The Limits of Price Discrimination

Dirk Bergemann, Ben Brooks and Stephen Morris

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a classic issue in the analysis of monopoly is the impact of discriminatory pricing on consumer and producer surplus

• a monopolist engages in third degree price discrimination if he uses additional information - beyond the aggregate distribution - about consumer characteristics to offer different prices to different segments
additional information leads to segmentation of the population

different segments are offered different prices

with additional information about the valuations of the consumers seller can match / tailor prices to consumer characteristics

what are then the possible (consumer surplus, producer surplus) pairs (for some information)?
in other words, what are possible welfare outcomes from *third degree price discrimination*?
if market segmentations are exogenous (location, time, age), then only specific segmentations may be of interest,
but, increasingly, data intermediaries collect and distribute information, and in consequence segmentations become increasingly endogeneous, choice variables
for example, if data is collected directly by the seller, then as much information about valuations as possible might be collected, consumer surplus is extracted
by contrast, if data is collected by an intermediary, to increase consumer surplus, or for some broader business model, then the choice of segmentation becomes an instrument of design
implications for privacy regulations, data collection, data sharing, etc....
A Classical Economic Problem: A First Pass

- Fix a demand curve
- Interpret the demand curve as representing single unit demand of a continuum of consumers
- If a monopolist producer is selling the good, what is producer surplus (monopoly profits) and consumer surplus (area under demand curve = sum of surplus of buyers)?
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- Interpret the demand curve as representing single unit demand of a continuum of consumers
- If a monopolist producer is selling the good, what is producer surplus (monopoly profits) and consumer surplus (area under demand curve = sum of surplus of buyers)?
- If the seller cannot discriminate between consumers, he must charge uniform monopoly price
• Write $u^*$ for the resulting consumer surplus and $\pi^*$ for the producer surplus ("uniform monopoly profits")
• But what if the producer could observe each consumer’s valuation perfectly?
• Pigou (1920) called this "first degree price discrimination"
• In this case, consumer gets zero surplus and producer fully extracts efficient surplus $w^* > \pi^* + u^*$
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• Pigou (1920) called this "third degree price discrimination"
• What can happen?
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Pigou (1920) called this "third degree price discrimination"

What can happen?

A large literature (starting with Pigou (1920)) asks what happens to consumer surplus, producer surplus and thus total surplus if we segment the market in particular ways
Our main question:

- What could happen to consumer surplus, producer surplus and thus total surplus for all possible ways of segmenting the market?
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- What could happen to consumer surplus, producer surplus and thus total surplus for all possible ways of segmenting the market?

Our main result
- A complete characterization of all (consumer surplus, producer surplus) pairs that can arise...
1. Voluntary Participation: Consumer Surplus is at least zero
Payoff Bounds: Voluntary Participation

Consumer surplus is at least zero

Consumer surplus

Producer surplus

$w^*$

$\pi^*$

$u^*$
Three Payoff Bounds

1. Voluntary Participation: Consumer Surplus is at least zero
2. Non-negative Value of Information: Producer Surplus bounded below by uniform monopoly profits $\pi^*$
Payoff Bounds: Nonnegative Value of Information

Producer gets at least uniform price profit

$\pi^*$

$w^*$

$0$

Consumer surplus
Three Payoff Bounds

1. Voluntary Participation: Consumer Surplus is at least zero
2. Non-negative Value of Information: Producer Surplus bounded below by uniform monopoly profits $\pi^*$
3. Social Surplus: The sum of Consumer Surplus and Producer Surplus cannot exceed the total gains from trade
Total surplus is bounded by efficient outcome
Beyond Payoff Bounds

1. Includes point of uniform price monopoly, \((u^*, \pi^*)\),
2. Includes point of perfect price discrimination, \((0, w^*)\)
3. Segmentation supports convex combinations
Payoff Bounds and Convexity

1. Includes point of uniform price monopoly, \((u^*, \pi^*)\),
2. Includes point of perfect price discrimination, \((0, w^*)\)
3. Segmentation supports convex combinations

What is the feasible surplus set?
Main Result: Payoff Bounds are Sharp

Main result

\[ \pi^* \]

\[ w^* \]

Producer surplus

Consumer surplus
Main Result

- For any demand curve, any (consumer surplus, producer surplus) pair consistent with three bounds arises with some segmentation / information structure....
Main Result

- For any demand curve, any (consumer surplus, producer surplus) pair consistent with three bounds arises with some segmentation / information structure....in particular, there exist ...

1. a consumer surplus maximizing segmentation where
   - the producer earns uniform monopoly profits,
   - the allocation is efficient,
   - and the consumers attain the difference between efficient surplus and uniform monopoly profit.
Main Result

- For any demand curve, any (consumer surplus, producer surplus) pair consistent with three bounds arises with some segmentation / information structure. In particular, there exist...

1. A consumer surplus maximizing segmentation where
   1. the producer earns uniform monopoly profits,
   2. the allocation is efficient,
   3. and the consumers attain the difference between efficient surplus and uniform monopoly profit.

2. A social surplus minimizing segmentation where
   1. the producer earns uniform monopoly profits,
   2. the consumers get zero surplus,
   3. and so the allocation is very inefficient.
• convex combination of any pair of achievable payoffs as binary segmentation between constituent markets
• it suffices to obtain the vertices of the surplus triangle
Main Result

- Setup of Finite Value Case
- Proof for the Finite Value Case
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- Constructions (and a little more intuition?)
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- The Relation to the Classical Literature on Third Degree Price Discrimination, including results for output and prices
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Methodology of Bayes correlated equilibrium

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- A solution concept, "Bayes correlated equilibrium," characterizes what could happen in (Bayes Nash) equilibrium for all information structures.

Advantages:
- Do not have to solve for all information structures separately.
- Nice linear programming characterization.
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Papers Related to this Agenda

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   1. ...today...
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3. Linear Normal Symmetric
   1. Stylised applications within continuum player, linear best response, normally distributed games with common values (aggregate uncertainty) ("Robust Predictions in Incomplete Information Games", Econometrica 2013)
   2. "Information and Volatility" (with Tibor Heumann): economy of interacting agents, agents are subject to idiosyncratic and aggregate shocks, how do shocks translate into individual, aggregate volatility, how does the translation depend on the information structure?
   3. "Market Power and Information" (with Tibor Heumann): adding endogeneous prices as supply function equilibrium
• continuum of consumers
• finite set of valuations:

\[ 0 < v_1 < v_2 < \ldots < v_k < \ldots < v_K \]

• constant marginal cost normalized to zero
- continuum of consumers
- finite set of valuations:
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- a *market* is a probability vector
  \[ x = (x_1, \ldots, x_k, \ldots, x_K) \]
  where \( x_k \) is the proportion of consumers with valuation \( v_k \)
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\[ x = (x_1, \ldots, x_k, \ldots, x_K) \]

where \( x_k \) is the proportion of consumers with valuation \( v_k \)

• set of possible markets \( X \) is the \( K \)-dimensional simplex,

\[ X \triangleq \left\{ x \in \mathbb{R}_+^K \mid \sum_{k=1}^{K} x_k = 1 \right\}. \]
the price $v_i$ is \textit{optimal} for a given market $x$ if and only if

$$v_i \sum_{j \geq i} x_j \geq v_k \sum_{j \geq k} x_j, \quad \forall k$$
Markets and Monopoly Prices

- the price $v_i$ is *optimal* for a given market $x$ if and only if
  \[ v_i \sum_{j \geq i} x_j \geq v_k \sum_{j \geq k} x_j, \quad \forall k \]
- write $X_i$ for the set of markets where price $v_i$ is optimal,
  \[ X_i \triangleq \left\{ x \in X \left| v_i \sum_{j \geq i} x_j \geq v_k \sum_{j \geq k} x_j, \quad \forall k \right. \right\}. \]
Markets and Monopoly Prices

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  \]

- each \( X_i \) is a convex polytope in the probability simplex
there is an "aggregate market" \( x^* \):

\[
x^* = (x_1^*, ..., x_k^*, ..., x_K^*)
\]
• there is an "aggregate market" $x^*$:

$$x^* = (x_1^*, ..., x_k^*, ..., x_K^*)$$

• define the uniform monopoly price for aggregate market $x^*$:

$$p^* = v_{i^*}$$

such that:

$$v_{i^*} \sum_{j \geq i^*} x_j^* \geq v_k \sum_{j \geq k} x_j^*, \ \forall k$$
• given aggregate market $x^*$ as point in probability simplex
• here $x^* = (1/3, 1/3, 1/3)$ uniform across $v \in \{1, 2, 3\}$
composition of aggregate market $x^* = (x_1^*, ..., x_k^*, ..., x_K^*)$ determines optimal monopoly price: $p^* = 2$
Segmentation of Aggregate Market

- segmentation: $\sigma$ is a simple probability distribution over the set of markets $X$,

$$\sigma \in \Delta (X)$$

- $\sigma (x)$ is the proportion of the population in segment with composition $x \in X$
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• $\sigma (x)$ is the proportion of the population in segment with composition $x \in X$

• a segmentation is a two stage lottery over values $\{v_1, ..., v_K\}$ whose reduced lottery is $x^*$:

$$\left\{ \sigma \in \Delta (X) \left| \sum_{x \in \text{supp} (\sigma)} \sigma (x) \cdot x = x^*, \ |\text{supp} (\sigma)| < \infty \right. \right\}.$$
Segmentation of Aggregate Market

- Segmentation: \( \sigma \) is a simple probability distribution over the set of markets \( X \),
  \[
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- \( \sigma (x) \) is the proportion of the population in segment with composition \( x \in X \)
- A segmentation is a two-stage lottery over values \( \{v_1, ..., v_K\} \) whose reduced lottery is \( x^* : \)
  \[
  \left\{ \begin{array}{c}
  \sigma \in \Delta (X) \\
  \sum_{x \in \text{supp}(\sigma)} \sigma (x) \cdot x = x^*, \quad |\text{supp}(\sigma)| < \infty
  \end{array} \right\} .
  \]
- A pricing strategy for segmentation \( \sigma \) specifies a price in each market in the support of \( \sigma \),
  \[
  \phi : \text{supp}(\sigma) \rightarrow \Delta \{v_1, ..., v_K\}
  \]
• consider the uniform market with three values
• a segmentation of the uniform aggregate market into three market segments:

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the segments of the aggregate market form a joint distribution over market segmentations and valuations

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<td>market 3</td>
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<td>$\frac{1}{6}$</td>
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• additional information (signals) can generate the segmentation
• likelihood function

\[ \lambda : V \rightarrow \Delta(S) \]

• in the uniform example

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this segmentation was special

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price 2 is optimal in all markets
this segmentation was special

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price 2 is optimal in all markets

in fact, seller is always indifferent between all prices in the support of every market segment, "unit price elasticity"
extremal segment $x^S$: seller is indifferent between all prices in the support of $S$
• an optimal policy: always charge lowest price in the support of every segment:

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• another optimal policy: always charge highest price in each segment:

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for any support set \( S \subseteq \{1, ..., K\} \neq \emptyset \), define market \( x^S \):

\[
x^S = (\ldots, x_k^S, \ldots) \in X,
\]

with the properties that:

1. no consumer has valuations outside the set \( \{v_i\}_{i \in S} \);
2. the monopolist is indifferent between every price in \( \{v_i\}_{i \in S} \).
for every $S$, this uniquely defines a market

$$x^S = \left(\ldots, x_k^S, \ldots\right) \in X$$

writing $S$ for the smallest element of $S$, the unique distribution is:

$$x^S_k \triangleq \begin{cases} \frac{v_S}{v_k} - \sum_{k'>k} x_{k'} & \text{if } k \in S \\ 0, & \text{if } k \notin S. \end{cases}$$

(a discrete version of the Pareto distribution)

for any $S$, market $x^S$ is referred to as extremal market
• extremal markets
set of markets $X_{i\ast}$ where uniform monopoly price $p^* = v_{i\ast}$ is optimal:

$$X_{i\ast} = \left\{ x \in X \mid v_{i\ast} \sum_{j \geq i\ast} x_j \geq v_k \sum_{j \geq k} x_j, \ \forall k \right\}$$
Convex Representation

- set of markets \( X_{i*} \) where uniform monopoly price \( p^* = v_{i*} \) is optimal:

\[
X_{i*} = \left\{ x \in X \left| v_{i*} \sum_{j \geq i*} x_j \geq v_k \sum_{j \geq k} x_j, \forall k \right. \right\}
\]

- \( S^* \) is subset of subsets \( S \subseteq \{1, ..., i^*, ..., K\} \) containing \( i^* \)
Convex Representation

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$$X_{i^*} = \left\{ x \in X \mid v_{i^*} \sum_{j \geq i^*} x_j \geq v_k \sum_{j \geq k} x_j, \forall k \right\}$$

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Lemma (Extremal Segmentation)

$X_{i^*}$ is the convex hull of $(x^S)_{S \in S^*}$
Lemma (Extremal Segmentation)

\[ X_{i^*} \text{ is the convex hull of } (x^S)_{S \in S^*} \]
Extremal Segmentations

- $S^*$ is subset of subsets $S \subseteq \{1, \ldots, i^*, \ldots, K\}$ containing $i^*$

**Lemma (Extremal Segmentation)**

$X_{i^*}$ is the convex hull of $(x^S)_{S \in S^*}$

**Sketch of Proof:**

- pick any $x \in X$ where price $v_{i^*}$ is optimal (i.e., $x \in X_{i^*}$) but there exists $k$ such that valuation $v_k$ arises with strictly positive probability (so $x_k > 0$) but is not an optimal price
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**Sketch of Proof:**

- pick any $x \in X$ where price $v_{i^*}$ is optimal (i.e., $x \in X_{i^*}$) but there exists $k$ such that valuation $v_k$ arises with strictly positive probability (so $x_k > 0$) but is not an optimal price
- let $S$ be the support of $x$
Extremal Segmentations

- $S^*$ is subset of subsets $S \subseteq \{1, \ldots, i^*, \ldots, K\}$ containing $i^*$

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- so $x$ is not an extreme point of $X_{i^*}$
• Split $x^*$ into any extremal segmentation
• There is a pricing rule for that one segmentation that attains any point on the bottom of the triangle, i.e., producer surplus $\pi^*$ anything between 0 and $w^* - \pi^*$.
• The rest of the triangle attained by convexity
A pricing rule specifies how to break monopolist indifference

1. "Minimum pricing rule" implies efficiency (everyone buys)

So minimum pricing rule maximizes consumer surplus (bottom right corner of triangle)

So maximum pricing rule minimizes total surplus (bottom left corner of triangle)
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Theorem (Minimum and Maximum Pricing)

1. In every extremal segmentation, minimum and maximum pricing strategies are optimal;
2. producer surplus is $\pi^*$ under every optimal pricing strategy;
3. consumer surplus is zero under maximum pricing strategy;
4. consumer surplus is $w^* - \pi^*$ under minimum pricing strategy.
We first report a simple direct construction of a consumer surplus maximizing segmentation (bottom right hand corner):

1. first split:
   1. We first create a market which contains all consumers with the lowest valuation $v_1$ and a constant proportion $q_1$ of valuations greater than or equal to $v_2$
   2. Choose $q_1$ so that the monopolist is indifferent between charging price $v_1$ and the uniform monopoly price $v_{i^*}$
   3. Note that $v_{i^*}$ continues to be an optimal price in the residual market

2. Iterate this process
We first report a simple direct construction of a consumer surplus maximizing segmentation (bottom right hand corner):

1. first split:
2. Iterate this process
3. thus at round \( k \),

1. first create a market which contains all consumers with the lowest remaining valuation \( v_k \) and a constant proportion \( q_k \) of valuations greater than or equal to \( v_{k+1} \)
2. Choose \( q_k \) so that the monopolist is indifferent between charging price \( v_k \) and the uniform monopoly price \( v_{i*} \) in the new segment
3. Note that \( v_{i*} \) continues to be an optimal price in the residual market
In our three value example, we get:

<table>
<thead>
<tr>
<th></th>
<th>( v = 1 )</th>
<th>( v = 2 )</th>
<th>( v = 3 )</th>
<th>price</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>first segment</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{1}{4} )</td>
<td>1</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>second segment</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>2</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>total</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
A Simple "Direct" Construction
Advice for the Consumer Protection Agency?

- Allow producers to offer discounts (i.e., prices lower the uniform monopoly price)
- Put enough high valuation consumers into discounted segments so that the uniform monopoly price remains optimal
A Dual Purpose Segementation: Greedy Algorithm

1. Put as many consumers as possible into extremal market $x^{\{1,2,...,K\}}$

2. Generically, we will run out of consumers with some valuation, say, $v_k$

3. Put as many consumers as possible into residual extremal market $x^{\{1,2,...,K\}}/\{k\}$

4. Etc....
Greedy Algorithm

- In our three value example, we get first:

<table>
<thead>
<tr>
<th></th>
<th>$v = 1$</th>
<th>$v = 2$</th>
<th>$v = 3$</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1, 2, 3}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>${2, 3}$</td>
<td>0</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>total</td>
<td>$\frac{1}{3}$</td>
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Then we get

<table>
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<th>$v = 1$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>market 1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>market 2</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>market 3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>total</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td></td>
</tr>
</tbody>
</table>
• extremal markets $x^{\{\ldots\}}$
• splitting the aggregate market $x^*$ into extremal markets $x\{\ldots\}$

Split off $x^{\{1,2,3\}}$

Diagram:
- $x^{\{2\}}$
- $x^{\{2,3\}}$
- $x^{\{1,2,3\}}$
- Residual
- $x^*$
A Visual Proof: Splitting and Greedy Algorithm

- splitting greedily: maximal weight on the maximal market
splitting the aggregate market $x^*$ into extremal market segments all including $p^* = 2$
minimal and maximal pricing rule maintained $\pi^*$

first degree price discrimination resulted in third vertex

**Theorem (Surplus Triangle)**

*There exists a segmentation and optimal pricing rule with consumer surplus $u$ and producer surplus $\pi$ if and only if $(u, \pi)$ satisfy $u \geq 0$, $\pi \geq \pi^*$ and $\pi + u \leq \pi^*$*

- convexity of information structures allows to establish the entire surplus triangle
Continuous Demand Case

- All results extend
- Main result can be proved by a routine continuity argument
- Constructions use same economics, different math (differential equations)
- Segments may have mass points
Third Degree Price Discrimination

- **classic topic:**
  - Pigou (1920) *Economics of Welfare*
  - Robinson (1933) *The Economics of Imperfect Competition*

- **middle period:** e.g.,
  - Schmalensee (1981)
  - Varian (1985)
  - Nahata et al (1990)

- **latest word:**
  - Aguirre, Cowan and Vickers (AER 2010)
  - Cowan (2012)
Existing Results: Welfare, Output and Prices

- examine welfare, output and prices
- focus on two segments
- price rises in one segment and drops in the other if segment profits are strictly concave and continuous: see Nahata et al (1990))
- Pigou:
  - welfare effect = output effect + misallocation effect
  - two linear demand curves, output stays the same, producer surplus strictly increases, total surplus declines (through misallocation), and so consumer surplus must strictly decrease
- Robinson: less curvature of demand \((-\frac{p \cdot q''}{q'})\) in "strong" market means smaller output loss in strong market and higher welfare
Our Results (across all segmentations)

- **Welfare:**
  - Main result: consistent with bounds, anything goes
  - Non first order sufficient conditions for increasing and decreasing total surplus (and can map entirely into consumer surplus)

- **Output:**
  - Maximum output is efficient output
  - Minimum output is given by conditionally efficient allocation generating uniform monopoly profits as total surplus (note: different argument)

- **Prices:**
  - all prices fall in consumer surplus maximizing segmentation
  - all prices rise in total surplus minimizing segmentation
  - prices might always rise or always fall whatever the initial demand function (this is sometimes - as in example - consistent with weakly concave profits, but not always)
our results concerned a special "screening" problem: each consumer has single unit demand
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can ask the same question: .... look for feasible (information rent, principal utility) pairs... in general screening problems
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- no complete characterization
Beyond Linear Demand and Cost

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- can ask the same question.... look for feasible (information rent, principal utility) pairs... in general screening problems
- no complete characterization
- we study what drives our results by seeing what happens as we move towards general screening problems by adding a little non-linearity

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Re-interpret our Setting and adding small concavity

- Our main setting: Consumer type $\nu$ consuming quantity $q \in \{0, 1\}$ gets utility $\nu \cdot q$
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Note that efficient allocation for all types is 1
Three Types and Three Output Levels

- Suppose $v \in \{1, 2, 3\}; q \in \{0, \frac{1}{2}, 1\}$
- Always efficient to have allocation of 1
- Note that in this case, utilities are given by

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>$\frac{1}{2}$</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2} + \varepsilon$</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$1 + \varepsilon$</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$\frac{3}{2} + \varepsilon$</td>
<td>3</td>
</tr>
</tbody>
</table>

- contract $q = (q_1, q_2, q_3)$ specifies output level for each type
- six contracts which are monotonic and efficient at the top:
  - $(0, 0, 1), (0, \frac{1}{2}, 1), (0, 1, 1), (\frac{1}{2}, \frac{1}{2}, 1), (\frac{1}{2}, 1, 1)$ and $(1, 1, 1)$
- Now we can look at analogous simplex picture
- Illustrates geometric structure in the general case
- richer partition of probability simplex

- additional allocations beyond binary appear as optimal
Now restrict attention to $v \in \{1, 2\}$

- probability simplex becomes unit interval
- denote by $x$ probability of low valuation:

$$x \triangleq \Pr(v = 1)$$

- extremal markets are $x$ and $\bar{x}$
Now it is natural to plot consumer surplus and producer surplus as a function of $x$, the probability of type 1.
Now solving for feasible (consumer surplus, producer surplus pairs) for \( x = \frac{1}{2} \) comes from concavifying weighted sums of these expressions.
Now allow any $q \in [0, 1]$. If $x$ is the proportion of low types, the optimal contract is now:

$$
\tilde{q}(x) = \begin{cases} 
\frac{1}{2} - \frac{1}{8\epsilon} \left(2 - \frac{1}{x}\right), & \text{if } x \leq \frac{1}{2+4\epsilon} \\
\frac{1}{2+4\epsilon}, & \text{if } \frac{1}{2+4\epsilon} \leq x \leq \frac{1}{2-4\epsilon} \\
1, & \text{if } x \geq \frac{1}{2-4\epsilon}
\end{cases}
$$
Two Types, Continuous Output

- **Producer surplus ($\pi$)**
  - Graph showing the relationship between $\pi$ and the proportion of low types ($x$).

- **Consumer surplus ($u$)**
  - Graph showing the relationship between $u$ and the proportion of low types ($x$).

- **$\frac{3}{4} \pi + u$**
  - Graph showing the relationship between $\frac{3}{4} \pi + u$ and the proportion of low types ($x$).
Two Types, Continuous Output
1. The set of prior distributions of types where it is possible to attain bottom left and bottom right corner will shrink fast as the setting gets more complex.

2. As long as there are a finite set of output levels,
   1. There is an analogous restriction to extreme points of best response regions of the simplex (geometric approach translates).
   2. The "bottom flat" survives: there is an open set of information rents consistent with principal getting uninformed profit.

3. With continuum output levels
   1. The "bottom flat" goes.
   2. Multiple information rents consistent with other levels of consumer profit, approaching the triangle continuously as we approach a linear case.
Kamenica and Gentzkow (2010): Suppose that a sender could commit (before observing his type) to cheap talk signals to send to a receiver. What would he send?
defacto, this is what happened in Aumann and Maschler (1995) repeated games with one sided information who showed sender "concavifies" payoffs
We can solve for feasible surplus pairs by this method if the "sender" were a social planner maximizing a arbitrary weighted sum of consumer and producer surplus and the "receiver" were the monopolist
Very helpful in two type case, implicit in many type case
- robust predictions research agenda....
- the set of all outcomes that could arise in Bayes Nash equilibrium in given "basic game" for all possible information structures = "Bayes correlated equilibria"
  - "The comparison of information structures in games: Bayes correlated equilibrium and individual sufficiency" (general theory)
  - "Robust predictions in games with incomplete information games" (applications in symmetric continuum player linear best response games, Ecta (2013))
- seller problem here is single player application
- this paper is by-product of many player application:
  - Bergemann, Brooks and Morris: "Extremal Information Structures in First Price Auction"
Auction Teaser

- First price auction
- Bidder $i$’s valuations drawn according to cdf $F_i$
- Lower bound on interim bidder surplus of bidder with valuation $v$ is
  \[ u_i(v) = \max_b (v - b) \prod_{j \neq i} F_j(b) \]
- Lower bound on ex ante expected surplus of bidder $i$ is
  \[ U_i = \int_{v=0}^{1} u_i(v) f_i(v) \, dv \]
- Upper bound on expected revenue is total expected surplus minus each bidder’s surplus lower bound
- Claim: there is an information structure where these bounds are attained in equilibrium
Tell each bidder if he has the highest value or not

Losing bidders bid their values and lose (undominated strategy)

Winning bidder’s "uniform monopoly profit" (maximum profit if he knows nothing about the losing bid) is now the lower bound $U_i$

Our main result states that we can provide (partial) information to the winner about highest losing bid in just such a way that he is still held down to his uniform monopoly profit and always wins
Two Bidders: Information and Revenue

- 2 bidders, valuations uniform on $[0, 1]$
- Ex ante expected surplus is $\frac{2}{3}$
- No information:
  - bid $\frac{1}{2} v$, each bidder surplus $\frac{1}{6}$, revenue $\frac{1}{3}$
- Complete information $=$ Bertrand:
  - each bidder surplus $\frac{1}{6}$, revenue $\frac{1}{3}$
- Our intermediate information structure:
  - each bidder surplus $\frac{1}{12}$, revenue $\frac{1}{2}$
- distribution of bidders (surplus) and implications for revenue equivalence, ...

The Payoff Space of the Bidders
It is feasible and interesting to see what happens under many information structures at once.

This methodology generates striking new answers for classical economic questions.

In mechanism design we design the payoffs of the game, assuming the information structure is fixed.

In information design, we design the information received by the players, assuming the game is fixed.
Do We Care about Extremal Segmentations?

- extremal segmentations are "extreme"...
- might not arise exogenously....
- but suppose someone could choose segments endogenously?
• extremal segmentations are "extreme"
• might not arise exogenously
• but suppose someone could choose segments endogenously?
• Google knows everyone’s values of everything (pretty much)
• Google wants to "do no evil"
• Operationalization of "do no evil": report noisy signals of values to sellers in such a way that sellers choose to price discriminate in a way that attains efficiency and gives all the efficiency gains to consumers