Revenue Maximizing Mechanisms with Strategic Customers and Unknown Demand: Name-Your-Own-Price.

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1. The primary aim of Revenue Management is selling the right product to the right customer at the right time for the right price (Wikipedia)

2. Revenue Management is widely used:
   Airline industry, hotel booking, freight transportation, car rentals and holiday resorts, the retail of seasonal and style goods (e.g., groceries and apparel), electricity generation, e-business (online advertising and broadcasting, allocation of bandwidth), and event management (sports, concerts, etc...)

3. Could be used for spectrum auctions
1. Consider a finite supply of objects that loses all value after some point in time (airline ticket, hotel capacity, etc)

2. Typical trade-off:
   - selling today to a customer with a lower willingness to pay
   - waiting for a customer with a higher willingness to pay and incurring the risk of not selling at all

3. As the risk of not selling increases over time, prices decrease over time when no object is sold

4. As the risk of not selling decreases every time an object is sold; prices increase after sales
Long vs Short-Lived Consumers

1. As prices potentially decrease in the future, buyers might have an incentive to delay the purchase to profit from lower prices.

2. Large parts of the literature assume that buyers are “short lived”, either buy immediately or never.

3. How realistic this assumption is depends on the application.

4. Until recently the execution of such strategic timing was encumbered by the fact that pricing algorithms are opaque, and by the absence of reliable historical data: strategizing customers were basically playing a lottery.

5. This dramatically changed with the advent of price comparison websites such as Bing/Travel and Kayak.
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BOS 8:04a → SFO 11:46a 6h 42m  nonstop
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Revenue Losses from Strategic Buyers

- Mantin and Rubin (WP 2013) estimate a 3% revenue loss on routes where information from Bing/Travel is available.

- Soysal and Krishnamurthi (2012) estimate a significant 11% decrease in revenue in the market for fashion goods.

- Note that revenue may be potentially harmed both by the shift in demand to lower prices, and by the indirect effect on the seller’s ability to learn about residual demand.
Literature

- Buyers are short-lived:
  - With Learning about the arrival rate: Mason and Välimäki (2011), Aviv and Pazgal (WP)
  - With Learning about valuations: Gershkov and Moldovanu (2009 and 2010)

- Buyers are long-lived:

- We study long-lived strategic buyers and learning about the arrival process
Buyers are short-lived:
- With Learning about the arrival rate: Mason and Välimäki (2011), Aviv and Pazgal (WP)
- With Learning about valuations: Gershkov and Moldovanu (2009 and 2010)

Buyers are long-lived:

We study long-lived strategic buyers and learning about the arrival process

Learning is important in this setup as a common theme in this literature is that without learning constant prices are almost optimal (see Gallego and Van Ryzin (1994) or Gallien (2006))
Results

- It might be impossible to implement the welfare maximizing policy with strategic customers and learning
- We characterize the revenue maximizing mechanism if the process is Markov
- The revenue maximizing mechanism is not a posted price mechanism, but involves a name your own price feature
- We show that the revenue that in the optimal mechanism with strategic and long-lived consumers is always higher than with short-lived consumers
- We show that it is always optimal to reveal all information to potential buyers
The Model
Buyers Preferences

1. Time is continuous in \([0, T]\), deadline \(T \leq 0\)

2. Buyers arrive according to a counting process \((N(t))_{t \in [0, T]}\)
   - Renewal and Poisson processes are special cases

3. Each buyer demands one unit of the good

4. Buyer \(i\)'s valuation \(v_i \in \mathbb{R}_+\) is iid. distributed according to the distribution \(F\) with density \(f\)

5. Denote \(\tau_i \in [0, T] \cup \{\infty\}\) the (potentially random) time an object is allocated to buyer \(i\)

6. and \(P_i\) the payment she makes

7. Buyers are risk-neutral and discount at a constant rate
   \[\mathbb{E} \left[ e^{-r \tau_i} (v_i - P_i) \right]\]
Allocations

1. Denote $a_i$ the arrival time of agent $i$

2. No agent can receive an object before he arrives $a_i \leq \tau_i$

3. We set $\tau_i = \infty$ if buyer $i$ does not receive an object

4. There are $k < \infty$ objects, i.e. $\sum_{i=1}^{\infty} 1\{\tau_i < \infty\} \leq k$

5. An allocation rule $\tau = (\tau_1, \tau_2, \ldots)$ is a vector of allocation times

6. We denote a history at time $t$ by $h(t) = ((a_j)_{j \leq N(t)}, (v_j)_{j \leq N(t)})$. 
Direct Revelation Mechanism

- We assume that the seller can commit to allocations and future prices
- At the time of her arrival agent $i$ reports her value $v_i$ and her arrival time $a_i$
- A mechanism is a mapping from a history of reports $\tilde{h}(t)$ into a vector of allocation times $\tau$ and payments $P$
- We look at mechanisms which are:
  1. Bayesian incentive compatible: it is optimal for each agent to report truthfully
  2. Ex-post individually rational: no agent pays more than he values the objects
     \[ v_i - P_i \geq 0 \]
  3. Online: the payment of agent $i$ does not depend on any information that arrives after time $\tau_i$
- By the Revelation Principle for dynamic environments we restrict attention to truthful mechanisms
Revenue Maximization

1. Denote by $J(v) = v - \frac{1-F(v)}{f(v)}$ the virtual valuation (assumption: increasing)

2. and by $W^\circ(\tau)$ the expected discounted virtual valuation of bidders who get an object

$$W^\circ(\tau) = \mathbb{E} \left[ \sum_{i=1}^{\infty} e^{-r\tau_i} J(v_i) \right].$$

3. Denote by $\tau^\circ = \arg \max_{\tau} W(\tau)$ the virtual valuation maximizing policy.
Revenue Maximization

**Theorem (Generalized Revenue Equivalence)**

Assume an agent with a valuation of zero receives no transfer. The revenue generated in any incentive compatible, ex-post individually rational mechanism is given by

$$W^\circ(\tau) = \mathbb{E} \left[ \sum_{i=1}^{\infty} e^{-r_i} P_i \right],$$

This result holds for any signal structure such that the buyer observes less information than the seller, i.e. the agent could observe reported arrivals and valuations of other agents, past allocation decisions.

This result shows that the seller’s revenue depends only on the implemented allocation, but not on the information of the buyers.
Revenue Maximization

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  - reported arrivals and valuations of other agents
  - past allocation decisions
- This result shows that the seller’s revenue depends only on the implemented allocation, but not on the information of the buyers.
Example: Learning About the Arrival Rate

- Buyers arrive according to a Poisson process
- The arrival rate $\lambda$ is either high or low $\lambda \in \{l, h\}$
- Neither the seller nor the buyers know the arrival rate
- The posterior expected arrival rate equals

$$
\lambda(t) = \mathbb{E} [\lambda \mid h(t)] = l + \frac{h - l}{1 + \frac{h - \mathbb{E} [\lambda \mid h(0)]}{\mathbb{E} [\lambda \mid h(0)] - l} \exp((h - l)t - \mathcal{N}(t) \log(\frac{h}{l}))}
$$

- The optimal policy is characterized by a vector of cut-offs

$$
c(t, \lambda(t), k(t))
$$

depending on time $t$, the remaining number of objects $k(t)$ and the posterior expected arrival rate $\lambda(t)$
Optimal Policy

Figure: $k = 1, l = 1, h = 5, r = 0.05, F(v) = 2v - 1, T = 1, \mathbb{E}[\lambda|h(0)] = 2, \hat{c} = 0.854, \text{gain} = 48\%$
Implementation with Observable Arrivals

Definition

Consider two histories \( h(a_i) \) and \( h'(a_i) \) that differ only in the valuation of agent \( i \), such that \( v'_i \), the valuation of \( i \) in \( h'(a_i) \), is larger than \( v_i \), the valuation in \( h(a_i) \). A stopping time \( \tau \) is monotone with respect to valuations if for any agent \( i \), and for any two such histories it holds that

\[
\mathbb{E} \left[ e^{-r \tau_i(a_i,(v'_i,v_{-i}))} \mid h'(a_i) \right] \geq \mathbb{E} \left[ e^{-r \tau_i(a_i,(v_i,v_{-i}))} \mid h(a_i) \right].
\]

Proposition

Assume that arrivals are observable. If the allocation \( \tau \) is implementable then it is monotone with respect to valuations. Conversely, if the allocation \( \tau \) is monotone with respect to valuations, then \( \tau \) can be implemented using the payment

\[
P_i(a,v) = 1_{\{\tau_i(a,v)<\infty\}} \frac{\mathbb{E} \left[ \int_0^{v_i} (e^{-r \tau_i(a,v)} - e^{-r \tau_i(a,z,v_{-i})}) \, dz \mid h(a_i) \right]}{\mathbb{E} \left[ e^{-r \tau_i(a,v)} \mid h(a_i) \right]}.
\]
Proposition

The welfare maximizing $\tau^*$ and the virtual valuation maximizing allocation $\tau^\circ$ are monotone in valuations, and thus implementable when arrivals are observable. The virtual valuation maximizing allocation and the associated payments maximize revenue.
How Does the Mechanism Work

- When an agent arrives, there are three cases
  1. gets allocated an object immediately if his valuation is high
  2. he gets on a waiting list from which he is potentially recalled
  3. he never gets allocated a good if his valuation is low

- At time $T$ the remaining objects are allocated to the agents with the highest valuation on the waiting list

- This allocation can be implemented by in an **indirect mechanism**:  
  1. There is a buy it now price
  2. Alternatively agents could name their own (lower) price at which the seller can recall them
Step 2: Choose the star level for your hotel

The minimum Guaranteed Amenities are shown for select star levels in your chosen area(s). Star levels may not be available in all areas.

- **5-Star Luxury**: Which Hotel?
- **4½-Star Deluxe-Plus**: Which Hotel?
- **4-Star Deluxe**: Which Hotel? Best deal
- **3½-Star Upscale-Plus**: Which Hotel?
- **3-Star Upscale**: Which Hotel?
- **2½-Star Moderate-Plus**: Which Hotel?
- **2-Star Moderate**: Which Hotel?
- **1-Star Economy**: Which Hotel?

Step 3: Name Your Own Price® (per room night)

Total charges, including taxes and service fees, are shown on the next page. You’re protected by our Best Price Guarantee.

Name Your Own Price®
Per Room, Per Night (USD)
$ __________.00

Median retail price for a 4 star hotel in the area selected is $327. Name a lower price, or click here to shop and compare prices.
Step 2: Choose the star level for your hotel

The minimum Guaranteed Amenities are shown for select star levels in your chosen area(s). Star levels may not be available in all areas.

<table>
<thead>
<tr>
<th>Star Level</th>
<th>Amenities Rating</th>
<th>Which Hotel?</th>
<th>Best deal</th>
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<tbody>
<tr>
<td>5-Star Luxury</td>
<td>⭐⭐⭐⭐⭐⭐</td>
<td></td>
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<tr>
<td>4½-Star Deluxe-Plus</td>
<td>⭐⭐⭐⭐⭐1/2</td>
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<td>⭐⭐⭐½</td>
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<tr>
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<td></td>
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Step 3: Name Your Own Price® (per room, per night)

Total charges, including taxes and service fees, are shown on the next page. You're protected by our Best Price Guarantee.

Name Your Own Price®
Per Room, Per Night (USD)
$[ ] 0.00

Median retail price for a 4-star hotel in the area selected is $[327]. Name a lower price, or click here to shop and compare prices.

See more great deals in nearby areas:
- San Jose, CA
- Marin County, CA
- Oakland, CA - Eastbay

Priceline hotel participants are major national brands or quality independent hotels. We screen participating hotels carefully and update our information constantly to ensure you always receive quality accommodations. We will find you a room in a hotel willing to agree to your price in the shaded area.
Unobservable Arrivals
Monotonicity with Respect to Arrivals

Definition

A deterministic allocation rule $\tau$ is monotone in the arrival times iff buyers who arrive earlier get the object earlier, i.e. for all $i$, $a_i < \tilde{a}_i$ and all $a_{-i}$, $v \in \mathbb{R}_+^\infty$

$$\tau_i((a_i, a_{-i}), v) \leq \tau_i((\tilde{a}_i, a_{-i}), v).$$
Monotonicity with Respect to Arrivals

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**Theorem**

Assume that the arrival process is Markov. Consider a vector of deterministic and Markovian allocation times $\tau$ and a vector of payments $P$ such that:

1. the payment $P$ implements $\tau$ under observable arrivals
2. $\tau$ is monotone with respect to arrivals
3. $(\tau, P)$ is a winner-pay mechanism

Then $P$ implements $\tau$ with unobservable arrivals.
Relevance Maximization for Markov Processes

**Proposition**

Assume that the arrival process is Markov. The payment defined before implements the revenue maximizing policy under unobservable arrivals.

---

1. As the process is Markov, the continuation value is independent of the precise timing of arrivals.
2. Hence, when an buyer arrives later, the principal uses the same continuation strategy as he would have used conditional on not allocating the object to the buyer.
3. It thus follows that $\tau$ is monotone with respect to arrivals.
4. Thus, the payments implement the same allocation.
5. By the revenue equivalence principle the seller makes the same revenue.
6. As the seller can always ignore unwanted information, the revenue with observable arrivals is an upper bound for the revenue with unobservable arrivals.
Revenue Maximization for Markov Processes

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Corollary

Assume that the arrival process is Markov. The revenue in the optimal mechanism for short-lived buyers (that only buy upon arrival) is lower than the revenue in the optimal mechanism for buyers who can strategically time their buying decision, and whose arrivals are unobservable.
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Proof.

1. As shown before the seller can achieve the same revenue with observable and unobservable arrivals.
2. The seller can always achieve the same revenue in a situation with long-lived buyers and observable arrivals as with short lived buyers, by restricting to policies that allocate the object upon arrival.
Conclusion

1. We have analyzed dynamic allocation and pricing in a continuous time, discounted model where arrivals are governed by a general counting process, and where buyers are privately informed both about values and arrival times.

2. We have shown that the seller can always profit from long-lived strategic buyers, if he uses the right mechanism.

3. The Optimal Mechanism offers buyers to name their own price.
Thank You!