The Comparison of Information Structures in Games: Bayes Correlated Equilibrium and Individual Sufficiency

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Robust Predictions

- game theoretic predictions are very sensitive to "information structure" a.k.a. “higher order beliefs" a.k.a "type space"
  - Rubinstein’s email game
- information structure is hard to observe - no counterpart to revealed preference
- what can we say about (random) choices if we do not know exactly what the information structure is?
- robust predictions: predictions that are robust (invariant) to the exact specification of the private information
- partially identifying parameters independent of knowledge of information structure

Basic Question

- fix a game of incomplete information
- which (random) choices could arise in Bayes Nash equilibrium in this game of incomplete information *or one in which players observed additional information*
- begin with a **lower bound** on information (possibly a zero lower bound)

Basic Answer: Bayes Correlated Equilibrium

- set of (random) choices consistent with Bayes Nash equilibrium given any additional information the players may observe $=$
- set of (random) choices that could arise if a mediator who knew the payoff state could privately make action recommendations

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- set of incomplete information correlated equilibrium (random) choices

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- set of incomplete information correlated equilibrium (random) choices
- we refer to this very permissive version of incomplete information correlated equilibrium as "Bayes correlated equilibrium (BCE)"

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- set of (random) choices that could arise if a mediator who knew the payoff state could privately make action recommendations
- set of incomplete information correlated equilibrium (random) choices
- we refer to this very permissive version of incomplete information correlated equilibrium as "Bayes correlated equilibrium (BCE)"
- and we will prove formal equivalence result between BCE and set of (random) choices consistent with Bayes Nash equilibrium given any additional information the players may observe

Many Applied Uses for Equivalence Result

- robust predictions and robust identification
  - "Robust Predictions in Games with Incomplete Information" (linear best response games with continuum of agents), Econometrica, forthcoming;
- tractable solutions
  - "The Limits of Price Discrimination" (joint with Ben Brooks);
- optimal information structures
  - "Extremal Information Structures in First Price Auctions" (joint with Ben Brooks);
- volatility and information in macroeconomics (joint with Tibor Heumann)
  - "Information, Interdependence and Interaction: Where does the Volatility come from ?"

Today's Paper and Talk: Foundational Issues

1. basic equivalence result
2. more information can only increase the set of **feasible** (random) choices...
   - ..what is the formal ordering on information structures that supports this claim?
3. more information can only reduce the set of **optimal** (random) choices...
   - ..what is the formal ordering on information structures that supports this claim?
4. "individual sufficiency" generalizes Blackwell's (single player) ordering on experiments
   - how does our novel ordering on information structures relate to other orderings?

Outline of Talk: Single Player Case

- Bayes correlated equilibrium with single player:
  what predictions can we make in a one player game ("decision problem") if we have just a lower bound on the player's information structure ("experiment")?
- we suggest a partial order on experiments:
  one experiment is more *incentive constrained* than another if it gives rise to smaller set of possible BCE (random) choices across all decision problems

Single Player Ordering and Blackwell (1951/53)

- an experiment S is *sufficient* for experiment S' if signals in S are sufficient statistic for signals in S'
- an experiment S is *more informative* than experiment S' if *more* interim payoff vectors are supported by S than by S'
- an experiment S is *more incentive constrained* than experiment S' if, for every decision problem, S supports *fewer Bayes correlated equilibria*

Notions Related to Blackwell (1951/1953)

- an experiment S is *more informative* than experiment S' if *more* interim payoff vectors are supported by S than by S'
- an experiment S is *more permissive* than experiment S' if more random choice functions are supported by S than by S'
- an experiment S is *more valuable* than experiment S' if, in every decision problem, ex ante utility is higher under S than under S' (Marschak and Radner)

Blackwell's Theorem Plus: One Player

Theorem

The following are equivalent:

1. *Experiment S is sufficient for experiment S' (statistical ordering);*
2. *Experiment S is more incentive constrained than experiment S' (incentive ordering);*
3. *Experiment S is more permissive than experiment S' (feasibility ordering).*

Theorem

The following are equivalent:

1. *Information structure S is individually sufficient for information structure S' (statistical ordering);*
2. *Information structure S is more incentive constrained than information structure S' (incentive ordering);*
3. *Information structure S is more permissive than information structure S' (feasibility ordering).*

Related Literature

1. Forges (1993, 2006): many notions of incomplete information correlated equilibrium
2. Lehrer, Rosenberg and Shmaya (2010, 2012): many multi-player versions of Blackwell's Theorem
3. Gossner and Mertens (2001), Gossner (2000), Peski (2008): Blackwell's Theorem for zero sum games
4. Liu (2005, 2012): one more (important for us) version of incomplete information correlated equilibrium and a characterization of correlating devices that relates to our ordering

- single decision maker
- finite set of *payoff states* $\theta \in \Theta$,
- finite set of *actions* $a \in A$,
- a *decision problem* $G = (A, u, \psi)$,

$$u : A \times \Theta \to \mathbb{R}$$

is the agent's (vNM) utility and

$$\psi \in \Delta(\Theta)$$

is a prior.

- an *experiment* $S = (T, \pi)$, where T is a finite set of *types* (i.e., signals) and likelihood function

$$\pi : \Theta \to \Delta(T)$$

- a choice environment (one player game of incomplete information) is (G, S)

- a *decision rule* is a mapping

$$\sigma : \Theta \times T \rightarrow \Delta(A)$$

- a *random choice rule* is a mapping

$$\nu : \Theta \rightarrow \Delta(A)$$

- random choice rule ν is *induced* by decision rule σ if

$$\sum_{t \in T} \pi(t|\theta)\, \sigma(a|t, \theta) = \nu(a|\theta)$$

Definition (Obedience)

Decision rule $\sigma : \Theta \times T \to \Delta(A)$ is obedient for (G, S) if

$$\sum_{\theta \in \Theta} \psi(\theta)\, \pi(t|\theta)\, \sigma(a|t,\theta)\, u(a,\theta) \geq \sum_{\theta \in \Theta} \psi(\theta)\, \pi(t|\theta)\, \sigma(a|t,\theta)\, u(a',\theta) \tag{1}$$

for all $a, a' \in A$ and $t \in T$.

Definition (Bayes Correlated Equilibrium)

Decision rule σ is a Bayes correlated equilibrium (BCE) of (G, S) if it is obedient for (G, S).

- random choice rule ν is a *BCE random choice rule* for (G, S) if it is induced by a BCE σ

Blackwell Triple

- with the decision rule

$$\sigma : \Theta \times T \rightarrow \Delta(A)$$

  we are interested in a triple of random variables

$$\theta, t, a$$

- an elementary property of a triple of random variable, as a property of conditional independence, was stated in Blackwell (1951) as Theorem 7
- as it will be used repeatedly, we state it formally

Blackwell Triple: A Statistical Fact

- consider a triple of variables $(x, y, z) \in X \times Y \times Z$ and a joint distribution:

$$P \in \Delta(X \times Y \times Z).$$

Lemma

The following three statements are equivalent:

1. $P(x|y,z)$ *is independent of* z;
2. $P(z|y,x)$ *is independent of* x;
3. $P(x,y,z) = P(y)\,P(x|y)\,P(z|y)$.

- if these statements are true for the ordered triple (x, y, z), we refer to it as Blackwell triple
- "a Markov chain $P(x|y,z) = P(x|y)$ is also a Markov chain in reverse, namely $P(z|y,x) = P(z|y)$"

Definition (Belief Invariance)

A decision rule σ is belief invariant for (G, S) if for all $\theta \in \Theta, t \in T$, $\sigma(a|t, \theta)$ is independent of θ.

- belief invariance captures decisions that can arise from a decision maker randomizing conditional on his signal t but not state θ...
- ... now (a, t, θ) are a Blackwell triple, hence $\sigma_\psi(\theta|t, a)$ is independent of a ...
- ...motivates the name: chosen action a does not reveal anything about the state beyond that contained in signal t
- a decision rule σ could arise from a decision maker with access only to the experiment S if it is belief invariant

Definition (Bayes Nash Equilibrium)

Decision rule σ is a Bayes Nash Equilibrium (BNE) for (G, S) if it is obedient and belief invariant for (G, S).

- we want to ask what happens when decision maker observes more information than contained in S
- introduce a language to combine and compare experiments

Combined Experiment

- consider separate experiments,

$$S^1 = \left(T^1, \pi^1\right), \qquad S^2 = \left(T^2, \pi^2\right)$$

- join the experiments S^1 and S^2 into $S^* = \left(T^*, \pi^*\right)$:

$$T^* = T^1 \times T^2, \quad \pi^* : \Theta \to \Delta\left(T^1 \times T^2\right)$$

Definition

S^* is a *combined experiment* of S^1 and S^2 if:

1. $T^* = T^1 \times T^2, \ \pi^* : \Theta \to \Delta\left(T^1 \times T^2\right)$
2. marginal of S^1 is preserved:

$$\sum_{t^2 \in T^2} \pi^*\left(\left(t^1, t^2\right)|\theta\right) = \pi^1\left(t^1|\theta\right), \quad \forall t^1, \ \forall \theta.$$

3. marginal of S^2 is preserved:

$$\sum_{t^1 \in T^1} \pi^*\left(\left(t^1, t^2\right)|\theta\right) = \pi^2\left(t^2|\theta\right), \quad \forall t^2, \ \forall \theta.$$

Combining Experiments and Expanding Information

- there are multiple combined experiments S^* for any pair of experiments, since only the marginals have to match
- If S^* is combination of S and another experiment S', we say that S^* is an *expansion* of S.

(One Person) Robust Predictions Question

- fix (G, S)
- which (random) choices can arise under optimal decision making in (G, S^*) where S^* is any expansion of S?
- as a special case, information structure may be the null information structure:

$$S^\circ = \{T^\circ = \{t^\circ\}, \quad \pi^\circ(t^\circ|\theta) = 1\}$$

Theorem

An (random) choice ν is a BCE (random) choice of (G, S) if and only if there is an expansion S^* of S such that ν is a Bayes Nash equilibrium (random) choice for (G, S^*)

Idea of Proof:

- ($\Leftarrow$) S^* has "more" obedience constraints than S
- ($\Rightarrow$) let ν be BCE of (G, S) supporting σ and consider expansion S^* with $T^* = T \times A$ and $\pi^*(t, a|\theta) = \sigma(t, a|\theta)$.

Example: Bank Run

- a bank is solvent or insolvent:

$$\Theta = \{\theta_I, \theta_S\}$$

- each event is equally likely:

$$\psi(\cdot) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

- running (r) gives payoff 0
- not running (n) gives payoff -1 if insolvent, y if solvent:

$$0 < y < 1$$

- $G = (A, u)$ with $A = \{r, n\}$ and u given by

	θ_S	θ_I
r	0	0^*
n	y^*	-1

Bank Run: Common Prior Only

- suppose we have the prior information only - the null information structure:

$$S^{\circ} = \left(T^{\circ}, \pi^{\circ}\right), \quad T^{\circ} = \left\{t^{\circ}\right\}$$

- parameterized consistent (random) choices:

$\nu(\theta)$	θ_S	θ_I
r	ρ_S	ρ_I
n	$(1-\rho_S)$	$(1-\rho_I)$

- $\rho_S = \nu[\theta_S](r)$: (conditional) probability of running if solvent
- $\rho_I = \nu[\theta_I](r)$: (conditional) probability of running if insolvent

Bank Run: Obedience

- agent may not necessarily know state θ but makes choices according to $\nu(\cdot)$
- if "advised" to run, run has to be a best response:

$$\begin{aligned} 0 &\geq \rho_S y - \rho_I \Leftrightarrow \\ \rho_I &\geq \rho_S y \end{aligned}$$

- if "advised" not to run, not run has to be a best response

$$\begin{aligned} (1-\rho_S)\, y - (1-\rho_I) &\geq 0 \Leftrightarrow \\ \rho_I &\geq (1-y) + \rho_S y \end{aligned}$$

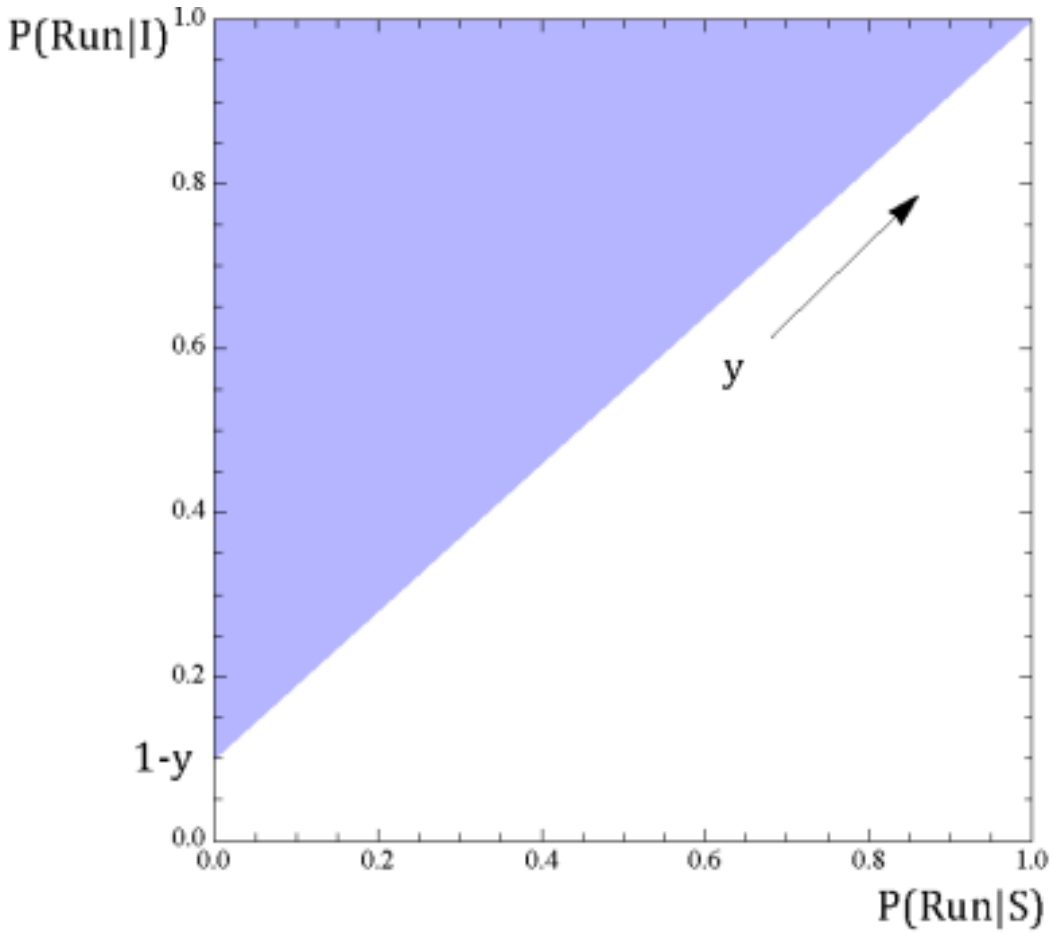
- here, not to run provides binding constraint:

$$\rho_I \geq (1-y) + \rho_S y$$

- never to run, $\rho_I = 0, \rho_S = 0$, cannot be a BCE

Bank Run: Equilibrium Set

- set of BCE described by (ρ_I, ρ_S)



- never to run, $\rho_I = 0, \rho_S = 0$, is not be a BCE

Bank Run: Extremal Equilibria

- BCE minimizing the probability of runs has:

$$\rho_I = 1 - y, \quad \rho_S = 0$$

- Noisy stress test $T = \left\{ t^I, t^S \right\}$ implements BNE via informative signals:

$\pi(t \mid \theta)$	θ_I	θ_S
t^I	$1 - y$	0
t^S	y	1

- the bank is said to be healthy if it is solvent (always) **and** if it is insolvent (sometimes)
- solvent and insolvent banks are bundled

Bank Run: Positive Information

- suppose player observes conditionally independent private binary signal of the state with accuracy:

$$q > \frac{1}{2}$$

- $S = (T, \pi)$ where $T = \{t^S, t^I\}$:

π	θ_S	θ_I
t^S	q	$1-q$
t^I	$1-q$	q

- strictly more information than null information $q = \frac{1}{2}$

Bank Run: Additional Obedience Constraints

- conditional probability of running now depends on the signal: $t \in \left\{t^S, t^I\right\}$
- ρ_I, ρ_S become $\left(\rho_I^I, \rho_S^I\right), \left(\rho_I^S, \rho_S^S\right)$
- conditional obedience constraints, say for t^S :

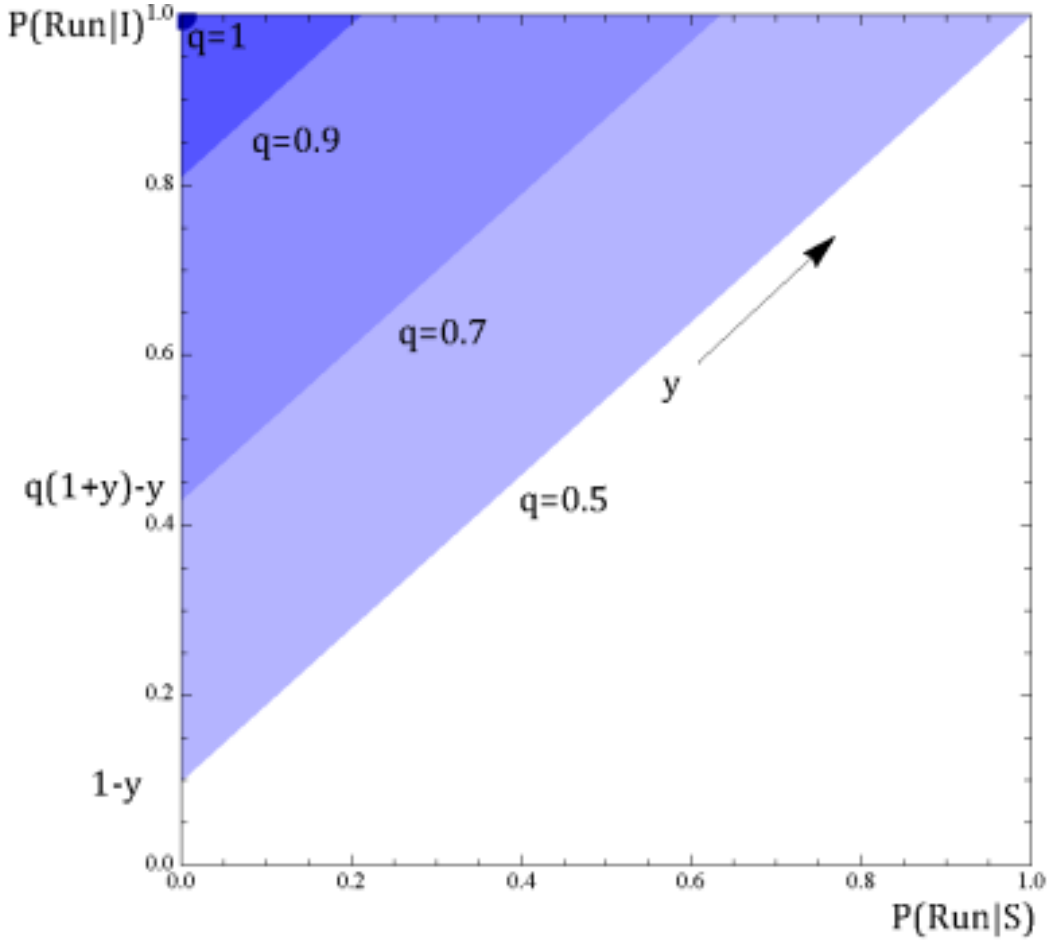
$$\begin{aligned} r &: 0 \geq q\rho_S^S y - (1-q)\,\rho_I^S \\ n &: q\left(1-\rho_S^S\right) y - (1-q)\left(1-\rho_I^S\right) \geq 0 \end{aligned}$$

or

$$\begin{aligned} r &: \rho_I^S \geq \frac{q}{1-q}\rho_S^S y \\ n &: \rho_I^S \geq 1 - \frac{q}{1-q} y + \frac{q}{1-q}\rho_S^S y \end{aligned}$$

Bank Run: Equilibrium Set

- set of BCE described by (ρ_I, ρ_S)



- $\rho_I = 1, \rho_S = 0$, is complete information BCE

Incentive Compatibility Ordering

- Write $BCE\left(G,S\right)$ for the set of BCE (random) choices of (G,S)

Definition

Experiment S is more *incentive constrained* than experiment S' if, for all decision problems G,

$$BCE\left(G,S\right) \subseteq BCE\left(G,S'\right).$$

- Note that "more incentive constrained" corresponds, intuitively, to having more information

Definition (Feasible Random Choice Rule)

A random choice rule ν is feasible for (G, S) if it is induced by a decision rule σ which is belief invariant for (G, S).

- write $F(G, S)$ for the set of feasible (random) choices of (G, S)

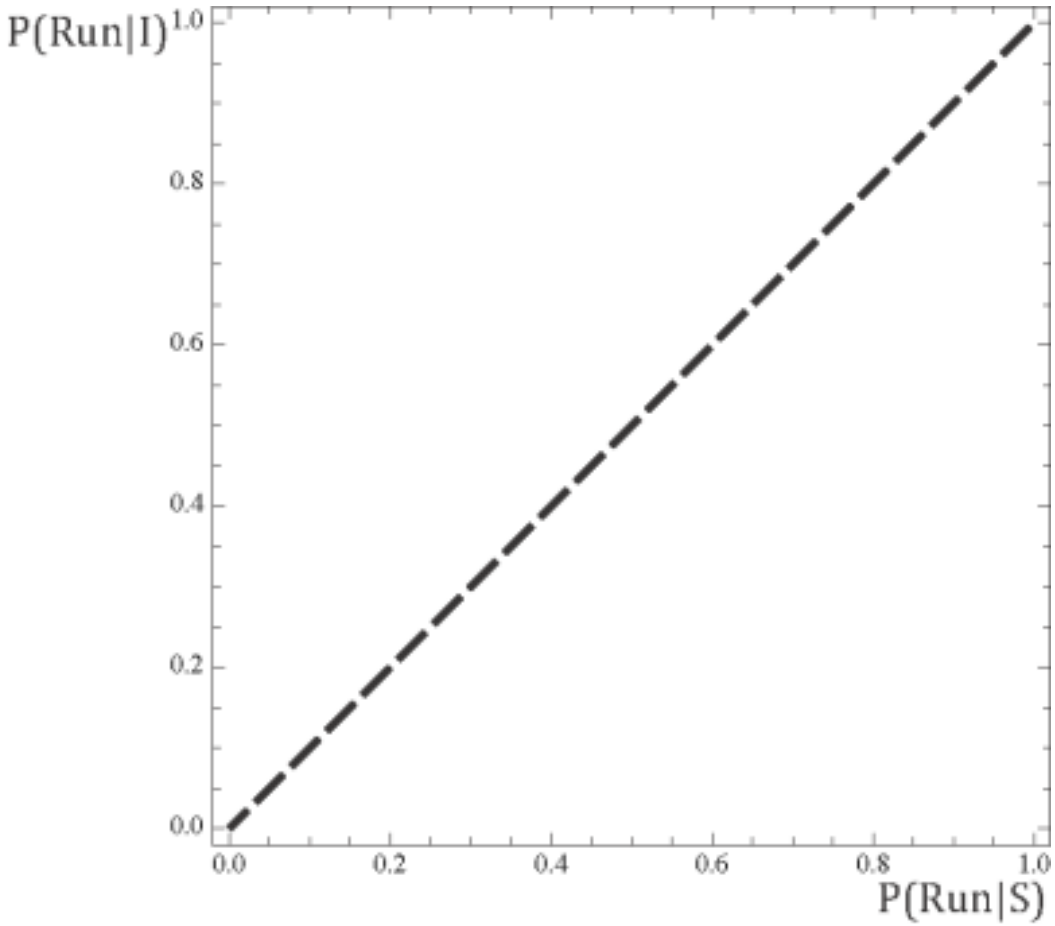
Definition (More Permissive)

Experiment S is more permissive than experiment S' if, for all decision problems G,

$$F(G, S) \supseteq F\left(G, S'\right).$$

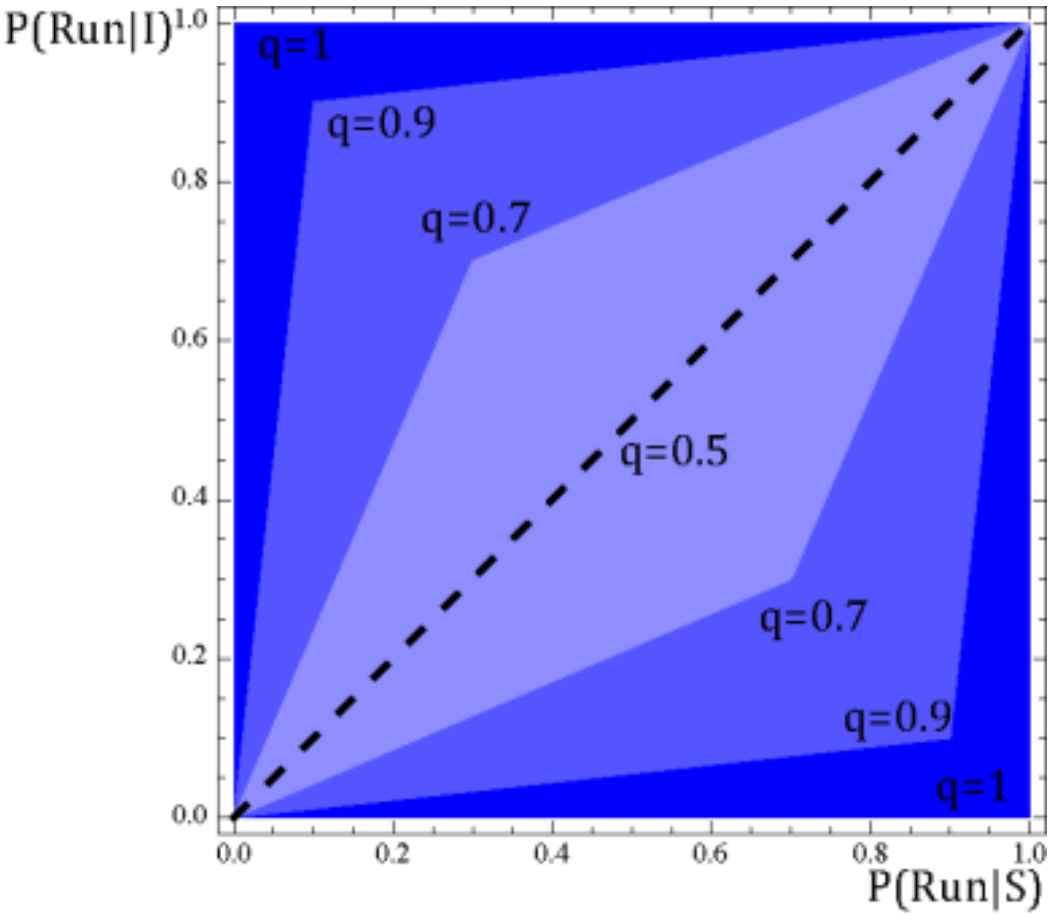
Back to the Example: Feasibility

- suppose we have the prior information only - the null information structure: $S_0 = (T_0, \pi), \quad T_0 = \{t_0\}$
- feasible (random) choices $\nu(\theta)$ can be described by (ρ_I, ρ_S):



Back to the Example: Feasibility

- suppose player observes conditionally independent private binary signal of the state with accuracy $q \geq \frac{1}{2}$:
- feasible (random) choices $\nu(\theta)$ can be described by (ρ_I, ρ_S):



- Experiment S is sufficient for experiment S' if there exists a combination S^* of S and S' such that

$$\Pr\left(t'|t,\theta\right) = \frac{\pi^*\left(t,t'|\theta\right)}{\sum_{\widetilde{t}'\in\Theta}\pi^*\left(t,\widetilde{t}'|\theta\right)}$$

  is independent of θ.

Sufficiency: Two Alternative Statements

1. (following from statistical fact): for any $\psi \in \Delta_{++}(\Theta)$,

$$\Pr\left(\theta|t,t'\right) = \frac{\psi\left(\theta\right)\pi^{*}\left(t,t'|\theta\right)}{\sum_{\theta'\in\Theta}\psi\left(\theta'\right)\pi^{*}\left(t,t'|\theta'\right)}.$$

   is independent of t'.

2. (naming the θ-independent conditional probability) there exists $\phi : T \to \Delta\left(T'\right)$ such that

$$\pi'\left(t'|\theta\right) = \sum_{t\in T}\phi\left(t'|t\right)\pi\left(t|\theta\right).$$

Aside: Belief Invariance = Sufficiency of Signals

- An (random) choice $\nu : \Theta \to \Delta(A)$ embeds an experiment (A, π) where

$$\pi(a|\theta) = \frac{\nu[\theta](a)}{\sum_{\widetilde{a}} \nu[\theta](\widetilde{a})}$$

- An (random) choice can be induced by a belief invariant decision rule if and only if S is sufficient for (A, ν).

Theorem

The following are equivalent:

1. *Experiment S is sufficient for experiment S' (statistical ordering);*
2. *Experiment S is more incentive constrained than experiment S' (incentive ordering);*
3. *Experiment S is more permissive than experiment S' (feasibility ordering).*

Proof of Blackwell's Theorem Plus

- Equivalence of (1) "sufficient for" and (3) "more permissive" is due to Blackwell
- (2) "more incentive constrained" $\Rightarrow$ (3) "more permissive":

1. take the stochastic transformation ϕ that maps S into S'
2. take any BCE $\nu \in \Delta(A \times T \times \Theta)$ of (G, S) and use ϕ to construct $\nu' \in \Delta(A \times T' \times \Theta)$
3. show that ν' is a BCE of (G, S')

Proof of Blackwell's Theorem Plus

- (3) "more permissive" $\Rightarrow$ (2) "more incentive constrained" by contrapositive
- suppose S is not more permissive than S'
- so $F(G,S) \not\supseteq F(G,S')$ for some G
- so there exists G' and $\nu' \in \Delta(A \times T' \times \Theta)$ which is feasible for (G',S') and gives (random) choice $\nu \in \Delta(A \times \Theta)$, with ν not feasible for (G,S)
- can choose G' so that the value V of ν' in (G',S') is V and the value every feasible ν of (G',S) is less than V
- now every there all BCE of (G',S') will have value at least V and some BCE of (G',S) will have value strictly less than V
- so $BCE(G',S) \subsetneq BCE(G',S')$

- players $i = 1, ..., I$
- (payoff) states Θ
- actions $(A_i)_{i=1}^I$
- utility functions $(u_i)_{i=1}^I$, each $u_i : A \times \Theta \to \mathbb{R}$
- state distribution $\psi \in \Delta(\Theta)$
- $G = \left((A_i, u_i)_{i=1}^I, \psi\right)$
- "decision problem" in the one player case

Information Structure

- signals (types) $(T_i)_{i=1}^I$
- signal distribution $\pi : \Theta \longrightarrow \Delta(T_1 \times T_2 \times ... \times T_I)$
- $S = \left((T_i)_{i=1}^I, \pi\right)$
- "experiment" in the one player case

Statistical Ordering: Individual Sufficiency

- Experiment S is individually sufficient for experiment S' if there exists a combination S^* of S and S' such that

$$\Pr\left(t'_i | t_i, t_{-i}, \theta\right) = \frac{\sum_{t'_{-i} \in T'_{-i}} \pi^*\left(t, \left(t'_i, t'_{-i}\right) | \theta\right)}{\sum_{\widetilde{t}'_i \in T'_i} \sum_{t'_{-i} \in T'_{-i}} \pi^*\left(t, \left(\widetilde{t}'_i, t'_{-i}\right) | \theta\right)}$$

  is independent of (t_{-i}, θ).

Sufficiency: Two Alternative Statements

- following from statistical fact applied to triple $(t_i', t_i, (t_{-i}, \theta))$ after integrating out t'_{-i}
- for any $\psi \in \Delta_{++}(\Theta)$,

$$\Pr\left(t_{-i}, \theta \left| t_i, t_i'\right.\right) = \frac{\sum_{t'_{-i} \in T'_{-i}} \psi(\theta)\, \pi^*\left(\left(t_i, t_{-i}\right), \left(t_i', t'_{-i}\right) | \theta\right)}{\sum_{\widetilde{t}_{-i} \in T_{-i}} \sum_{\widetilde{\theta} \in \Theta} \sum_{t'_{-i} \in T'_{-i}} \psi\left(\widetilde{\theta}\right) \pi^*\left(\left(t_i, \widetilde{t}_{-i}\right), \left(t_i', t'_{-i}\right) | \widetilde{\theta}\right.}$$

is independent of t_i'.

Sufficiency: Two Alternative Statements

- letting $\phi : T \times \Theta \to \Delta(T')$ be conditional probability for combined experiment π^*
- there exists $\phi : T \times \Theta \to \Delta(T')$ such that

$$\pi'\left(t'|\theta\right) = \sum_{t \in T} \phi\left(t'|t,\theta\right)\pi\left(t|\theta\right)$$

and

$$\Pr_{\phi}\left(t_i' \,|t_i, t_{-i}, \theta\right) = \sum_{t'_{-i} \in T'_{-i}} \phi\left(\left(t_i', t'_{-i}\right)|\left(t_i, t_{-i}\right), \theta\right)$$

is independent of (t_{-i}, θ)

Nice Properties of Ordering

- Transitive
- Neither weaker or stronger than sufficiency (i.e., treating signal profiles as multidimensional signals)
- Two information structures are each sufficient for each other if and only if they share the same higher order beliefs about Θ
- S is individually sufficient for S' if and only if S is higher order belief equivalent to an expansion of S'
- S is individually sufficient for S' if and only if there exists a combined experiment equal to S' plus a correlation device

Example

- Compare null information structure S°...
- ...with information structure S with $T_1 = T_2 = \{0, 1\}$

$\pi(\cdot\|0)$	0	1
0	$\frac{1}{2}$	0
1	0	$\frac{1}{2}$

$\pi(\cdot\|1)$	0	1
0	0	$\frac{1}{2}$
1	$\frac{1}{2}$	0

- Each information structure is individually sufficient for the other

Theorem

The following are equivalent:

1. *Information structure S is individually sufficient for information structure S' (statistical ordering);*
2. *Information structure S is more incentive constrained than information structure S' (incentive ordering);*
3. *Information structure S is more permissive than information structure S' (feasibility ordering);*

Proof of Blackwell's Theorem Plus

- (1) $\Rightarrow$ (3) directly constructive argument
- (1) "sufficient for" $\Rightarrow$ (2) "more incentive constrained" works as in the single player case

1. take the stochastic transformation ϕ that maps S into S'
2. take any BCE $\nu \in \Delta(A \times T \times \Theta)$ of (G, S) and use ϕ to construct $\nu' \in \Delta(A \times T' \times \Theta)$
3. show that ν' is a BCE of (G, S')

- need a new argument to show (3) $\Rightarrow$ (2)

New Argument: Game of Belief Elicitation

- Suppose that S is more incentive constrained than S'
- Consider game where players report types in S
- Construct payoffs such that (i) truthtelling is a BCE of (G, S) (ii) actions corresponding to reporting beliefs over $T_{-i} \times \Theta$ with incentives to tell the truth
- In order to induce the truth-telling (random) choice of (G, S), there must exist $\phi : T \times \Theta \to \Delta(T')$ corresponding to one characterization of individual sufficiency

Incomplete Information Correlated Equilibrium

- Forges (1993): "Five Legitimate Definitions of Correlated Equilibrium"
- BCE = set of (random) choices consistent with (common prior assumption plus) common knowledge of rationality and that players have observed at least information structure S.
- Not a solution concept for a fixed information structure as information structure is in flux

Other Definitions: Stronger Feasibility Constraints

- Belief invariance: information structure cannot change, so players cannot learn about the state and others' types from their action recommendations
  - Liu (2011) - belief invariant Bayes correlated equilibrium: obedience and belief invariance
  - captures common knowledge of rationality and players having exactly information structure S.
- Join Feasibility: equilibrium play cannot depend on things no one knows given S
  - Forges (1993) - Bayesian solution: obedience and join feasibility
  - captures common knowledge of rationality and players having at least information structure S and a no correlation restriction on players' conditional beliefs
- belief invariant Bayesian solution - imposing both belief invariance and join feasibility - has played prominent role in the literature

Other Definitions: the rest of the Forges' Five

1. More feasibility restrictions: agent normal form correlated equilibrium
2. More incentive constraints: communication equilibrium: mediator can make recommendations contingent on players' types only if they have an incentive to truthfully report them.
3. Both feasibility and incentive constraints: strategic form correlated equilibrium

Generalizing Blackwell's Theorem

- we saw - in both the one and the many player case - that "more information" helps by relaxing feasibility constraints and hurts by imposing incentive constraints
- Lehrer, Rosenberg, Shmaya (2010, 2011) propose family of partial orders on information structures, refining sufficiency
  - LRS10 kill incentive constraints by showing orderings by focussing on common interest games. Identify right information ordering for different solution concepts
  - LRS 11 kill incentive constraints by restriction attention to info structures with the same incentive constraints. Identify right information equivalence notion for different solution concepts
- We kill feasibility benefit of information by looking at BCE. Thus we get "more information" being "bad" and incentive constrained ordering characterized by individual sufficiency.
- Same ordering corresponds to a natural feasibility ordering (ignoring incentive constraints)

Conclusion

- a permissive notion of correlated equilibrium in games of incomplete information: Bayes correlated equilibrium
- BCE renders robust predicition operational, embodies concern for robustness to strategic information
- leads to a natural multi-agent generalization of Blackwell's single agent information ordering