expression which is a true sentence in one language can be false or meaningless in another. (§2, 'The Semantic Conception')

This leads him to define the concepts of true-in-L, for different languages L, wholly independently of one another.

There is, however, another explanation of the multiplicity of uses of 'true', one which is much better. It is an application of Aristotle's theory of non-chance homonymy, or *focal meaning*, as it has been called.[1] Aristotle gives focal-meaning accounts for multiple uses of 'be', 'healthy', and 'medical':

> There are many senses in which a thing may be said to 'be', but all that 'is' is related to one central point, one definite kind of thing, and is not said to 'be' by a mere ambiguity. Everything which is healthy is related to health, one thing in the sense that it preserves health, another in the sense that it produces it, another in the sense that it is a symptom of health, another because it is capable of it. And that which is medical is relative to the medical art, one thing being called medical because it possesses it, another because it is naturally adapted to it, another because it is a function of the medical art. And we shall find other words used similarly to these.
>
> As, then, there is one science which deals with all healthy things, the same applies in the other cases also. For not only in the case of things which have one common notion does the investigation belong to one science, but also in the case of things which are related to one common nature; for even these in a sense have one common notion.
>
> But everywhere science deals chiefly with that which is primary, and on which the other things depend, and in virtue of which they get their names. (*Metaphysics*, book Gamma, 1003^a3–1003^b18)

A focal-meaning explanation of the multiple uses of 'true' goes as follows. The fact that English sentences can be said to be true in their way (i.e., true-in-English), that French sentences can be said to be true in another (i.e., true-in-French), etc. is not a matter of a "mere ambiguity". Rather they are said to be true in virtue of the fact that they are all related to "one central point", namely, a central concept of truth. Specifically, for each given language L, a sentence is true-in-L if and only if what it expresses in L is true in the primary sense. Since what a sentence expresses is a thought, thoughts are the things that are true primarily. So the central concept of truth is the concept of a true thought. Sentences in the various languages are called true only secondarily, by virtue of the fact that they are related via their respective meaning relations to true thoughts. Accordingly, a theory of truth should "deal chiefly" with the concept of a true thought. For it is on this concept that the constellation of secondary truth concepts depends; it is in virtue of their relation to this concept that sentences may be called true secondarily.[2]

Defects in the semantic conception of truth become evident as soon as one sees truth for sentences as dependent upon the central concept of a true thought. Its most glaring fault is that it completely by-passes the primary concept of a true thought, which is that in virtue of which the indefinitely many semantical truth concepts qualify as truth concepts at all. Doing so, it abandons the possibility of explaining why they are all called truth concepts. Matters are worsened for Tarski's semantic conception by its being stated in terms of the theory of reference rather than the theory of meaning. It attempts to define a sentence's truth in terms of relations among the "references" of its primitive predicates and names. But if a sentence is true because of the truth of the thought it expresses, then the "references" of the sentence's predicates would be only indirectly related to the sentence's truth; for the "references" of the predicates could not determine which proposition a sentence expresses. What the predicates *express* is what is relevant to the thought expressed and, in turn, to the sentence's truth. Furthermore, predicates do not refer to anything in the first place; they only express. Tarski's semantic conception of truth thus has the added trouble of resting on a questionable theory of the fundamental relations between words and things. (See §23 and §38 for an extended critique of referential semantics.) A final problem in Tarski's theory of truth is that it is framed within set theory. But set theory is an artifice without ground in our naturalistic ontology or natural logic and without pragmatic justification either. (See chapter 5 for a critique of set theory.) The theory of qualities and concepts is the proper theory within which to frame a theory of truth.

As Aristotle says, 'everywhere science deals chiefly with that which is primary, and on which the other things depend, and in virtue of which they get their names.' In view of the foregoing discussion, one may conclude that the chief task of a theory of truth is to define the concept of a true thought, for this is the central concept upon which the other truth concepts depend and in virtue of which they are called truth concepts. At the outset of his search for a commonsense theory of truth, Bertrand Russell gives the following reasonable advice:

> ...[We] have to seek a theory of truth which (1) allows truth to have an opposite, namely falsehood, (2) makes truth a property of beliefs, but (3) makes it a property wholly dependent upon the relation of the beliefs to outside things. (p. 123, *The Problems of Philosophy*)

Thus according to Russell, truth is a property of beliefs (i.e., thoughts) that depends upon a relation with things outside, i.e., upon 'a correspondence of thought with something outside thought' (p. 121, *Problems*). And this view is, to the naive eye, a virtual truism. Even those generally unsympathetic to this account of truth acknowledge its privileged position:

> There can be no denying the attractiveness of this view: it seems to be just right. It struck the great philosophers who first considered the problem of truth—viz., Plato and Aristotle—as so obviously the correct one that the question of possible alternatives to it never occurred to them. And certainly if there were such a thing as the common-sense view of truth, it would be the correspondence theory. Common-sense views of this sort may all, in the end, be correct, once they are properly understood; and to call them "common-sense views" is to claim that at the outset they appear to be straightforwardly and undeniably correct. But between the outset and the end (when they are at last "properly understood")—that is to say when they are in the hands of the philosophers—they inevitably run into tough sailing. (p. 4, George Pitcher, *Truth*)

Now if a commonsense view can be made fully clear and precise and if at the same time it can be economically integrated into a larger body of accepted theory, then it is to be preferred over views that clash with it. From Plato and Aristotle down to Russell and Tarski, the correspondence theory of truth has been almost universally acknowledged as the commonsense view. Indeed, the only cogent objection to it has been that it has defied clear and precise formulation. To be sure, a thought is true if and only if it corresponds to a condition in the world, i.e., to a condition that obtains. But what is a thought? What is a condition? What is it for a thought to correspond to a condition? And what is it for a condition to obtain? These are questions that modern correspondence theorists have attempted in vain to answer. The reason for their failure is that they, like nearly all modern philosophers, have been under the spell of representationalism. Their representationalism has blinded them to the fact that the classical correspondence theory is based on realism. The only way to give the correspondence theory a clear and precise formulation is within a realistic framework such as that embraced by Plato and Aristotle.

This is where the theory of qualities and concepts comes in. For, as shown in §42, it harks back to the realism of Plato and Aristotle. Indeed, within the framework provided by this logical theory I have already been able to give definitions of the concepts of a thought and a condition and also of the correspondence relation itself.

Therefore, it remains only to define what it is for a condition to obtain. However, this may be easily done, e.g., as follows:

> x obtains $\mathit{iff}_{\mathrm{df}}$ for some property which has an instance, x is just the condition that this property does have an instance.

In symbols, x obtains $\mathit{iff}_{\mathrm{df}}$ $(\exists y)((\exists z)z \,\Delta\, y \;\&\; x = \lvert(\exists z)z \,\Delta\, y\rvert^{y})$.[3] Using these definitions, I then define the single central concept of truth:

> x is true $\mathit{iff}_{\mathrm{df}}$ x corresponds to a condition that obtains.[4]

What about the semantical and intentional paradoxes? In the ramified type theories proposed by Russell and by Church these paradoxes necessitate an infinite hierarchy of non-equivalent truth concepts for thoughts. However, in the setting of the theory of qualities and concepts such a hierarchy of truth concepts is not needed, for the paradoxes can instead be diagnosed and avoided by means related to those described in §26.[5] Thus, in line with the theory of Plato and Aristotle we have a definition of a single, central concept of truth. Since this definition can be written out entirely in the canonical logical language $\mathscr{L}$ with Δ, here is a clear and precise expression of the classical correspondence theory of truth.

46. Necessity

> ...[T]he terms of *efficacy*, *agency*, *power*, *force*, *energy*, *necessity*, *connexion*, and *productive quality*, are all nearly synonimous; and therefore 'tis an absurdity to employ any of them in defining the rest.
>
> ...[W]hen we speak of a necessary connexion betwixt objects, and suppose, that this connexion depends upon an efficacy or energy, with which these objects are endow'd; in all these expressions, *so apply'd*, we have really no distinct meaning, and make use only of common words, without any clear and determinate ideas. ('Of the idea of necessary connexion', *A Treatise of Human Nature*)

David Hume thus called into question the existence of the concept of a necessary connection. In recent years W. V. O. Quine has called into question the existence of the concept of analyticity. These doubts might seem unrelated. However, like Carnap and many logical positivists, Quine has made a practice of writing as though he thinks analyticity and necessity are the same. In this way Quine may be heard as a contemporary echo of Hume. In fact, the doubts of Hume and Quine have substantially the same origin, being founded in each case on the same sort of argument about definability. The argument goes as follows: if it were to exist, the concept of necessity (analyticity) ought to have a non-circular definition, and yet all the candidate definitions appear upon analysis to be circular. In this section the Humean doubt about the concept of necessity will be met head-on by means of non-circular definitions of necessity and necessary connection. In the subsequent section the Quinean doubt about the concept of analyticity will be met. This two-step strategy is needed, for despite the attitudes of Carnap and Quine, necessity and analyticity have significantly different definitions. It should go without saying that what makes my definitions possible is the theory of qualities and concepts, a logical theory that in relevant respects is much stronger than either Hume's psychology of impressions and ideas or Quine's set-theoretic materialism. Before getting to my definitions of necessity and necessary connection, however, I should like to take a moment to comment on Carnap's popular possible-worlds approach to the problem of necessity. This alternate approach, whose origins may be traced to the writings of Leibniz, has been at once hailed as a worthwhile formal tool and condemned as circular. It would be good, therefore, to get straight on this issue.

In my view both of these judgments of the possible-worlds approach are sound. The reason for this is that there are two quite distinct uses to which this approach is put. First, it is used in formal semantics to characterize validity for certain languages containing necessity and possibility operators. In this application the possible-worlds approach is free of circularity, for the construction is just a part of set theory; in particular, it is a part of model theory. To be sure, certain set-theoretic objects are sometimes spoken of as possible worlds, possible individuals, etc. However, such talk is heuristic in character. In the formal statement of the theory all such talk disappears, and only the vocabulary of first-order extensional set theory remains. Now this application of the possible-worlds approach has been successful within limits. A critic, once he understands the nature of the project, must acknowledge that the class of valid sentences in certain formal languages can be characterized within set theory by means of the possible-worlds technique.

There is, however, another use to which the possible-worlds approach is put; namely, it is used to define necessity itself:

x is necessary $\mathit{iff}_{\mathrm{df}}$ x is true in all possible worlds.

Here, the talk of possible worlds is not a mere heuristic, eliminable in favor of set-theoretic talk. On the contrary, 'is a possible world' and 'is true in' are actually primitive constants in the theory. But what is a possible world? And what is it to be true in one? We are as much in the dark on these questions as we are about the nature of necessity. Indeed, the circularity of the possible-worlds definition of necessity can be made explicit. For given the theory of qualities and concepts and the theory of ordinary aggregates (§27)—two theories for which there is independent justification—the primitive terms of the possible-worlds definition of necessity can be defined in terms of necessity. These definitions might go as follows:

w is a world iff_{df} for every proposition x, x is in w or the negation of x is in w.

x is-true-in w iff_{df} x is a proposition and x is in w.

w is a possible world iff_{df} w is a world and no necessarily false proposition is-true-in w.[6]

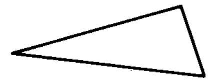
Thus, a "world" is a maximal aggregate of propositions, and a possible world is just one that does not contain necessary falsehoods. Notice, however, that the circularity just exposed in the possible-worlds definition of necessity is precisely the sort that Hume and Quine would criticize. And in this instance at least the criticism seems justified. The conclusion, therefore, is that the possible-worlds approach is not of use in solving the classical problem of necessity.

Let us try another approach. Thoughts are very finely differentiated things; in fact, there is no limit to the number of thoughts that are necessarily true. The following examples will help to indicate the variety there is among the thoughts that are necessary. The thoughts that triangles are triangles, that triangles are three sided, that $5 + 7 = 12$, that the continuum hypothesis is independent of the axioms of Zermelo-Fraenkel set theory, that colors are incompatible, that aesthetic qualities are supervenient: these thoughts are distinct from one another, and they are all necessary. If one is to solve the problem of necessity, one must frame the definition so that every such thought comes out as necessary. Now, recall that a thought is true if and only if it corresponds to a condition that obtains. By analogy, a thought is necessary if and only if it corresponds to a condition that must obtain, i.e., to a condition that is necessary. But I have already given a definition of what it is for a thought to correspond to a condition. Thus, the problem of defining what it is for a thought to be necessary will have been solved if one can define what it is for a condition to be necessary.

So far so good. But now let me consider the matter of necessary connection. Humeans have been unable to make progress on this problem largely as a result of their nominalism and ontological extensionalism. Let me explain. Suppose that there is a necessary connection x between two things y and z. The Humean, being a nominalist, first looks to the individual pair of objects y and z in order to discover what it is for them to have a necessary connection. And, of course, he must fail there. For, as Russell makes clear, relations are located nowhere, and hence, they cannot be discovered in the objects that they relate. Next, the Humean, being an ontological extensionalist, looks to various representative samples of pairs of objects resembling y and z. And, of course, he must fail here too. For connections are intensional and, therefore, can never be adequately characterized by means of samples of their instances, no matter how complete the samples.

The underlying source of the Humean's problem is that he allows himself virtually no *logic*. Now the only reasons that the Humean has for this frugal practice are questionable epistemological ones. I say questionable, for beings possessing so little logical facility could not be deemed rational. We certainly are not beings of that sort. With an appropriate logic, one with which we have a native facility, the problem which only baffles the Humean submits to solution. What I have in mind is the logic for qualities and concepts. Within this logic one is able to define the key logical concept of a connection. The definition is given entirely in terms of the fundamental logical operations. Connections are the special relations from which thoughts can be built up by means of these fundamental logical operations. Given this, if one can also define what it is for a condition to be necessary, then the definition of necessary connection is immediate: x is a necessary connection between y and z if and only if the condition that x is a connection between y and z is a necessary condition. Thus, one has only to consider the matter of necessary condition.

Recall that conditions conform to the first traditional conception of intensional entities, conception 1. Conditions thus are identical if and only if they are necessarily equivalent. Consider, for example, a condition that involves the little black object below:

(1) 

The condition that (1) is three-angled, the condition that (1) is three-sided, the condition that (1) is triangular and such that $5 + 7 = 12$, the condition that (1) is triangular and such that the continuum hypothesis is independent of the axioms of Zermelo-Fraenkel set theory, etc.: these conditions are all the same condition. They are all identical to the same condition right here in the world; it is a condition that I am observing right now. True enough, there is no limit to the number of necessarily equivalent ways I can think about (1) and its shape. However, here in the world there is only one condition to which all these distinct thoughts correspond. Now the same thing goes for necessary conditions. Consider, e.g., the necessary condition that triangles are triangles. What condition is this? It is a condition that must obtain no matter what. It is a way the world must be, come what may. However, there is one and only one way the world must be: it is the way the world necessarily is. There is one and only one condition that must obtain: it is the necessary condition of the world. Thus, the condition that triangles are triangles is a condition in the world that coincides with, e.g., the condition that triangles are three-sided; the condition that $5 + 7 = 12$; the condition that colors are incompatible; the condition that aesthetic qualities are supervenient; etc. These are all the same condition in the world. To be sure, there is no limit to the number of ways one can think about how the world must be; there is no limit to the number of necessary contents of mind (i.e., necessary thoughts) one might have. But they all correspond to the same condition in the world, the condition that a thing is what it is.

Thus, I arrive at the following three definitions:

x is a necessary condition $\mathit{iff}_{\mathrm{df}}$ $x =$ the condition that $x = x$.[7]

x is a necessary thought $\mathit{iff}_{\mathrm{df}}$ x corresponds to a necessary condition.

x is a necessary connection between y and z *iff*$_{\mathrm{df}}$ the condition that x is a connection between y and z is a necessary condition.

And in the same vein:

$x \approx_{\mathrm{N}} y$ *iff*$_{\mathrm{df}}$ the condition that x is equivalent to y is necessary, and x and y are the same degree.

These definitions can all be written out fully within the purely logical language $\mathscr{L}$ with Δ.

According to this analysis, then, necessity is neither a naturalistic nor an empirical nor a mysterious intuitive concept. It is a logical concept, and a fairly simple one at that. To define necessity, one must only appeal to the fundamental logical operations by means of which conditions and thoughts are formed. Intuitively, the analysis works because these logical operations, together with the genuine qualities and connections upon which they ultimately operate, are the things that determine what is necessary.

It is natural at this point to wonder what the relationship is between necessity, as I have analysed it, and the special kind of necessity known as *logical necessity*. The answer is a truism: logical necessity is that species of necessity having to do with *logic*. Specifically, a thought is logically necessary if and only if it is necessary by virtue of logic alone. It would seem that not all necessary thoughts are logically necessary. For example, it would seem that some are necessary by virtue of metaphysics, and perhaps others are necessary by virtue of causal law. In the next section I will try to define this special concept of necessity. For the present, however, the important point is that logical necessity and metaphysical necessity (and perhaps necessity arising from causal law) are species of the general concept of necessity that was analysed here.

I will close this section with a remark about my Russellian semantic method (defended in §38). Notice that all algebraic model structures $\mathscr{M}$ contain an element $\mathscr{K}$ which, as I indicated in §14, might be thought of as determining various alternate or possible extension functions for the universe of discourse $\mathscr{D}$. Some people might see $\mathscr{K}$ as a vestige of the possible-worlds semantic method and conclude on that basis that my Russellian semantic method must, after all, appeal to possible worlds at least vestigially.[8] However, this would be an error, as I will now explain.

I mentioned in §28 that all the metatheory done in this work should finally be understood, not as part of set theory, but rather as part of a theory of intensional entities. This comment applies to my Russellian semantics: it should be understood as a construction within a general background theory of qualities and concepts. (For example, when one gets around to constructing a serious semantics for fragments of natural language, the universe of discourse will contain intensional entities of the sort provided by the background theory.) Now this background theory will be stated in a first-order extensional language akin to the canonical language $\mathscr{L}$ in which one can express the predication relation and the fundamental logical operations. However, this logical machinery renders the use of algebraic model structures unnecessary. In order to do a Russellian semantics for a fragment of natural language, one need only specify a Russellian interpretation $\mathscr{I}$ and then give a recursive definition of the Russellian meaning function $\mathbf{M}_{\mathscr{I}}$. This can all be done straightaway without algebraic model structures (and without $\mathscr{K}$s); one simply uses the special logical machinery provided by the background theory. This, then, is my final proposal for semantics; it makes no allusion to possible worlds.

But where does this procedure leave us on the matters of necessity and possibility? How is one to do the semantics for natural-language talk of necessity and possibility? The answer is that one should do the Russellian semantics for this fragment of natural language just as one would any other fragment. Of course, in this fragment there are expressions whose ordinary meanings are the properties necessity and possibility. Therefore, one will want to choose the interpretation $\mathscr{I}$ so that the relevant expressions are mapped onto these properties. No problems arise here, however, for necessity and possibility are, as I have shown, definable within the theory of qualities and concepts. This is all that is required to obtain a perfectly adequate semantics for the language of necessity and possibility, and it is free of all vestiges of possible worlds.[9]

47. Analyticity

In all judgments in which the relation of a subject to the predicate is thought (I take into consideration affirmative judgments only, the subsequent application to negative judgments being easily made), this relation is possible in two different ways. Either the predicate *B* belongs to the subject *A*, as something which is (covertly) contained in this concept *A*; or

B lies outside the concept *A*, although it does indeed stand in connection with it. In the one case I entitle the judgment analytic, in the other synthetic. (Immanuel Kant, *Critique of Pure Reason*)

By use of this metaphorical language Kant introduced his famous concept of analyticity, a concept that would play a central role in his own system of philosophy and in much of the subsequent philosophy in the European tradition. The concept was by no means original with Kant. Closely related concepts had played a role in the thought of other modern philosophers: Locke's trifling propositions, Leibniz's identical propositions, Hume's relations of ideas. The problem of analyticity confronting philosophers today is that of giving a precise, non-circular definition of the concept, a definition which facilitates answers to the fundamental epistemological questions that troubled Kant originally.

The Quinean position is that the problem has no solution:

But for all its *a priori* reasonableness, a boundary between analytic and synthetic statements simply has not been drawn. That there is such a distinction to be drawn at all is an unempirical dogma of empiricists, a metaphysical article of faith. (p. 37, W. V. O. Quine, 'Two Dogmas of Empiricism')

Now some might think that a solution to the Humean problem of necessity can double as a solution to the problem of analyticity. Indeed, following in the footsteps of Carnap, Quine often writes as though the concepts of analyticity and necessity were interchangeable. I will argue that they are not, however, and if I am right we must look elsewhere for a definition of the Kantian concept.

In 'Notes on the Theory of Reference' Quine paints the following pessimistic picture:

In Tarski's technical construction... we have an explicit general routine for defining truth-in-*L* for individual languages which conform to a certain standard pattern and are well specified in point of vocabulary. We have indeed no similar single definition of 'true-in-*L*' for variable '*L*'; but what we do have suffices to endow 'true-in-*L*', even for variable '*L*', with a high enough degree of intelligibility so that we are not likely to be averse to using the idiom.

See how unfavorably the notion of analyticity-in-*L*, characteristic of the theory of meaning, compares with that of truth-in-*L*. [We have no]... systematic routine for constructing definitions of 'analytic-in-*L*', even for the various individual choices of *L*; definition of 'analytic-in-*L*' for each *L* has seemed rather to be a project unto itself. The most evident principle of unification, linking analyticity-in-L for one choice of L with analyticity-in-L for another choice of L, is the joint use of the syllables 'analytic'. (p. 138)

But what Quine says here does not hold up. There is a general routine for defining analyticity-in-L, one that is fully comparable to Tarski's routine for defining truth-in-L. In order to arrive at this routine for defining analyticity-in-L, one must only combine the formal algebraic semantics of §14—which leads to a definition of validity-in-L—and the Russellian semantics of §38—which yields a definition of meaning-in-L. The resulting routine yields a neat little definition of analyticity-in-L:

> A is analytic-in-L relative to interpretation $\mathscr{I}$ and algebraic model structure $\mathscr{M}$ *iff*$_{\text{df}}$ there is a valid L-formula B such that, for some interpretation $\mathscr{I}'$, the meaning of B relative to $\mathscr{I}'$ and $\mathscr{M}$ is the same as the meaning of A relative to $\mathscr{I}$ and $\mathscr{M}$.

i.e.,

$$\mathrm{An}_{\mathscr{I}\mathscr{M}}(A) \ \textit{iff}_{\text{df}} \ (\exists B)(\vDash B \ \& \ (\exists \mathscr{I}')\mathrm{M}_{\mathscr{I}\mathscr{M}}(A) = \mathrm{M}_{\mathscr{I}'\mathscr{M}}(B)).^{10}$$

Though the doubt raised by Quine in 'Notes on the Theory of Reference' can be resolved in this way, Quine would not feel that the original problem of defining analyticity had been solved. For to employ the above routine in the case of a particular spoken language L, one must first know the correct interpretation $\mathscr{I}$ for the primitive predicates and names in L. Yet Quine thinks there are insurmountable barriers to such empirical semantic knowledge.[11] Now although I believe such skepticism can be overcome, this epistemological controversy is a side issue. As I will show later, the concept of an analytic proposition is directly definable without recourse to any semantical concepts. Thus, the solution to the original problem of defining analyticity does not ride on the possibility of empirical semantic knowledge; it depends only on whether one has an adequate theory of propositions.

Quine links the problem of analyticity to a problem in the theory of empirical knowledge because he embraces what may be called the logical positivist conception of analyticity. This conception is distinctive in two ways: (1) it treats interpreted sentences as the primary bearers of analyticity, and (2) it treats an interpreted sentence as analytic if and only if the sentence is alike in meaning to a valid sentence, where validity is understood in Tarski's model-theoretic way. Quite apart from Quine's skepticism about empirical semantic knowledge, there are reasons not to embrace the logical positivist conception. First, I have shown (§45) that propositions, not sentences, are the primary bearers of truth (falsehood); sentences are true (false) only secondarily through their meanings. It would seem by analogy that propositions should be identified as the primary bearers of analyticity and that sentences should be counted as analytic (synthetic, contradictory) only secondarily through their meanings. Secondly, there are, I will argue, difficulties in the Tarskian model-theoretic conception of validity. Before I consider that issue, however, it will be helpful to explore the implications of the thesis that propositions are the primary bearers of analyticity.

Since the time Kant introduced his concept of analyticity there have been only a few attempts to find a clear and precise definition for it. One attempt is found in Carnap's *Meaning and Necessity*. In this work, however, Carnap in effect identifies analyticity with necessity: '"*L-true*" is meant as an explicatum for what Leibniz called necessary truth and Kant analytic truth' (p. 8). (Note that Carnap defines 'L-true' in terms of his state descriptions, which he indicates (p. 9) are intended to '...represent Leibniz' possible worlds or Wittgenstein's possible states of affairs'.) Continuing the Carnapian approach, David Lewis also in effect identifies analyticity with necessity. (P. 174, *Convention*. Note that Lewis identifies necessity with truth-in-all-possible-worlds.) In my view, this approach to analyticity is not what is wanted. Kant held that whatever is knowable *a priori* is necessary and, hence, that whatever is synthetic *a priori* is a synthetic necessity; the greater part of the *Critique of Pure Reason* is devoted to the study of these synthetic necessities. However, given the possible-worlds definition, it follows by a one-step inference that synthetic necessities could not exist.[12] Kant might have been mistaken about the existence of synthetic necessities, but if so, the mistake was a deep theoretical one. We do not want to undercut trivially Kant's philosophy and the tradition surrounding it simply by the way we define analyticity. Rather, we want to sharpen Kant's informal, metaphorical definition in a way that enhances the investigation of the existence of synthetic necessities.

The problem with the possible-worlds definition of analyticity stems from its reliance on the traditional conception of intensions according to which they are identical if and only if they are necessarily equivalent—that is, its reliance on conception 1. Since on this conception there is only one necessity and since all analyticities are necessary, it follows all too quickly that there can be no synthetic necessity. The way out of this problem is to give due weight to the fact that in the Kantian scheme *judgements* are primary bearers of analyticity. 'Judgement', like 'thought', is ambiguous. It can be used to mean a kind of intentional act and can also be used to mean a type of proposition that is the characteristic object of that kind of intentional act. In turn, there are two uses of 'analytic', one for intentional acts of judging and one for the propositional objects of those intentional acts. These two uses of 'analytic' are related by the following elementary equivalence: the intentional act of judging the proposition x is analytic if and only if the proposition x is analytic. Since the relation between the two uses is so direct, one need not be concerned here with the issue of whether the intentional act or the propositional object should be taken as the primary bearer of analyticity,[13] one should feel free to adopt either alternative. And since the latter alternative leads to a simpler treatment, I will adopt it as a matter of convenience. This practice is consonant with much, though not all, of Kant's own usage. So what type of 0-ary intensions are typically the objects of these intentional acts (judgements, thoughts, denials, hunches, recollections, etc.)? They are, of course, the type that fall under traditional conception 2; that is to say, they are thoughts. Thoughts may thus be identified as the primary bearers of analyticity.[14]

Since Kant's time the theory of logical form has undergone significant change. We entertain a much richer system of formal classification of thoughts than Kant did. Kant mentions only two categories of thoughts in his definition of analyticity, affirmative and negative subject/predicate thoughts, and he explicitly defines analyticity only for affirmative subject/predicate thoughts. But he does say, 'I take into consideration affirmative judgments only, the subsequent application to negative judgments being easily made'. The implication is that there are negative analytic thoughts, too, and that the general notion of analyticity is to be obtained by appropriately adapting the circumscribed definition to this further category of thought. (So, e.g., if it is analytic that all A are B, then it is also analytic that all things that are not B are not A; etc.) In view of this, it is natural to ask what Kant would say about the still further categories of thoughts entertained by our logical theory.[15]

Presumably, he would adopt the same attitude toward these further categories as he did toward the category of elementary negative thoughts. (If not, what good reason could he have for drawing the line here, just after admitting negative analytic thoughts?[16]) In that event, the general Kantian concept of analyticity must be obtained, not by piecemeal extensions of the original circumscribed definition, but rather by generalization on the essential underlying feature he was trying to get at in the original.

When one performs this kind of generalization, one arrives at more or less the following. Analytic thoughts are those that must be true by virtue of logic alone; their particular non-logical content is immaterial. But notice, thoughts that must be true are necessary, and thoughts that are necessary by virtue of logic alone, independently of their non-logical content, are necessary logically. Now, for Kant, not all necessary thoughts are necessary logically. Some are necessary by virtue of their non-logical content. For Kant, the necessities that are logical are analytic, and the necessities that are non-logical are synthetic.

When we say of a thought that it must be true by virtue of logic alone independently of its non-logical content, what we mean is that the thought is one that must be true because of its logical form and that its non-logical content is immaterial. However, thoughts that must be true by virtue of their logical form are none other than those that are valid. Thus I arrive at the following conclusion. An analytic thought is just a thought that is valid, where a valid thought is one whose necessity is logical rather than non-logical in nature. The problems of defining analyticity, validity, and logical necessity are consequently one and the same. Now a thought is made necessary by its logical form (independently of its non-logical content) if and only if any proposition having the same logical form (though perhaps a different non-logical content) is necessary. So the problem of defining validity (analyticity, logical necessity) turns on the question of what the logical form of a thought is.

There are two opposing views on this question. According to the first, the logical form of a thought is simply the abstract shape (form) of its complete thought-building tree, i.e., the tree determined by the inverses of the fundamental thought-building operations. Besides these fundamental logical operations, the identity of the other nodes (i.e., the content) in the tree is immaterial. The second view of logical form is just like the first except that the identity of all purely logical nodes is counted in. That is, the logical form of a thought is the form of its complete thought-building tree when the purely logical content is held constant.

The second view is, I maintain, clearly the right one. For it is only on this view that elementary validities involving, say, identity and necessary equivalence (e.g., $[(\forall x)x = x]$ and $[(\forall x)x \approx_{\mathrm{N}} x]$) qualify as valid. However, if the purely logical relations of identity and necessary equivalence are counted in, then so must the purely logical relation predication. The predication relation is no less a logical relation than are identity and necessary equivalence. In fact, I have shown that identity and necessary equivalence are definable in terms of the predication relation (together with the fundamental logical operations).[17] And just as there are highly intuitive elementary validities involving identity and necessary equivalence, there are highly intuitive elementary validities involving the predication relation (e.g., $[(\forall x)(x \text{ lives} \equiv x \text{ is living})]$).[18] In this matter, the predication relation is for Kant the paradigm of a purely logical relation; it is central throughout his considerations of what it takes for a thought to be analytic. Thus, I conclude that the logical form of a thought is determined by all the purely logical elements that show up in its analysis under the inverses of the thought-building operations; in particular, the predication relation is to be counted as one of these purely logical elements.

With this conclusion in hand, let us consider the Tarskian model-theoretic account of validity, which underlies the logical positivist conception of analyticity. The goal of the Tarskian account is to define in exclusively set-theoretic terms what it takes for a sentence in a given language to be valid. Everyone must admit that the Tarskian account of validity achieves its goal for a number of interesting languages and that this is of considerable value. Nevertheless, a significant obstacle seems to stand in the way of a general Tarskian account of validity.[19] This obstacle is tied to the fact that the logical form of a thought is determined in part by occurrences in its analysis of the purely logical relation *predication*.[20] Let me explain. On the usual Tarskian account, the validity of a sentence is just truth-in-*all*-possible-models. However, when a predicate (e.g., $=$, $\approx_{\mathrm{N}}$, Δ) is singled out as a distinguished logical predicate, the Tarskian must appropriately narrow the class of models if he is to get the right result. For example, when $=$ is singled out as a distinguished logical predicate, the class of models is narrowed down so as to include only those models $\langle \mathscr{D}, \mathscr{R} \rangle$ in which the "reference" of the predicate $=$ is just the extensional identity relation on $\mathscr{D}$, i.e., $\mathscr{R}(=) = \{xy \in \mathscr{D} : x = y\}$. In §17 I showed how a comparable narrowing of the admissible models can be attained for the distinguished logical predicate $\approx_N$. The problem facing the Tarskian account of validity is to do the same thing for the distinguished logical predicate Δ, which on the intended interpretation expresses the predication relation. Gödel's first incompleteness theorem shows that there is no syntactical solution to the problem. And in §26 I showed that a model-theoretic version of Russell's paradox results if one attempts to solve the problem by naively requiring that $\mathscr{G}(\bar{\Delta}) = \{xy : x \in \mathscr{G}(y)\}$. The only way I know to guarantee a correct model-theoretic account is to *use* a Δ-predicate in the metatheory itself and, thereby, explicitly to transgress the set-theoretic limits imposed by Tarski. But if I am right about this, why bother to go to all the trouble to give a model-theoretic account? It is far more simple and natural to dismantle the model-theoretic superstructure and to give the definition of validity within the object-theory itself, i.e., within the full theory of qualities and concepts with the predication relation. In so doing, one should take the theory at face value on its intended standard interpretation. There is no shame in this strategy, for set theory—not the theory of qualities and concepts—is the theory that one should be happy to do without. By the stage when one is framing a general definition of validity, the Tarskian model-theoretic approach has outlived its usefulness.

The best strategy, therefore, is to define straightaway what it is for a thought to be valid. Then and only then can a general definition of validity for sentences be given: a sentence is valid in (a fragment of) a language *iff*$_{\mathrm{df}}$ it expresses a valid thought in (the fragment of) the language. Thus, there can be an adequate general definition of validity for sentences if and only if there is an adequate definition of the Kantian concept of analyticity. Ironically, by attacking the possibility of an adequate definition of analyticity, philosophers unwittingly attack the possibility of an adequate general definition of validity for sentences.

Let me turn now to the definition of analyticity. A thought, I have said, is analytic if and only if every thought having the same logical form is necessary. What I must do now is to turn this informal definition into a formal one. Assume for a moment that we know what a *purely logical object* is (i.e., the sort of object that is a purely logical node in a thought-building tree). Let t and t' be two complete thought-building trees (i.e., thought-building trees that cannot be analysed further by means of the inverses of the thought-building operations). Suppose that t and t' can be obtained from one another by making replacements among their non-logical nodes in accordance with the following rule: for all objects v and v', if v is a node in t and v' is the associated node in t', then v is found in t at all and only those places where v' is found in t'. Let u be the thought that has tree t, and let u' be the thought that has tree t'. In this case u and u' are defined to have the same logical form. Then, analyticity may be defined in the intuitive way:

> A thought is analytic iff_{df} every thought having the same logical form is necessary.

Suppose that a predicate for purely logical objects is adjoined to the purely logical language $\mathscr{L}$ with Δ. Then, given that there are not complete thought-building trees having infinitely long branches, one can write out this definition of analyticity in a Zermelo-style theory for Δ.

It would be nice to have a definition of what a purely logical object is. I will suggest one. It should be noted, though, that the correctness of the definition of analyticity does not ride on the correctness of this definition, for if worst comes to worst, the notion of a purely logical object could be taken as undefined. Thus far I have seemed successful in defining a variety of purely logical concepts in terms of the predication relation and the fundamental condition-building and thought-building operations. This prompts the conjecture that all and only purely logical objects can be built up from the fundamental logical relation of predication by means of these two types of fundamental logical operations. This conjecture is in the spirit of this work. And given the role predication and these fundamental logical operations play in the world and in thought, I do not see any clear-cut counterexamples. However, the conjecture is distinctly philosophical and has no positive proof. In any case, if it is correct, then the conjecture can be converted into a definition of the concept of a purely logical object.[21] And, if correct, this definition suggests, in turn, a definition of the concept of a *logical constant*: a logical constant is an expression whose meaning is a purely logical object.

From the definitions in this chapter emerges an intuitive picture of three central logical concepts that have loomed large in modern philosophy: truth, necessity, and analyticity. A thought is true if and only if it corresponds to a condition that obtains; a thought is necessary if and only if it corresponds to a condition that must obtain, and a thought is analytic if and only if every thought having the same logical form corresponds to a condition that must obtain. I have also argued that validity and analyticity are one and the same. If I am right, then the definition of analyticity doubles as a definition of validity:

> A thought is valid iff_{df} every thought having the same logical form corresponds to a condition that must obtain.[22]

I will bring the chapter to a close with some remarks on the conception of logic that emerges from this definition of validity. This is called for since logic, by definition, is concerned with validity and valid thinking.

According to Aristotle's conception, logic is primarily a *tool*, or *organon*, for valid thinking. True, Aristotle believed that logic inevitably touches questions about the basic components of and structure of thoughts and that, in this, logic overlaps certain fundamental parts of metaphysics. But this substantive dimension was viewed by Aristotle as incidental. At the onset of modern philosophy Francis Bacon sought to revise the Aristotelian conception of logic by expanding its scope so as to include certain forms of inductive reasoning. It is doubtful that the expanded Baconian conception is warranted; in any event it did not call into question the underlying Aristotelian view that logic is a tool for valid thinking. Perhaps the first truly major alteration in the Aristolelian conception came with Frege. Initially, Frege too approached logic as a tool for valid thinking. However, in time he realized that an adequate formulation of logic required positing a wide range of purely logical objects. Indeed, (numerical) mathematics turned out to be nothing but a science that deals with a special kind of purely logical object. Thus it was that logic came to have a legitimate subject matter of its own, a subject matter that was not merely incidental as in the case of Aristotle's organon.[23] Nevertheless, someone could still sustain the belief that all the valid thoughts, and, hence, all the valid ways of thinking, could be captured by a well constructed organon. And thus, someone could still hold that logic, at least as defined by its purpose, is a tool for valid thinking. In this, the Aristotelian conception still seemed viable. However, this situation was brought to an abrupt end by Gödel. Given Gödel's first incompleteness theorem and given the logicist thesis that all truths of (numerical) mathematics are validities, logic had to be viewed as a full-fledged, evolving science in its own right. Its laws, i.e., the valid thoughts, could never all be captured by a tool. Valid thoughts—and, in turn, valid ways of thinking—would always be left out no matter how well the job is done. These valid thoughts would have to be discovered by some other means than the application of a tool. To be sure, among the by-products of logic there are numerous tools to aid valid thinking. But logic finally had to be considered a science primarily and a tool only secondarily through these by-products. And so it was that the Aristotelian conception of logic became untenable.

Few people today would accept this historical sketch. Yet I think that it is close to the truth. Two things are wrong with it, however. First, Frege's purely logical analysis of number broke down in some of its details. Secondly, by the time Gödel proved his result, it was hardly taken for granted that the truths of mathematics are part of logic; indeed, Gödel himself rejected the logicist thesis. Thus, the above sketch does not hold up as history. Despite this, the conclusions that logic has its very own ontology and that the Aristotelian organon conception of logic is untenable can be won by virtually the same route. One need only fill in the two gaps left by history. In effect, I have tried to do this. First, I gave a neo-Fregean analysis of number which, it has been argued, is free of the flaws present in Frege's original analysis. Secondly, I gave a defense of the logicist thesis that all truths of (numerical) mathematics are valid. This thesis follows immediately from the neo-Fregean analysis of number plus the proposed analysis of validity.

Logic, therefore, is not a tool; it is an open-ended, evolving science having an ontology of its own. Mathematics is but one of its parts, and its full scope is yet to be discovered. Given the definition of validity, every necessary thought whose analysis contains only purely logical objects is valid. So every time we discover a purely logical analysis of a concept previously thought to belong to a discipline outside logic, the acknowledged scope of logic must be expanded accordingly. The purely logical analysis of number is just one case in point, and I submit that there are many more. In this vein, I will in the closing chapter venture into an area that on the face of it might seem to some as foreign to logic as mathematics once did, namely, the area of intentionality, mind, and consciousness. If there is anything in the analyses I will offer, then the conception of logic that emerges is very far indeed from that of Aristotle, for whom logic is primarily a tool; instead, it is more like that of Plato, for whom logic is akin to reason itself.

10

Mind

48. Intentionality

> Every mental phenomenon is characterized by what the scholastics of the Middle Ages called the intentional (and also) mental inexistence of an object, and what we would call, although not in entirely unambiguous terms, the reference to a content, a direction upon an object (by which we are not to understand a reality ...), or an immanent objectivity. Each one includes something as an object within itself....
>
> This intentional inexistence is exclusively characteristic of mental phenomena. No physical phenomenon manifests anything similar. Consequently, we can define mental phenomena by saying that they are such phenomena as include an object intentionally within themselves. (Franz Brentano, *Psychologie vom empirischen Standpunkt*)

An intentional phenomenon, according to Franz Brentano, is one that makes reference to, is directed upon, or is about other objects, perhaps even objects that do not exist. Intentional phenomena can in this sense be said to 'include an object intentionally within themselves'. Intentionality, then, is that special property of being directed upon something (*Gerichtetsein*). Brentano used this concept of intentionality to formulate a two-part thesis that has come to be known as *the thesis of intentionality*:

(1) All and only mental phenomena are intentional.
(2) No purely physical phenomenon is intentional.

This is to say, (1) intentionality, i.e., the special property of directedness or aboutness, is the mark of the mental, and (2) it sunders the mental from the purely physical. I will return to Brentano's thesis in a while, but at the moment my concern is with the concept of intentionality itself. For though Brentano is to be credited with the modern rediscovery of intentionality, his analysis of it is inadequate. My immediate goal is to define intentionality without appealing to the metaphors of directedness and inexistence.

I will begin by giving a schematic summary of Brentano's theory of judgement as it is reported by Roderick Chisholm.[1] Brentano's theory differs sharply from the propositional/relational theory of judgement that I have been espousing in this work. On Brentano's theory, when one judges that $(\exists x)Ax$, one does not stand in relation to the proposition that $(\exists x)Ax$; nor does one stand in a relation to the concept of being an A. Instead, one affirms or accepts As. Likewise, when one judges that $\neg(\exists x)Ax$, one does not stand in a relation to the proposition that $\neg(\exists x)Ax$. Rather, one denies or rejects As. In the same vein, to judge that $(\exists x)(Ax \,\&\, Bx)$ is to accept As that are Bs. To judge that $\neg(\exists x)(Ax \,\&\, Bx)$ is to reject As that are Bs. To judge that $(\exists x)(Ax \,\&\, \neg Bx)$ is to accept As that are non-Bs, and to judge that $(\forall x)(Ax \supset Bx)$ is to reject As that are non-Bs. In increasingly awkward steps Brentano thus attempts to extend his theory to complex judgements.

Non-propositional/non-relational theories of judgement are not rare. Evidence of them is found in works ranging from Plato's *Sophist* (240D, 260C–263D) and *Theaetetus* (188E–189A) to Russell's *The Problems of Philosophy* (chapter 12) and his introduction to the first edition of *Principia Mathematica* (pp. 43–4). Yet all such theories share a flaw, indeed, the very flaw that spells defeat for adverbial and multiple-operator approaches to intensional logic. (See §6.) Even if by various awkward maneuvers these theories can handle statements concerning particular judgements, they cannot handle general statements concerning judgements. To handle general statements, one must be able to bring the theory within the scope of quantifier logic, and this is precisely what non-relational/non-propositional theories are unable to do in a credible way. Consider, e.g., the following intuitively valid arguments:

Whatever x believes is necessary.
Whatever is necessary is true.
—
∴ Whatever x believes is true.

Whatever x believes is true.
x believes that A.
—
∴ It is true that A.

x believes that A.
—
∴ x believes something.

I argued in §6 that on the canonical syntactic treatment of such