

# The Scope of Sequential Screening with Ex-Post Participation Constraints

Francisco Castro

Columbia University

Joint work with D. Bergemann (Yale) and G. Weintraub (Stanford)

Microsoft, March 2019

# Problem: Sequential Screening

- ▶ When and how to sell when a buyer learns her valuation over time?
- ▶ Classic example: Airline tickets
- ▶ Initial purchase is based on an imperfect estimate: buyer's type could be leisure/business travelers (**Period 1**)
- ▶ Buyer knows true willingness-to-pay only at date of travel(**Period 2**)

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## What is the revenue maximizing menu of contracts?

- ▶ Classic paper of Courty and Li (2000); also Akan et.al. (2015)
- ▶ Menu of upfront fees/refund contracts

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- ▶ Ex.1: in online shopping buyers can return purchases at low or no cost (Krähmer and Strausz 2015).
- ▶ Ex. 2: online display advertising markets: auction based and typical business constraint.



# Online Display Advertising Motivation

**FIVE CAME BACK**

**The New York Times**

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NARRATED BY ACADEMY AWARD WINNER  
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**FIVE CAME BACK**

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**GUILLERMO DEL TORO**  
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ON JOHN HUSTON

**STEVEN SPIELBERG**  
ON WILLIAM WYLER

**PAUL GREENGRASS**  
ON JOHN FORD

**LAWRENCE KASDAN**  
ON GEORGE STEVENS

ROLL OVER TO VIEW THE STORIES OF 5 LEGENDARY DIRECTORS TOLD BY 5 MODERN MASTERS

## 2 White House Officials Helped Nunes View Secret Reports

By MATTHEW ROSENBERG, MAGGIE HABERMAN and ADAM GOLDMAN  
1:16 PM ET

- White House officials helped provide Devin Nunes, the Republican chairman of the House Intelligence Committee, with reports that showed incidental surveillance of the Trump team.
- The revelation is likely to fuel criticism that Mr. Nunes has been too eager to do the bidding of the Trump administration.

■ 1127 Comments



## Desperate, on a Road to Nowhere

Times journalists spent weeks documenting the stories of people living along a desert highway in Niger, interviewing more than 100 residents scattered by Boko Haram.

By DIONNE SCHARCOT; Photographs by ADAM FERGUSON

The Daily 360: A View of the Highway

## California Today: Theater Company Has a Hit With a 'Zoot Suit' Revival

By JONATHAN ENGBEL BROMWICH  
Plus, the governor unveils an



## The Opinion Pages

### Ivanka Trump Is a Bad Ambassador for Working Women

By JAVIER CORRALES

Her policies are little more than lip service. And they could make real reform impossible.

- Editorial: Ignoring History and the Promise of Diplomacy
- Collins: Trump Remembers the Ladies
- Kristof: President Trump vs. Big Bird
- Join us on Facebook »

### The Blind Spots in Trump's Foreign Policy

By JAVIER CORRALES

The president is insensitive to the pro-United States political climate in Latin America.

- Edsall: Trump Is Ignorant of His Own Ignorance
- The Empty Supreme Court Confirmation Hearing
- Whatever Happened to France's Famed 'Liberté'?
- Suffer the Little Children: Church Cruelty in Ireland

**TIMES INSIDER** • Foreign Correspondents as They Live and Breathe

**THE CROSSWORD** • Play Today's Puzzle

16m One in three breast cancer patients under 45 removed the healthy breast along with the breast affected by cancer, a new study found.

# Online Display Advertising: Waterfall Auction



Preferred Deal  
SPA high reserve



Open Auction  
SPA low reserve



# Online Display Advertising: Waterfall Auction



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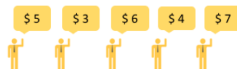


Think of period 1

# Online Display Advertising: Waterfall Auction



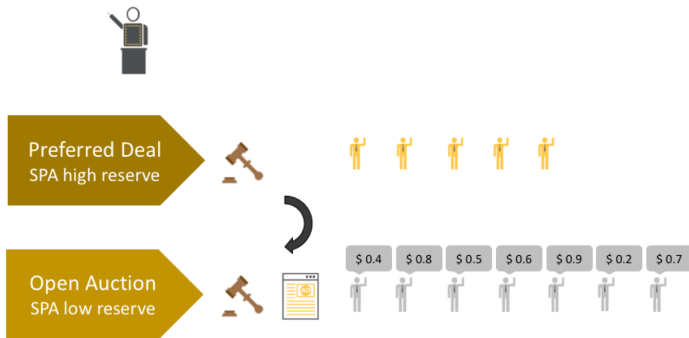
Preferred Deal  
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# Online Display Advertising: Waterfall Auction



Think of period 2

# This Paper

- ▶ What is the revenue maximizing sequential screening mechanism under ex-post participation constraints?
  - ▶ Classic solutions do not satisfy ex-post PC due to upfront fees.

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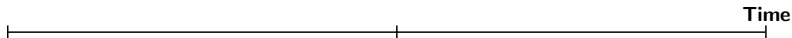
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- ▶ Use dual approach to unveil the structure of optimal mechanism
  - ▶ Cai et. al (2016) and Devanur & Weinberg (2017) dual approach also applies
- ▶ (Partially) Shed light on practical mechanisms as effective price discrimination devices such as Waterfall Auctions

# Model: Mechanism Design Formulation

**Seller:** single item  
**Single Buyer**

**Period 1**

**Period 2**



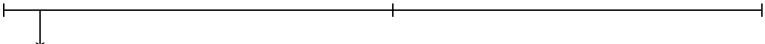
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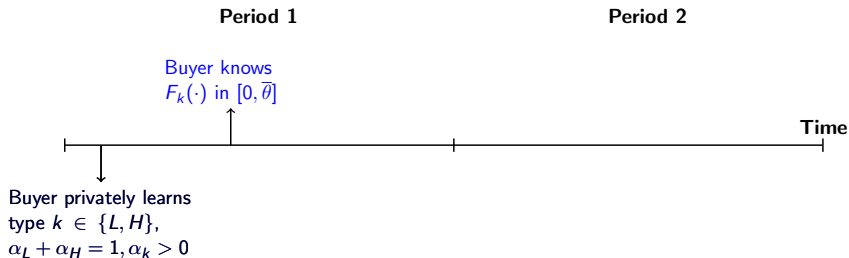
Time



Buyer privately learns  
type  $k \in \{L, H\}$ ,  
 $\alpha_L + \alpha_H = 1, \alpha_k > 0$

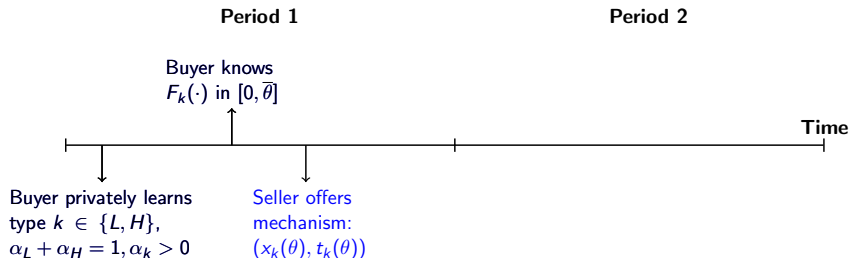
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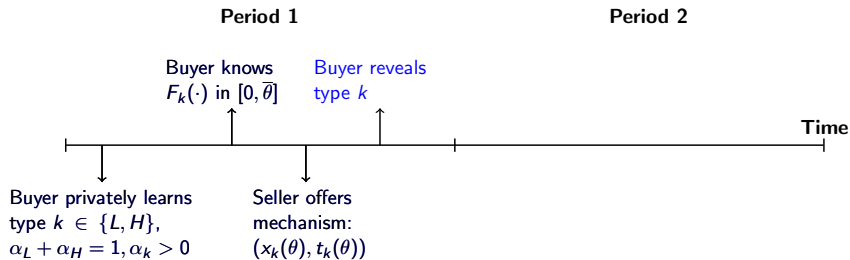
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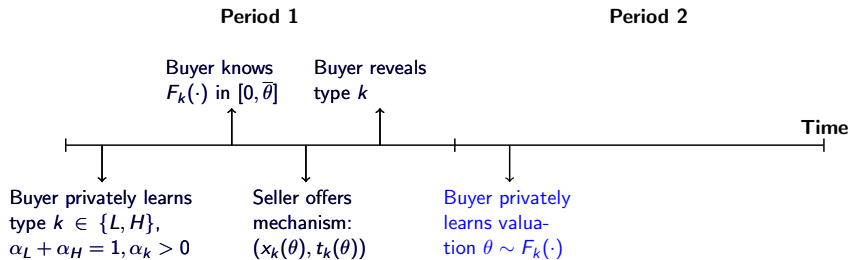
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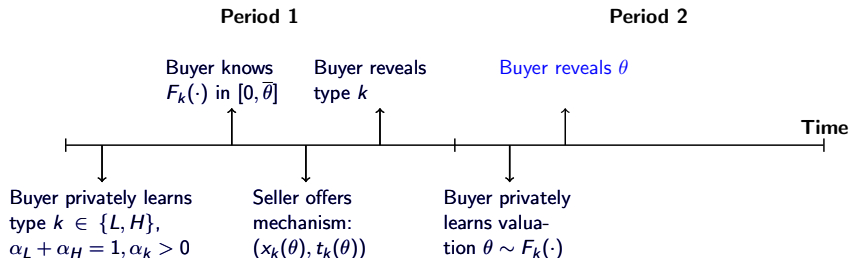
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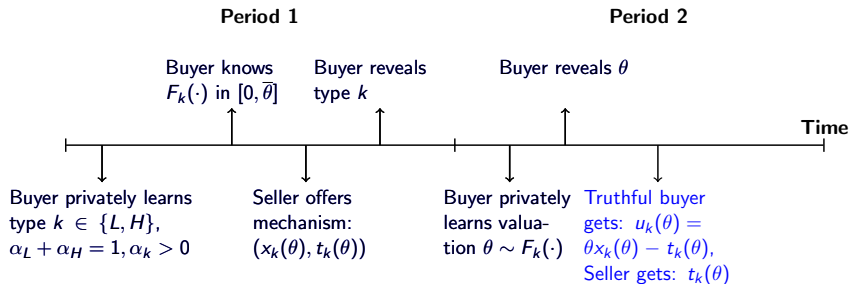
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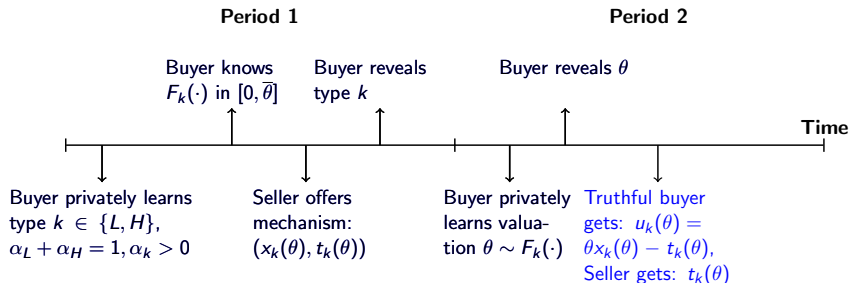
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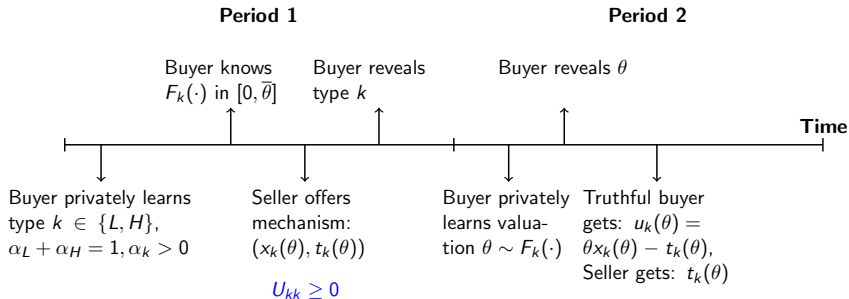
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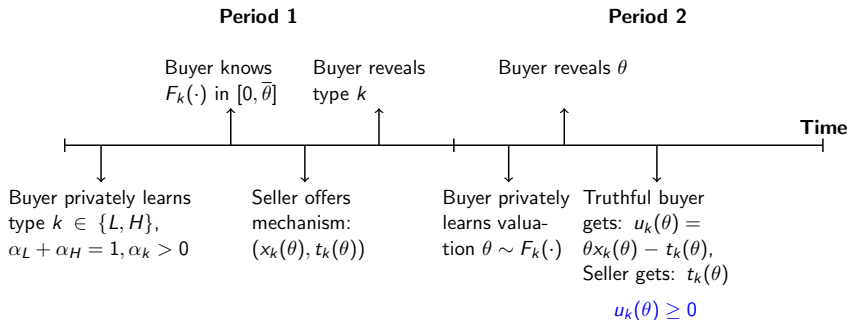
- ▶ Model primitives are common knowledge
- ▶ Parties are risk-neutral
- ▶ Non-increasing hazard rates. WLOG  $\hat{\theta}_L \leq \hat{\theta}_H$

# Revenue Maximizing Mechanisms



- **Courty and Li:** What is the revenue maximizing sequential screening mechanism under interim participation constraints?

# Revenue Maximizing Mechanisms



- **Our Question:** What is the revenue maximizing sequential screening mechanism under **ex-post participation constraints**?

$$[\text{Ex-post PC}] \quad u_k(\theta) \geq 0, \quad \forall k \in \{L, H\}, \quad \forall \theta$$

# Seller's Problem

The seller's problem is

$$(\mathcal{P}^d) \quad \max_{0 \leq x \leq 1, t} \quad \sum_{k \in \{L, H\}} \alpha_k \cdot \int_0^{\bar{\theta}} t_k(z) \cdot f_k(z) dz$$

$$\text{s.t.} \quad u_k(\theta) \geq \theta \cdot x_k(\theta') - t_k(\theta') \quad \forall k, \theta \text{ [Ex-post IC]}$$

$$\int_0^{\bar{\theta}} u_k(z) f_k(z) dz \geq \int_0^{\bar{\theta}} u_{k'}(z) f_k(z) dz, \forall k, k' \text{ [Interim IC]}$$

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$$u_k(\theta) \geq 0, \quad \forall k, \theta \text{ [Ex-post PC]}$$

- ▶ Ex-post IC: By the envelope theorem it is enough to solve for non-decreasing allocations  $x_k(\cdot)$  and the utility of the lowest ex-post types  $u_k(0)$
- ▶ Interim IC: More challenging (together with ex-post PC)

# Optimal Mechanisms

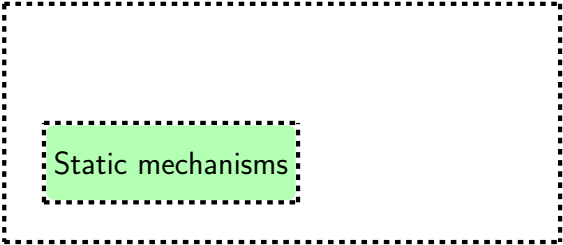
Screening mechanisms





# Optimal Mechanisms

Screening mechanisms



Static mechanisms

The diagram consists of a large rectangle with a dashed black border. Inside this rectangle, on the left side, is a smaller rectangle with a solid light green fill and a dashed black border. The text 'Static mechanisms' is centered within the green rectangle. The text 'Screening mechanisms' is positioned above the large dashed rectangle.

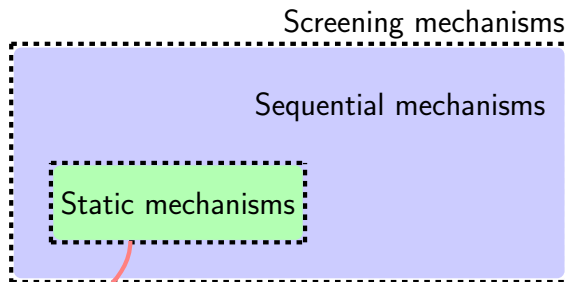
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## Screening mechanisms

Static mechanisms

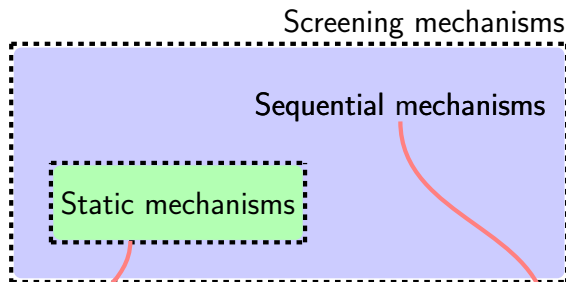
- A contract such that  $x_k(\cdot) \equiv x(\cdot)$  and  $t_k(\cdot) \equiv t(\cdot)$  for all  $k$  in  $\{L, H\}$
- Pooling of interim types
- Myerson for the mixture distribution:  
**posted price**  $\theta^s$

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- A contract such that  $x_L(\cdot) \neq x_H(\cdot)$  and  $t_L(\cdot) \neq t_H(\cdot)$

- Separate interim types

- Contract can be arbitrarily complex

# Research Questions/Contributions

## 1. **When is a static contract optimal? When it is not?**

- ▶ Krähmer and Strausz 2015: Sufficient condition
- ▶ Us: Necessary and sufficient condition

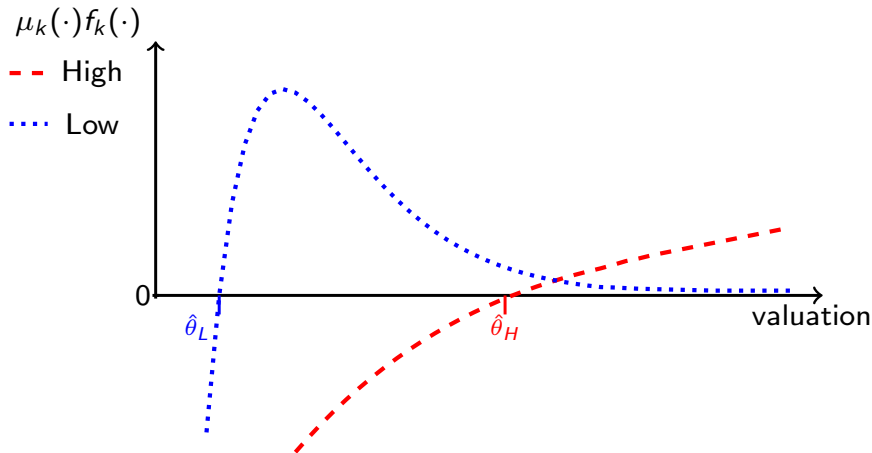
# Research Questions/Contributions

1. **When is a static contract optimal? When it is not?**
  - ▶ Krähmer and Strausz 2015: Sufficient condition
  - ▶ Us: Necessary and sufficient condition
2. **If a sequential contract is optimal, what does the optimal mechanism look like?**
  - ▶ Us: Full characterization
  - ▶ Very different to Courty and Li
  - ▶ Significant revenue improvement over static contract

# The "Simple Economics" of Optimal Sequential Contracts

Let's look at weighted virtual values ("marginal revenues");

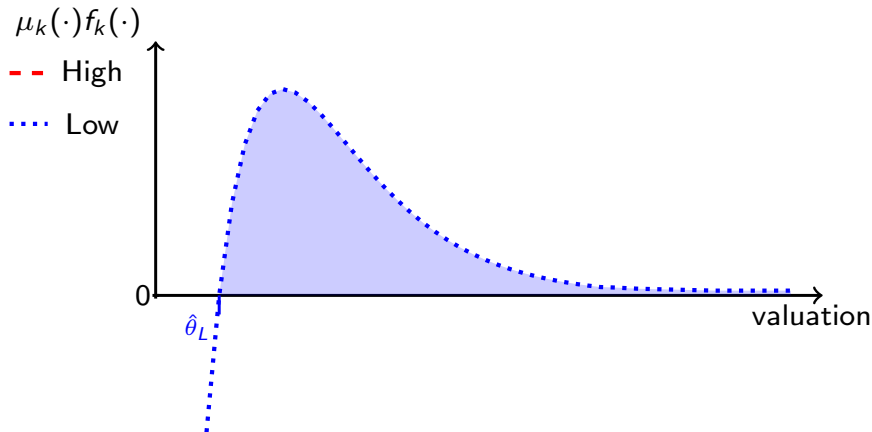
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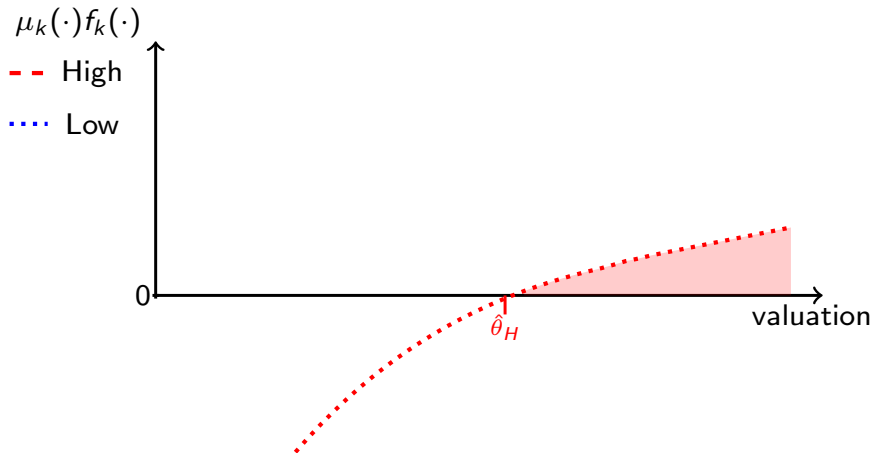




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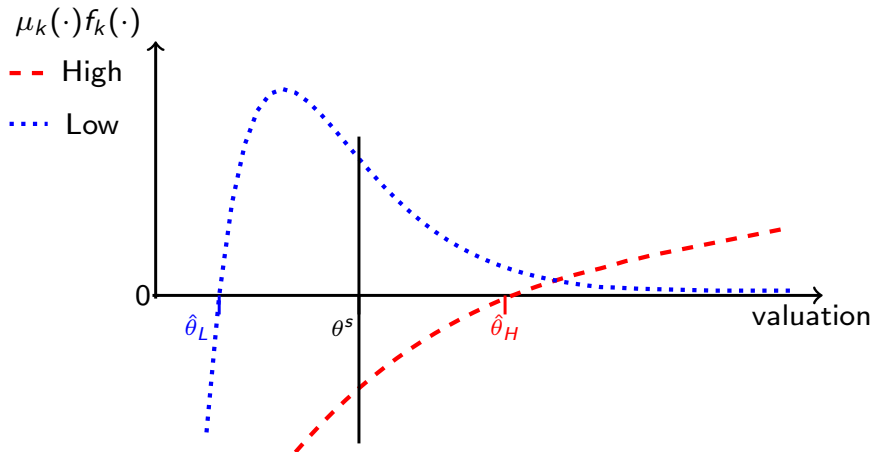
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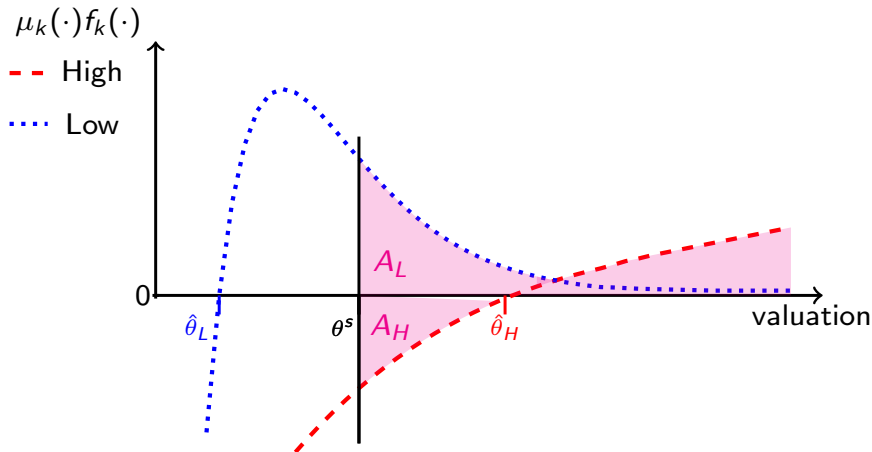


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$$\text{Rev (static)} = \alpha_L \cdot A_L + \alpha_H \cdot A_H$$

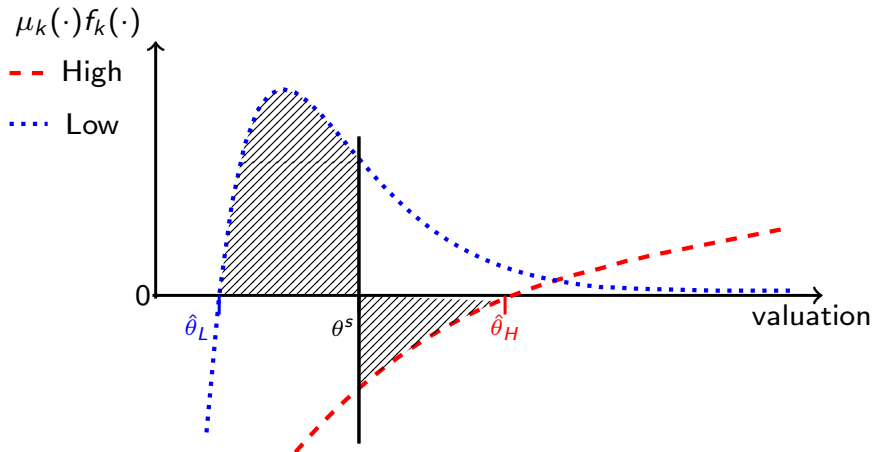


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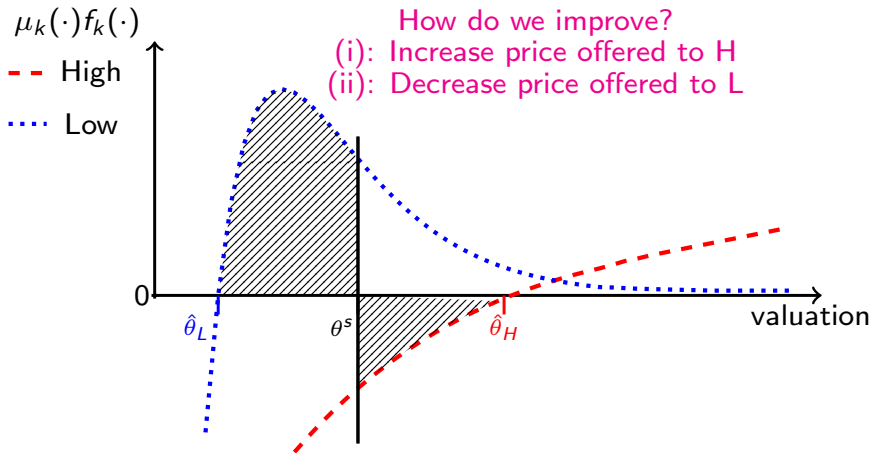
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How do we improve?

- (i): Increase price offered to H
- (ii): Decrease price offered to L



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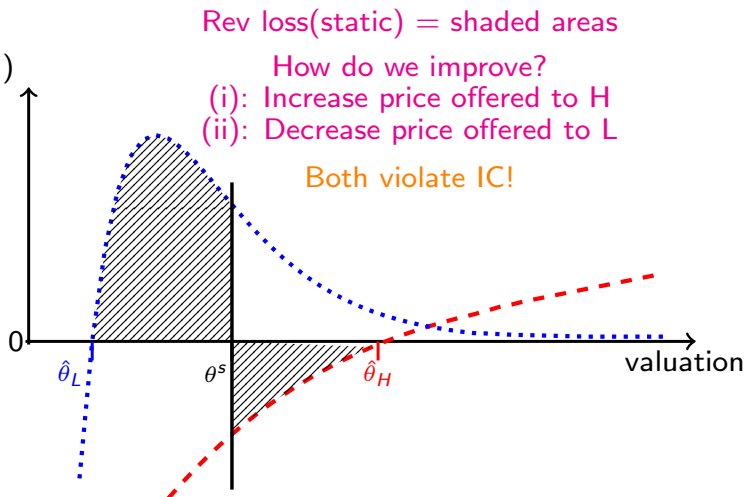
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$\mu_k(\cdot)f_k(\cdot)$

-- High

... Low

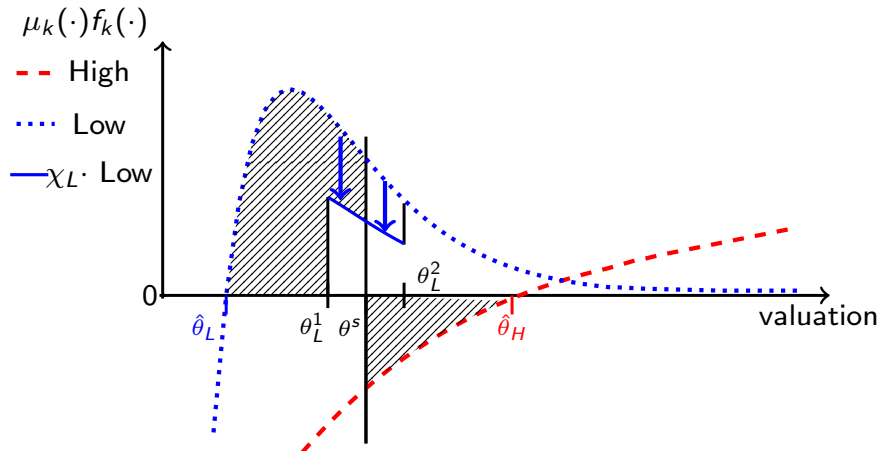


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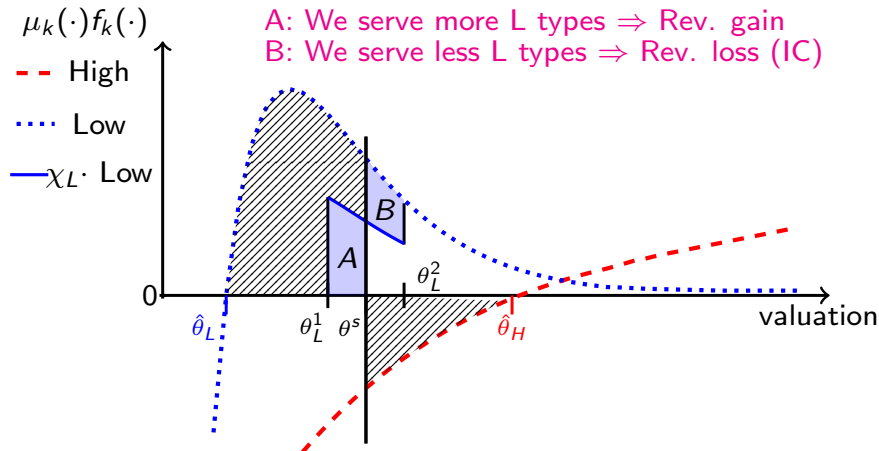
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A: We serve more L types  $\Rightarrow$  Rev. gain

B: We serve less L types  $\Rightarrow$  Rev. loss (IC)





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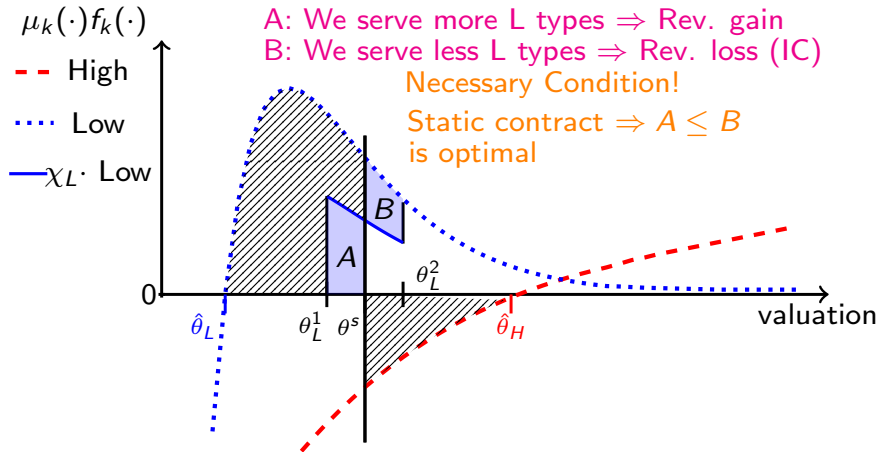
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Necessary Condition!

Static contract  $\Rightarrow A \leq B$   
is optimal



# General Necessity and Sufficiency

## Theorem

*The static contract is optimal if and only if*

$$\max \left\{ \underbrace{\text{Region A}}_{\text{revenue gain}} \right\} \leq \min \left\{ \underbrace{\text{Region B}}_{\text{revenue loss}} \right\}$$

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1. Condition can be rigorously stated in terms of primitives:

$$\max_{\theta \leq \theta^s} \frac{\int_{\theta}^{\theta^s} \mu_L(\theta) f_L(\theta) d\theta}{\int_{\theta}^{\theta^s} (1 - F_H(\theta)) d\theta} \leq \min_{\theta^s \leq \theta} \frac{\int_{\theta^s}^{\theta} \mu_L(\theta) f_L(\theta) d\theta}{\int_{\theta^s}^{\theta} (1 - F_H(\theta)) d\theta}$$

2. **Sharp intuitive characterization for optimality of static contract!**
3. Necessity formalizes picture above; sufficiency relaxes L to H IC and applies Lagrangian duality.

# Exponential Valuations

$$f_k(\theta) = \lambda_k e^{-\lambda_k \theta}, \quad k = \{L, H\} \quad \theta \geq 0, \quad \lambda_L > \lambda_H.$$

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## Proposition

*The static contract is optimal if and only if*

$$\lambda_L - \lambda_H \leq \frac{1}{\theta^s}$$

- ▶  $\theta^s$ : optimal Myerson price for mixture distribution
- ▶  $\lambda_L$  and  $\lambda_H$  close then screening is not optimal
- ▶  $\lambda_L$  and  $\lambda_H$  different then screening is optimal

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## Corollary

*Assume  $\lambda_L > 2\lambda_H$ , then there exists  $\bar{\alpha} \in (0, 1)$  such that the sequential contract is optimal iff  $\alpha_L \in (0, \bar{\alpha})$ .*

# General Necessity and Sufficiency

Krähmer and Strausz 2015 sufficient condition:

$$\frac{\mu_\ell(\theta)f_\ell(\theta)}{\bar{F}_k(\theta)}$$

is increasing for all  $\ell, k$



# General Necessity and Sufficiency

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$$\frac{\mu_\ell(\theta)f_\ell(\theta)}{\bar{F}_k(\theta)}$$

is increasing for all  $\ell, k$

- ▶ Stronger pointwise condition
- ▶ It implies our condition
- ▶ Ex.: does not necc. hold for exponential valuations.

# General Characterization

## Full characterization for optimal sequential contract!

### Theorem

Consider problem  $(\mathcal{P}^d)$  and assume profit-to-rent cond. does not hold, the optimal solution has allocations

$$x_L^*(\theta) = \begin{cases} 0 & \text{if } \theta < \theta_L^1 \\ \chi_L \in [0, 1] & \text{if } \theta_L^1 \leq \theta \leq \theta_L^2 \\ 1 & \text{if } \theta_L^2 < \theta, \end{cases} \quad x_H^*(\theta) = \begin{cases} 0 & \text{if } \theta < \theta_H \\ 1 & \text{if } \theta_H \leq \theta, \end{cases}$$

for some values  $\theta_L^1, \theta_H, \theta_L^2$  with  $\theta_L^1 \leq \theta_H \leq \theta_L^2$ , and  $u_L(0) = u_H(0) = 0$

# General Characterization

## Full characterization for optimal sequential contract!

### Theorem

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- ▶ Optimal contract is simple
- ▶ Optimal contract departs from bang-bang contract with one buyer and one item
- ▶  $\hat{\theta}_L \leq \theta_L^1, \theta_H \leq \hat{\theta}_H$

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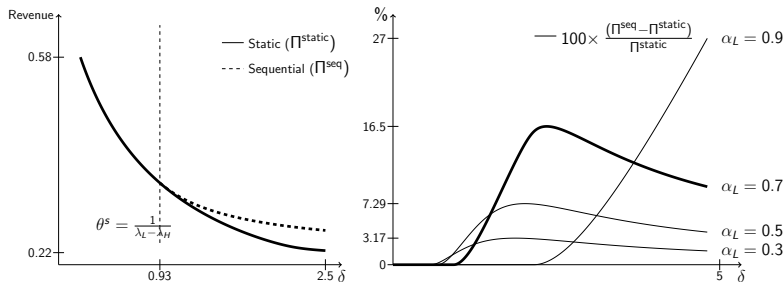
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# The Value of Sequential Screening: Optimal Revenues



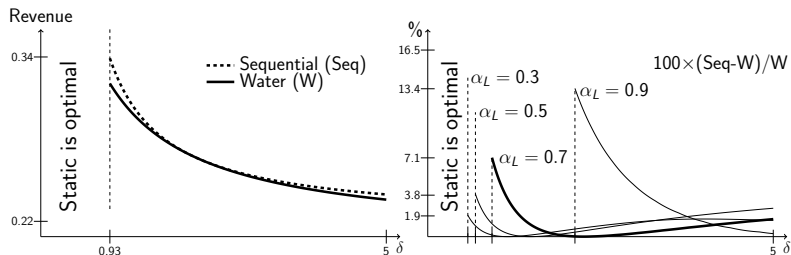
**Figure:** Left: Optimal expected revenue for static and sequential. Right: Percentage improvement of the sequential over the static contract. In both figures we set  $\lambda_L = \lambda_H + \delta$  with  $\lambda_H = 0.5$ .



# Back to Waterfall Auctions

- ▶ In Waterfall Auctions **low type buyers are randomized**: can only bid when high-reserve auction does not clear
- ▶ “high-reserve auction does not clear”  $\Leftrightarrow$  high type value  $\leq \theta_H$
- ▶ Seller revenue:

$$\max_{\theta_H \geq \theta_L \geq 0, (IC)} \underbrace{\alpha_L F_H(\theta_H)}_{\text{randomization}} \bar{F}_L(\theta_L)\theta_L + \alpha_H \bar{F}_H(\theta_H)\theta_H$$



**Figure:** *Left:* Optimal expected revenue for Waterfall and Sequential. *Right:* Percentage improvement of the Sequential over the Waterfall contract. In both figures we set  $\lambda_L = \lambda_H + \delta$  with  $\lambda_H = 0.5$ .

# Multiple Interim Types

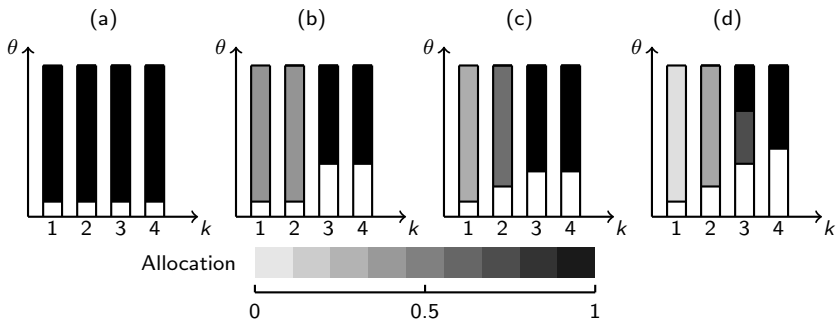
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- ▶ We prove that for exponential valuations there is at most one randomization step in optimal sequential contract. We partially extend this result.
- ▶ Multiple-type analysis is more complex because there is not an obvious relaxation of the math program.



**Figure:** Optimal allocations for 4 interim types with exponential valuations. In each panel the vertical axis corresponds to buyers' valuations and the horizontal axis corresponds to the type. Each bar represents the allocation for each type, lighter grey indicates lower probability of allocation while darker grey indicates higher probability of allocation. Different distributions of interim types across instances.

# Conclusions and Future Work

## Summary

- ▶ Complete characterization for the optimal mechanism for two interim types and one buyer
- ▶ Both static and sequential contracts can be optimal
- ▶ When the sequential contract is optimal, the seller has to randomize the low-type and give a deterministic allocation to the high-type
- ▶ Some extensions to multiple types

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## Current and Future work

- ▶ Study multiple buyers: may need ironing
- ▶ Connections with practical real-world mechanisms, such as waterfall auctions (randomization)
- ▶ Analyze performance guarantees of “simple mechanisms”