Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium

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In decision problems / games with imperfect information, additional information can influence / change behavior.

What is (maximal) extent by which a principal by providing information alone can change the behavior of an agent?

Bayesian persuasion: a single agent with zero information beyond common prior:

How does private information by (many) agents limit the scope of persuasion?
Information Design and Bayes Correlated Equilibrium

- how to optimally provide additional information to many, privately informed agents?
- an immediate difficulty: space of possible information structures, especially in many agent settings, is very large
- an apparent detour: first describe the set of all possible outcome across all possible information structures
- the solution concept, Bayes correlated equilibrium, characterizes outcomes independent of any specific information structure
- robust prediction: independent of details of any specific information structure
- information design: find the optimal information structure for a given objective function
Information Design and Mechanism Design

- **Mechanism Design**:  
  - Fix an economic environment and information structure  
  - Design the rules of the game to get a desirable outcome

- **Information Design**
  - Fix an economic environment and rules of the game  
  - Design an information structure to get a desirable outcome
Main Lessons

- information by the agents, public or private, limits the set of implementable outcomes by tightening the best response conditions
- yet, persuasion, with or without, elicitation of private information still substantially influences equilibrium outcomes
- suggests a ranking of (many agents) information structures in terms of equilibrium outcomes
- by contrast, information expands the set of feasible outcomes (before best response conditions)
"Basic Game"

- players $i = 1, \ldots, I$
- (payoff) states $\theta \in \Theta$
- actions $(A_i)_i^{i=1}$
- utility functions $(u_i)_i^{i=1}$, each $u_i : A \times \Theta \rightarrow \mathbb{R}$
- state distribution $\psi \in \Delta(\Theta)$
- $G = \left((A_i, u_i)_i^{i=1}, \psi\right)$
- games with information removed
- "decision problem" in the one player case
Information Structure

- signals (types) \((T_i)_{i=1}^i\)
- signal distribution \(\pi : \Theta \rightarrow \Delta(T_1 \times T_2 \times ... \times T_i)\)
- \(S = ( (T_i)_{i=1}^i , \pi) \)
- "experiment" in the one player case
Expanding Information Structures

- $S^* = \left( (T_i^*)_{i=1}^i, \pi^* \right)$ is a combined information structure for $S^1 = \left( (T_i^1)_{i=1}^i, \pi^1 \right)$ and $S^2 = \left( (T_i^2)_{i=1}^i, \pi^2 \right)$ if $T_i^* = T_i^1 \cup T_i^2$ for each $i$,

\[
\sum_{t^2 \in T^2} \pi^* ((t^1, t^2) | \theta) = \pi^1 (t^1 | \theta)
\]

for each $t^1 \in T^1$ and $\theta \in \Theta$ and

\[
\sum_{t^1 \in T^1} \pi^* ((t^1, t^2) | \theta) = \pi^2 (t^2 | \theta)
\]

for each $t^2 \in T^2$ and $\theta \in \Theta$.

- there are many combined information structures for any pair of information structures as only marginals have to match

- if $S^*$ is a combination of $S$ and another information structure, say that $S^*$ is an expansion of $S$. 

Games with Incomplete Information

- a pair \((G, S)\) is a standard game of incomplete information
- a *decision rule* is a mapping \(\sigma : T \times \Theta \rightarrow \Delta (A)\)
  - decision rule as a recommendation
- an outcome is a mapping \(\nu : \Theta \rightarrow \Delta (A)\)
  - outcome describes relationship between states and actions independent of information
Obedience

**DEFINITION.** A decision rule $\sigma : T \times \Theta \rightarrow \Delta (A)$ is *obedient* for $(G, S)$ if

$$
\sum_{a_{-i}, t_{-i}, \theta} \psi (\theta) \pi ((t_i, t_{-i}), \theta) \sigma ((a_i, a_{-i}) | (t_i, t_{-i}), \theta) u_i ((a_i, a_{-i}), \theta) \\
\geq \\
\sum_{a_{-i}, t_{-i}, \theta} \psi (\theta) \pi ((t_i, t_{-i}), \theta) \sigma ((a_i, a_{-i}) | (t_i, t_{-i}), \theta) u_i ((a'_i, a_{-i}), \theta)
$$

for each $i, a_i \in A_i, t_i \in T_i$ and $a'_i \in A_i$. 
**DEFINITION.** A decision rule $\sigma : T \times \Theta \rightarrow \Delta (A)$ is a Bayes correlated equilibrium if it is obedient.

- set of BCE describes everything what could happen under any expansion of information structure $T$ as a Bayes Nash equilibrium
- properties that hold across all BCE are robust predictions
an information designer has utility function $w : A \times \Theta \rightarrow \mathbb{R}$.

his ex ante utility from decision rule $\sigma$ is

$$W(\sigma) = \sum_{a,t,\theta} \psi(\theta) \pi(t|\theta) \sigma(a|t,\theta) w(a,\theta)$$

information designer to choose $\sigma$ to maximize $W$

argmax describes designer’s preferred information structure
Bank Run: one depositor and no initial information

- a bank depositor is deciding whether to run from the bank:
- binary state: $\theta \in \{B, G\}$, insolvent or solvent
- binary action: $a \in \{r, s\}$, run or stay
- payoffs

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$B$</th>
<th>$G$</th>
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</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>$-1$</td>
<td>$y$</td>
<td></td>
</tr>
</tbody>
</table>

with $0 < y < 1$

- prior probability of each state is $\frac{1}{2}$
- designer (regulator) seeks to minimize probability of bank run (independent of state)
Outcome Distribution

- outcome $\nu : \Theta \rightarrow \Delta(A)$ with no information:

<table>
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<td>$s$</td>
<td>0</td>
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- probability of run is 1
Bank Run: Common Prior Only

- Suppose we have the prior information only - the null information structure:

$$S^o = (T^o, \pi^o), \quad T^o = \{t^o\}$$

- Parameterized decision rule:

<table>
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<th>$\sigma(\theta)$</th>
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<td>$r$</td>
<td>$\sigma_B$</td>
<td>$\sigma_G$</td>
</tr>
<tr>
<td>$n$</td>
<td>$1 - \sigma_B$</td>
<td>$1 - \sigma_G$</td>
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- $\sigma_B = \sigma(r \mid B)$: probability of running if insolvent
- $\sigma_G = \sigma(r \mid G)$: probability of running if solvent
Bank Run: Obedience

- agent may not necessarily know state $\theta$ but makes choices according to $\sigma(\cdot)$
- if "advised" to run, run has to be a best response:
  \[ 0 \geq \sigma_G y - \sigma_B \iff \\
  \sigma_B \geq \sigma_G y \]
- if "advised" to stay, stay has to be a best response
  \[ (1 - \sigma_G) y - (1 - \sigma_B) \geq 0 \iff \\
  \sigma_B \geq (1 - y) + \sigma_G y \]
- here, stay provides binding constraint:
  \[ \sigma_B \geq (1 - y) + \sigma_G y \]
- never to run, $\sigma_B = 0, \sigma_G = 0$, cannot be a BCE
Bank Run: Equilibrium Set

- set of BCE described by outcome \((\sigma_B, \sigma_G) = (\nu_B, \nu_G)\)

- never to run, \(\sigma_B = 0, \sigma_G = 0\), is not be a BCE
Bank Run: Information Design

- BCE minimizing the probability of runs has:
  \[ \sigma_B = 1 - y, \quad \sigma_G = 0 \]

- noisy stress test \( S = \{s_B, s_G\} \) implements BNE via informative signals:
  \[
  \pi(s \mid \theta) \quad \begin{array}{ccc}
  B & G \\
  s_B & 1 - y & 0 \\
  s_G & y & 1 
  \end{array}
  \]

- the bank is said to be healthy if it is solvent (always) \textbf{and} if it is insolvent (sometimes)

- solvent and insolvent banks are bundled
suppose agent observes conditionally independent signal (type) of the state with accuracy:

\[ q > \frac{1}{2} \]

- \( S = (T, \pi) \) where \( T = \{b, g\} \):

<table>
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<th>( T ) / ( \mathcal{H} )</th>
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<th>( G )</th>
</tr>
</thead>
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<tr>
<td>( b )</td>
<td>( q )</td>
<td>( 1 - q )</td>
</tr>
<tr>
<td>( g )</td>
<td>( 1 - q )</td>
<td>( q )</td>
</tr>
</tbody>
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- strictly more information than null information \( q = \frac{1}{2} \)
Bank Run: Additional Obedience Constraints

- conditional probability of running now depends on the signal: \( t \in \{ b, g \} \)
- \((\sigma_B, \sigma_G)\) becomes \(((\sigma_b^B, \sigma_b^G), (\sigma_g^B, \sigma_g^G))\)
- conditional obedience constraints, say for \( t = g \):

  \[
  r : \quad 0 \geq q\sigma_g^G y - (1 - q) \sigma_B^g \\
  s : \quad q(1 - \sigma_g^G) y - (1 - q)(1 - \sigma_B^g) \geq 0
  \]

  or

  \[
  r : \quad \sigma_B^g \geq \frac{q}{1 - q} \sigma_G^g y \\
  s : \quad \sigma_B^g \geq 1 - \frac{q}{1 - q} y + \frac{q}{1 - q} \sigma_G^g y
  \]

- reversal of relevant constraint with precise private information
Bank Run: Equilibrium Set

- set of BCE described by outcome \((\nu_B, \nu_G)\)

\[
\sigma_B = 1, \sigma_G = 0, \text{ is complete information BCE}
\]
Incentive Compatibility Ordering

- Write $BCE(G, S)$ for the set of BCE (random) choices of $(G, S)$

**Definition**

Experiment $S$ is more *incentive constrained* than experiment $S'$ if, for all decision problems $G$,

$$BCE(G, S) \subseteq BCE(G, S').$$

- Note that "more incentive constrained" corresponds, intuitively, to having more information
Feasibility

Definition (Feasible Decision Rule)

A decision rule $\sigma$ is feasible for $(G, S)$ if it is induced by a decision rule $\sigma$ which is consistent (measurable) with $S$.

- write $F(G, S)$ for the set of feasible (random) choices of $(G, S)$

Definition (More Informative)

Experiment $S$ is more informative than experiment $S'$ if, for all decision problems $G$,

$$F(G, S) \supseteq F(G, S').$$
Back to the Example: Feasibility

- suppose we have the prior information only - the null information structure: $S_0 = (T_0, \pi), \ T_0 = \{t_0\}$
- feasible (random) choices $\nu(\theta)$ can be described by $(\sigma_B, \sigma_S)$:
Back to the Example: Feasibility

- suppose player observes conditionally independent private binary signal of the state with accuracy $q \geq \frac{1}{2}$:
- feasible (random) choices $\sigma (\theta)$ can be described by $(\sigma_B, \sigma_S)$:
Information Design with Private Information

- agent observes a type (signal) $t \in T$
- principal offers recommendation, a probability of running, as a function of reported type $t$ and state $\theta$:

$$\left(\sigma_B^t, \sigma_G^t\right)$$

- the constraints are: (i) each type has to truthfully report; (ii) each type has to be willing to follow the recommendation ("private persuasion")
- the truthtelling constraints are given by, say for type $g$:

$$q \left(1 - \sigma_G^g\right) y - (1-q) \left(1-\sigma_B^g\right) \geq q \left(1 - \sigma_G^b\right) y - (1-q) \left(1 - \sigma_B^b\right)$$

$$\iff q y \left(\sigma_G^b - \sigma_G^g\right) \geq (1-q) \left(\sigma_B^b - \sigma_B^g\right)$$

- the obedience constraints described earlier
- private persuasion (dark red), BCE (light red)
- truth-telling imposes constraint at the low and high end of running probabilities
The Limits of Private Persuasion

- equilibrium set shrinks with the precision of the private information $q$
Conclusion

- it is feasible and interesting to see what happens under many information structures at once.
- methodology generates new answers for classical economic questions
- In mechanism design we design the payoffs of the game, assuming the information structure is fixed
- In information design, we design the information received by the players, assuming the game is fixed.
agent observes a type (signal) \( t \in T \)

principal offers a recommendation, a probability of running, independent of the type \( t \), but dependent on the true state \( \theta \):

the constraints are: all types have to be willing to follow the recommendation

private persuasion restricts information design by incentive constraints

public persuasion restricts information design by uniform recommendation constraints
The regulator cannot stop the depositor withdrawing....
... but can choose what information is made available to prevent withdrawals

Best information structure:

tell the depositor that the state is bad exactly often enough so that he will stay if he doesn’t get the signal.....

<table>
<thead>
<tr>
<th>Outcome</th>
<th>$\theta_G$</th>
<th>$\theta_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay (intermediate signal)</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>Run (bad signal)</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
</tr>
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</table>

Think of the regulator as a mediator making an action recommendation to the depositor subject to an obedience constraint

Probability of run is $\frac{1}{3}$
1. Without loss of generality, can restrict attention to information structures where each player’s signal space is equal to his action space

- compare with the revelation principle of mechanism design:
  - without loss of generality, we can restrict attention to mechanisms where each player’s message space is equal to his type space
Bayesian Persuasion

- This is the leading example in Kamenica-Gentzkow 2011
- We are not exploiting "concavification" logic applicable in this case; will return to this later in lectures...
Bank Run: one depositor with initial information

- If the state is good, with probability $\frac{1}{2}$ the depositor will already have observed a signal $t_G$ saying that the state is good.
If the state is good, with probability $\frac{1}{2}$ the depositor will already have observed a signal $t_G$ saying that the state is good.

Outcome distribution with no additional information:

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<tbody>
<tr>
<td>Stay</td>
<td>$\frac{1}{6}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Run</td>
<td>0</td>
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<td>$\frac{2}{3}$</td>
</tr>
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</table>

- Probability of run is $\frac{5}{6}$
Optimal Information Design with one depositor with initial information

- Best information structure:
  - tell the depositor that the state is bad exactly often enough so that he will stay if he doesn’t get the signal.....

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<tr>
<th>Payoff</th>
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Optimal Information Design with one depositor with initial information

▶ Best information structure:
  ▶ tell the depositor that the state is bad exactly often enough so that he will stay if he doesn’t get the signal.....

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<tr>
<td>Run</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
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</table>

▶ Probability of run is $\frac{1}{2}$
Is initially more informed depositor good or bad?

- With no information design....
  - ...and no initial information, probability of run is 1
  - ...and initial information, probability of run is $\frac{5}{6}$
Is initially more informed depositor good or bad?

- With no information design....
  - ...and no initial information, probability of run is $\frac{1}{3}$
  - ...and initial information, probability of run is $\frac{5}{6}$

- With information design....
  - ...and no initial information, probability of a run is $\frac{1}{2}$
  - ...and initial information, probability of a run is $\frac{1}{2}$
Is initially more informed depositor good or bad?

- With no information design....
  - ...(in this example) more initial information is better for the regulator
Is initially more informed depositor good or bad?

- With no information design....
  - ...(in this example) more initial information is better for the regulator

- With information design....
  - ...more initial information is always bad for the regulator
Lessons

1. Without loss of generality, can restrict attention to information structures where each player’s signal space is equal to his action space

2. Prior information limits the scope for information design
Bank Runs: two depositors and no initial information (and strategic complements)

- A bank depositor would like to run from the bank if he assigns probability greater than $\frac{1}{2}$ to a bad state OR the other depositor running

<table>
<thead>
<tr>
<th>State $\theta_G$</th>
<th>Stay</th>
<th>Run</th>
</tr>
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<tbody>
<tr>
<td>Stay</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Run</td>
<td>0</td>
<td>0</td>
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</tbody>
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<th>State $\theta_B$</th>
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<tr>
<td>Stay</td>
<td>-1</td>
<td>-1</td>
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<td>Run</td>
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- Probability of the bad state is $\frac{2}{3}$
### Bank Runs: two depositors and no initial information

- **Outcome distribution with no information**

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- **Best information structure:**
  - **I** tell the depositors that the state is bad exactly often enough so that they will stay if they don’t get the signal.
  - With public signals optimal.
Bank Runs: two depositors and no initial information

- Outcome distribution with no information

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- Outcome distribution with no information

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- ...with public signals optimal
Bank Runs: two depositors, no initial information and strategic substitutes

- Previous example had strategic complements
- Strategic substitute example: a bank depositor would like to run from the bank if he assigns probability greater than $\frac{1}{2}$ to a bad state AND the other depositor staying

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</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Run</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>state $\theta_B$</th>
<th>Stay</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>$-1$</td>
<td>1</td>
</tr>
<tr>
<td>Run</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Bank Runs: two depositors, no initial information and strategic substitutes

- Previous example had strategic complements
- Strategic substitute example: a bank depositor would like to run from the bank if he assigns probability greater than $\frac{1}{2}$ to a bad state AND the other depositor staying

<table>
<thead>
<tr>
<th>state $\theta_G$</th>
<th>Stay</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Run</td>
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<td>0</td>
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<tbody>
<tr>
<td>Stay</td>
<td>−1</td>
<td>1</td>
</tr>
<tr>
<td>Run</td>
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</table>

- Probability of the bad state is $\frac{2}{3}$
Bank Runs: two depositors and no initial information

- Outcome distribution with no information: mixed strategy equilibrium
Bank Runs: two depositors and no initial information

- Outcome distribution with no information: mixed strategy equilibrium
- Best information structure:
  - tell the depositors that the state is bad exactly often enough so that they will stay if they don’t get the signal.....

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<tr>
<th>outcome $\theta_G$</th>
<th>Stay</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>$\frac{1}{3}$</td>
<td>0</td>
</tr>
<tr>
<td>Run</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>outcome $\theta_B$</th>
<th>Stay</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>$\frac{4}{9}$</td>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>Run</td>
<td>$\frac{1}{9}$</td>
<td>0</td>
</tr>
</tbody>
</table>
**Bank Runs: two depositors and no initial information**

- Outcome distribution with no information: mixed strategy equilibrium
- Best information structure:
  - tell the depositors that the state is bad exactly often enough so that they will stay if they don’t get the signal.....

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</tr>
<tr>
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</tr>
</tbody>
</table>

- ....with private signals optimal
Lessons

1. Without loss of generality, can restrict attention to information structures where each player’s signal space is equal to his action space
2. Prior information limits the scope for information design
3. Public signals optimal if strategic complementarities; private signals optimal if strategic substitutes
Bank Run: two depositors with initial information

have also analyzed elsewhere....
Bank Run with Imprecise Information 2