Dynamic Matching – Part 1

Leeat Yariv

Yale University

February 21, 2016

Two-sided One-to-one Matching Markets

- **Two classes of agents**: men and women, workers and firms, rabbis and congregations, patients and organ donors, etc.
- An agent from each class cares about whom they match with from the other class

Two-sided One-to-one Matching Markets

- **Two classes of agents**: men and women, workers and firms, rabbis and congregations, patients and organ donors, etc.
- An agent from each class cares about whom they match with from the other class
- **Today:**
  - Static matching for pedestrians
  - Why care about dynamic matching?
  - Some approaches to dynamic matching
- **Wednesday:** A particular framework for studying dynamic matching mechanisms

Static Matching on a Shoestring

Participants:

- A set of Squares S, with a typical firm $s \in S$
- A set of Rounds R, with a typical round $r \in R$
- Squares and rounds can stand for men and women, workers and firms, patients and organ donors, etc.

Static Matching on a Shoestring

Participants:

- A set of Squares S, with a typical firm $s \in S$
- A set of Rounds R, with a typical round $r \in R$
- Squares and rounds can stand for men and women, workers and firms, patients and organ donors, etc.

Preferences:

- Each square s has a complete ranking $\succ_s$ over $R \cup \{s\}$
- Each round r has a complete ranking $\succ_r$ over $S \cup \{r\}$
- **Today**: ignore indifferences (strict preferences)
- A round r is *acceptable* to square s if $r \succ_s s$, and *unacceptable* otherwise;

Static Matching on a Shoestring

Participants:

- A set of Squares S, with a typical firm $s \in S$
- A set of Rounds R, with a typical round $r \in R$
- Squares and rounds can stand for men and women, workers and firms, patients and organ donors, etc.

Preferences:

- Each square s has a complete ranking $\succ_s$ over $R \cup \{s\}$
- Each round r has a complete ranking $\succ_r$ over $S \cup \{r\}$
- **Today**: ignore indifferences (strict preferences)
- A round r is *acceptable* to square s if $r \succ_s s$, and *unacceptable* otherwise; Similarly for acceptable squares

Matchings and Stable Matchings

- A *matching* is a one-to-one mapping $\mu : S \cup R \longrightarrow S \cup R$ satisfying:
  - μ is of order 2: $\mu^2(x) = x$ for all x
  - $\mu(s) \in R \cup \{s\}$ for all $s \in S$
  - $\mu(r) \in S \cup \{r\}$ for all $r \in R$

Matchings and Stable Matchings

- A *matching* is a one-to-one mapping $\mu : S \cup R \longrightarrow S \cup R$ satisfying:
  - μ is of order 2: $\mu^2(x) = x$ for all x
  - $\mu(s) \in R \cup \{s\}$ for all $s \in S$
  - $\mu(r) \in S \cup \{r\}$ for all $r \in R$
- A matching μ is *stable* if:
  - It is *individually rational*: If $\mu(s) = r$, then $s \succ_r r$ and $r \succ_s s$
  - There are no *blocking pairs*, pairs (s', r') such that $r' \succ_{s'} \mu(s')$ and $s' \succ_{r'} \mu(r')$

Stable Matchings – Example 1

Suppose $S = \{s_1, s_2\}$, $R = \{r_1, r_2\}$, and

$$\begin{array}{llll} s_1: & r_1 \succ r_2 \succ s_1 & r_1: & s_2 \succ s_1 \succ r_1 \\ s_2: & r_2 \succ r_1 \succ s_2 & r_2: & s_1 \succ s_2 \succ r_2 \end{array}$$

Stable Matchings – Example 1

Suppose $S = \{s_1, s_2\}$, $R = \{r_1, r_2\}$, and

$$\begin{array}{llll} s_1: & r_1 \succ r_2 \succ s_1 & r_1: & s_2 \succ s_1 \succ r_1 \\ s_2: & r_2 \succ r_1 \succ s_2 & r_2: & s_1 \succ s_2 \succ r_2 \end{array}$$

- $\mu(s_1) = r_1, \mu(s_2) = r_2$ stable (squares are happy)

Stable Matchings – Example 1

Suppose $S = \{s_1, s_2\}$, $R = \{r_1, r_2\}$, and

$$\begin{array}{llcll} s_1: & r_1 \succ r_2 \succ s_1 & \qquad & r_1: & s_2 \succ s_1 \succ r_1 \\ s_2: & r_2 \succ r_1 \succ s_2 & & r_2: & s_1 \succ s_2 \succ r_2 \end{array}$$

- $\mu(s_1) = r_1, \mu(s_2) = r_2$ stable (squares are happy)
- $\mu(s_1) = r_2), \mu(s_2) = r_1$ stable (rounds are happy)

Stable Matchings – Example 2

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llll} s_1: & r_2 \succ r_1 \succ r_3 & \qquad r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & \qquad r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & \qquad r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

Stable Matchings – Example 2

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llcll} s_1: & r_2 \succ r_1 \succ r_3 & \quad & r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & & r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & & r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

$$\mu = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \end{pmatrix} \text{ stable}$$

Stable Matchings – Example 2

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llll} s_1: & r_2 \succ r_1 \succ r_3 & \qquad r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & \qquad r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & \qquad r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

$$\begin{aligned} \mu &= \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \end{pmatrix} \text{ stable} \\ \tilde{\mu} &= \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix} \text{ unstable} \end{aligned}$$

Stable Matchings – Example 2

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llcll} s_1: & r_2 \succ r_1 \succ r_3 & \qquad & r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & & r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & & r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

$$\mu = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \end{pmatrix} \text{ stable}$$

$$\tilde{\mu} = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix} \text{ unstable}$$

Stable Matchings – Example 2

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$
\begin{array}{llll}
s_1: & r_2 \succ r_1 \succ r_3 & \qquad r_1: & s_1 \succ s_3 \succ s_2 \\
s_2: & r_1 \succ r_2 \succ r_3 & \qquad r_2: & s_2 \succ s_1 \succ s_3 \\
s_3: & r_1 \succ r_2 \succ r_3 & \qquad r_3: & s_1 \succ s_3 \succ s_2
\end{array}
$$

$$
\begin{aligned}
\mu &= \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \end{pmatrix} \text{ stable} \\
\tilde{\mu} &= \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix} \text{ unstable}
\end{aligned}
$$

Stable Matchings – Example 2

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llll} s_1: & r_2 \succ r_1 \succ r_3 & \qquad r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & \qquad r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & \qquad r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

$$\mu = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \end{pmatrix} \text{ stable}$$

$$\tilde{\mu} = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix} \text{ unstable}$$

An Economist Goes to a Bar (Example 3)

An Economist Goes to a Bar (Example 3)

Becker (1992), "A Theory of Marriage"

- Society as a cocktail party with rational-minded daters searching for the most desirable viable partners

An Economist Goes to a Bar (Example 3)

Becker (1992), “A Theory of Marriage”

- Society as a cocktail party with rational-minded daters searching for the most desirable viable partners
- Each individual has an observable “pizzazz” parameter
- Everyone agrees that greater pizzazz is more desirable

An Economist Goes to a Bar (Example 3)

Becker (1992), "A Theory of Marriage"

- Society as a cocktail party with rational-minded daters searching for the most desirable viable partners
- Each individual has an observable "pizzazz" parameter
- Everyone agrees that greater pizzazz is more desirable
- **Unique stable matching:** *positive assortative matching*, men and women of similar desirability partner with one another.

Existence and Features of Stable Matchings

Theorem (Gale and Shapley, 1962)

There exists a stable matching in any one-to-one matching market.

Existence and Features of Stable Matchings

Theorem (Gale and Shapley, 1962)

There exists a stable matching in any one-to-one matching market.

Proof: By construction, using the Deferred Acceptance Algorithm.

The Deferred Acceptance Algorithm

- Each square proposes to highest round on her list

The Deferred Acceptance Algorithm

- Each square proposes to highest round on her list
- Rounds make a "tentative match" based on their preferred offer, and reject other offers, or all if none are acceptable

The Deferred Acceptance Algorithm

- Each square proposes to highest round on her list
- Rounds make a “tentative match” based on their preferred offer, and reject other offers, or all if none are acceptable
- Each rejected square removes round from her list, and makes a new offer

The Deferred Acceptance Algorithm

- Each square proposes to highest round on her list
- Rounds make a “tentative match” based on their preferred offer, and reject other offers, or all if none are acceptable
- Each rejected square removes round from her list, and makes a new offer
- Continue until no more rejections or offers, at which point implement tentative matches

The Deferred Acceptance Algorithm

- Each square proposes to highest round on her list
- Rounds make a "tentative match" based on their preferred offer, and reject other offers, or all if none are acceptable
- Each rejected square removes round from her list, and makes a new offer
- Continue until no more rejections or offers, at which point implement tentative matches

- This is the *square-proposing deferred acceptance algorithm*; Could analogously define the round-proposing version

The Deferred Acceptance Algorithm

Proof of the Gale-Shapley(1962) Theorem:

- Algorithm must end in a finite number of rounds

The Deferred Acceptance Algorithm

Proof of the Gale-Shapley(1962) Theorem:

- Algorithm must end in a finite number of rounds
- Suppose s, r are matched, but s prefers r'

The Deferred Acceptance Algorithm

Proof of the Gale-Shapley(1962) Theorem:

- Algorithm must end in a finite number of rounds
- Suppose s, r are matched, but s prefers r'
- At some point, s proposed to r' and was rejected

The Deferred Acceptance Algorithm

Proof of the Gale-Shapley(1962) Theorem:

- Algorithm must end in a finite number of rounds
- Suppose s, r are matched, but s prefers r'
- At some point, s proposed to r' and was rejected
- At that point, r' preferred his tentative match to s

The Deferred Acceptance Algorithm

Proof of the Gale-Shapley(1962) Theorem:

- Algorithm must end in a finite number of rounds
- Suppose s, r are matched, but s prefers r'
- At some point, s proposed to r' and was rejected
- At that point, r' preferred his tentative match to s
- As algorithm goes forward, r' can only do better

The Deferred Acceptance Algorithm

Proof of the Gale-Shapley(1962) Theorem:

- Algorithm must end in a finite number of rounds
- Suppose s, r are matched, but s prefers r'
- At some point, s proposed to r' and was rejected
- At that point, r' preferred his tentative match to s
- As algorithm goes forward, r' can only do better
- So r' prefers her final match to s

The Deferred Acceptance Algorithm

Proof of the Gale-Shapley(1962) Theorem:

- Algorithm must end in a finite number of rounds
- Suppose s, r are matched, but s prefers r'
- At some point, s proposed to r' and was rejected
- At that point, r' preferred his tentative match to s
- As algorithm goes forward, r' can only do better
- So r' prefers her final match to s
- Therefore, there are NO BLOCKING PAIRS

Features of Stable Matchings

- There is a square-optimal stable matching, which is the round-pessimal stable matching and vice versa

Features of Stable Matchings

- There is a square-optimal stable matching, which is the round-pessimal stable matching and vice versa
- The square-proposing stable matching implements the square-optimal stable matching

Features of Stable Matchings

- There is a square-optimal stable matching, which is the round-pessimal stable matching and vice versa
- The square-proposing stable matching implements the square-optimal stable matching
- In fact, the set of stable matchings has a lattice structure

Features of Stable Matchings

- There is a square-optimal stable matching, which is the round-pessimal stable matching and vice versa
- The square-proposing stable matching implements the square-optimal stable matching
- In fact, the set of stable matchings has a lattice structure
- Stable matchings are Pareto efficient (but not vice versa, see Example 2)

Stability in the Real World

In 1984, Al Roth discovered that the centralized clearinghouse used to match newly-minted doctors with residencies is the Gale-Shapley algorithm (discovered independently by doctors)!

Stability in the Real World

From Roth (2002), McKinney, Niederle, and Roth (2003), and Lee (2011)

	Still in use	No longer in use
Stable	**The NRMP:** over 40 specialty markets and submarkets for first year postgraduate positions, and 15 for second year positions **Specialty matching services:** over 30 subspecialty markets for advanced medical residencies and fellowships **British regional medical markets:** Edinburgh ($\geq$'69), Cardiff **Dental residencies:** 3 specialties **Other healthcare markets:** Osteopaths ($\geq$'94), Pharmacists, Clinical psychologists ($\geq$'99) **Canadian lawyers:** multiple regions	**Dental residencies:** Periodontists(<'97), Prosthodontists (<'00) **Canadian lawyers:** British Columbia(<'96)
Unstable	**British regional medical markets:** Cambridge, London Hospital	**British regional medical markets:** Birmingham, Edinburgh (<'67), Newcastle, Sheffield **Other healthcare markets:** Osteopaths (<'94)

"Folk Theorem" about the Success of Stable Mechanisms

- Suppose a centralized clearinghouse does not implement a stable matching

“Folk Theorem” about the Success of Stable Mechanisms

- Suppose a centralized clearinghouse does not implement a stable matching
- Then there would be some participants who would prefer to be unmatched, or form blocking pairs

“Folk Theorem” about the Success of Stable Mechanisms

- Suppose a centralized clearinghouse does not implement a stable matching
- Then there would be some participants who would prefer to be unmatched, or form blocking pairs
- They will try to undo the match (severing connections, poaching rounds from other squares)

"Folk Theorem" about the Success of Stable Mechanisms

- Suppose a centralized clearinghouse does not implement a stable matching
- Then there would be some participants who would prefer to be unmatched, or form blocking pairs
- They will try to undo the match (severing connections, poaching rounds from other squares)
- So ultimately, a stable matching would be implemented, but may entail some efficiency loss if there are any frictions

“Folk Theorem” about the Success of Stable Mechanisms

- Suppose a centralized clearinghouse does not implement a stable matching
- Then there would be some participants who would prefer to be unmatched, or form blocking pairs
- They will try to undo the match (severing connections, poaching rounds from other squares)
- So ultimately, a stable matching would be implemented, but may entail some efficiency loss if there are any frictions

A Note on Stable Matchings in the Lab

- **Decentralized Matching** (Echenique and Yariv, 2015):
  - Stable matchings are often implemented
  - The *median* stable matching has a strong draw
  - *Cardinal* presentation of preferences matters a lot

A Note on Stable Matchings in the Lab

- **Decentralized Matching** (Echenique and Yariv, 2015):
  - Stable matchings are often implemented
  - The *median* stable matching has a strong draw
  - *Cardinal* presentation of preferences matters a lot
- **Centralized Matching a-la DA** (Echenique, Wilson, and Yariv, 2015):
  - Stable matchings are implemented around 50% of the time
  - Many markets culminate in the *receiver-optimal* stable matching
  - Market features matter (cardinal presentation, number of stable matchings)

A Note on Mechanism Design in Static Settings

- No stable mechanism is incentive compatible

A Note on Mechanism Design in Static Settings

- No stable mechanism is incentive compatible
- Unless harsh restrictions on preferences (say, if there is always a unique stable matching)

A Note on Mechanism Design in Static Settings

- No stable mechanism is incentive compatible
- Unless harsh restrictions on preferences (say, if there is always a unique stable matching)
- When there are frictions (say, incomplete information) all hell breaks loose

A Note on Mechanism Design in Static Settings

- No stable mechanism is incentive compatible
- Unless harsh restrictions on preferences (say, if there is always a unique stable matching)
- When there are frictions (say, incomplete information) all hell breaks loose
- Stable matchings are not necessarily utilitarian efficient. For instance:

	r_1	r_2
s_1	**5**	4
s_2	3	**1**

- $\mu(s_i) = r_i$ unique stable matching, but $\mu(s_i) = r_{3-i}$ is efficient

A Note on Mechanism Design in Static Settings

- No stable mechanism is incentive compatible
- Unless harsh restrictions on preferences (say, if there is always a unique stable matching)
- When there are frictions (say, incomplete information) all hell breaks loose
- Stable matchings are not necessarily utilitarian efficient. For instance:

	r_1	r_2
s_1	**5**	4
s_2	3	**1**

- $\mu(s_i) = r_i$ unique stable matching, but $\mu(s_i) = r_{3-i}$ is efficient
- (but asymptotically, not necessarily an issue, Lee and Yariv, 2015)

A Note on Transfers

- Can define stability with transfers

A Note on Transfers

- Can define stability with transfers
- I consider here markets without transfers:
  - Transfers are "repugnant" (Roth, 2007) and illegal in many markets: cannot buy babies or organs in the U.S.
  - Even in labor markets, tailored wages are often not implemented:
    - Salaries for new medical residents exhibit low variance across the market and across years, and are uniform across specialties within a hospital
    - Hall and Krueger (2012): A large fraction of jobs in the U.S. have posted wages

Why Think about Dynamic Matching?

- Recall the "folk theorem" – If we do not implement a stable matching, the market will ultimately converge to a stable matching, possibly inducing inefficiencies
  - When would such a decentralized, and plausibly dynamic, process yield stable outcomes?

Why Think about Dynamic Matching?

- Recall the "folk theorem" – If we do not implement a stable matching, the market will ultimately converge to a stable matching, possibly inducing inefficiencies
  - When would such a decentralized, and plausibly dynamic, process yield stable outcomes?
- Many matching processes are inherently dynamic:
  - Fixed set of participants, but dynamic interactions: many labor markets
  - Changing sets of participants over time

Changing Sets of Participants over Time

- **Child Adoption:**
  - About 1.6 million, or 2.5%, of children in the U.S. are adopted
  - One adoption facilitator – 11 new potential parents and 13 newly relinquished children each month

Changing Sets of Participants over Time

- **Child Adoption:**
  - About 1.6 million, or 2.5%, of children in the U.S. are adopted
  - One adoption facilitator – 11 new potential parents and 13 newly relinquished children each month
- **Labor Markets:**
  - U.S. Bureau of Labor Statistics – approximately 5 million new job openings and slightly fewer than 5 million newly unemployed workers each month this year

Changing Sets of Participants over Time

- **Child Adoption:**
  - About 1.6 million, or 2.5%, of children in the U.S. are adopted
  - One adoption facilitator – 11 new potential parents and 13 newly relinquished children each month
- **Labor Markets:**
  - U.S. Bureau of Labor Statistics – approximately 5 million new job openings and slightly fewer than 5 million newly unemployed workers each month this year
- **Organ Donation:**
  - Organ Donation and Transplantation Statistics – a new patient added to the kidney transplant list every 14 minutes and about 3000 patients added each month
  - In 2013, about a third of ˜17,000 kidney transplants involved living donors

Modeling Dynamic Matching

- Fixed participants, dynamic interactions
- Dynamic participation and interactions

Dynamic Interactions with Fixed Participants

- Definition of *dynamic stability* – Doval (2015)
- Some references: Blum, Roth, and Rothblum (1997), Haeringer and Wooders (2009), Diamantoudi, Miyagawa, and Xue (2007), Alcade (1996), Alcalde, Pérez-Castrillo, and Romero-Medina (1998), Alcalde and Romero-Medina (2000), **Niederle and Yariv (2009)**

Dynamic Market Game

- $t = 0$: squares and rounds enter the market
- $t = 1, 2, \ldots$: two stages as follows

  Stage 1: Squares simultaneously decide whether and to whom to make an offer. Unmatched square can have at most one offer out

  Stage 2: Each round r who has received an offer from s can accept, reject, or hold the offer
- Once an offer is accepted, round r is matched to square s irreversibly

Payoffs

- Square s matched to round r at time $t \longrightarrow$ payoffs $\delta^t u_{sr}^S$ and $\delta^t u_{sr}^R$, where $\delta \leq 1$ is the *market discount factor*. Unmatched agents receive 0

Payoffs

- Square s matched to round r at time $t \longrightarrow$ payoffs $\delta^t u_{sr}^S$ and $\delta^t u_{sr}^R$, where $\delta \leq 1$ is the *market discount factor*. Unmatched agents receive 0
- To ease getting stable matching: focus on high δ

Market Monitoring

- Squares and rounds observe receival, rejection, and deferral only of own offers. When an offer is accepted, the whole market is informed of the match. Similarly, when there is market exit

Market Monitoring

- Squares and rounds observe receival, rejection, and deferral only of own offers. When an offer is accepted, the whole market is informed of the match. Similarly, when there is market exit

- **Equilibrium notion:** Nash equilibrium (Bayesian Nash Equilibrium if we introduce uncertainty about preferences)

Stable Matchings as Equilibrium Outcomes

Proposition: *When the underlying market has a unique stable matching, there exists a Nash equilibrium in strategies that are not weakly dominated that generates the unique stable matching.*

Stable Matchings as Equilibrium Outcomes

Proposition: *When the underlying market has a unique stable matching, there exists a Nash equilibrium in strategies that are not weakly dominated that generates the unique stable matching.*

Intuition:

Stable Matchings as Equilibrium Outcomes

Proposition: *When the underlying market has a unique stable matching, there exists a Nash equilibrium in strategies that are not weakly dominated that generates the unique stable matching.*

Intuition:

- $t = 1$: each square s makes offer to $\mu(s)$

Stable Matchings as Equilibrium Outcomes

Proposition: *When the underlying market has a unique stable matching, there exists a Nash equilibrium in strategies that are not weakly dominated that generates the unique stable matching.*

Intuition:

- $t = 1$: each square s makes offer to $\mu(s)$
- $t = 1$: each round r accepts square $\mu(r)$ or more preferred, or exits if no offers

Stable Matchings as Equilibrium Outcomes

Proposition: *When the underlying market has a unique stable matching, there exists a Nash equilibrium in strategies that are not weakly dominated that generates the unique stable matching.*

Intuition:

- $t = 1$: each square s makes offer to $\mu(s)$
- $t = 1$: each round r accepts square $\mu(r)$ or more preferred, or exits if no offers

But there can be other (unstable) equilibrium outcomes...

Example 2 (Continued)

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llll} s_1: & r_2 \succ r_1 \succ r_3 & r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

Example 2 (Continued)

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llll} s_1: & r_2 \succ r_1 \succ r_3 & \quad r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & \quad r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & \quad r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

$$\mu = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \end{pmatrix}, \tilde{\mu} = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix}$$

Example 2 (Continued)

Suppose $S = \{s_1, s_2, s_3\}$, $R = \{r_1, r_2, r_3\}$, all acceptable, and

$$\begin{array}{llcll} s_1: & r_2 \succ r_1 \succ r_3 & \qquad & r_1: & s_1 \succ s_3 \succ s_2 \\ s_2: & r_1 \succ r_2 \succ r_3 & & r_2: & s_2 \succ s_1 \succ s_3 \\ s_3: & r_1 \succ r_2 \succ r_3 & & r_3: & s_1 \succ s_3 \succ s_2 \end{array}$$

$$\mu = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & r_2 & r_3 \end{pmatrix}, \tilde{\mu} = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix}$$

μ unique stable matching, can implement $\tilde{\mu}$

Multiplicity (Continued)

In "sub-market" without (s_3, r_3), multiple stable matchings (Example 1...):

$$\begin{array}{ll} s_1: & r_2 \succ r_1 \\ s_2: & r_1 \succ r_2 \end{array}, \qquad \begin{array}{ll} r_1: & s_1 \succ s_2 \\ r_2: & s_2 \succ s_1 \end{array}$$

$$\mu = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & s_2 & r_3 \end{pmatrix}, \quad \tilde{\mu} = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix}$$

Multiplicity (Continued)

In "sub-market" without (s_3, r_3), multiple stable matchings (Example 1...):

$$\begin{array}{lccc} s_1: & r_2 & \succ & r_1 \\ s_2: & r_1 & \succ & r_2 \end{array}, \qquad \begin{array}{lccc} r_1: & s_1 & \succ & s_2 \\ r_2: & s_2 & \succ & s_1 \end{array}$$

$$\mu = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_1 & s_2 & r_3 \end{pmatrix}, \quad \tilde{\mu} = \begin{pmatrix} s_1 & s_2 & s_3 \\ r_2 & r_1 & r_3 \end{pmatrix}$$

Stage 1 : s_3 and r_3 match, Stage 2: follow $\tilde{\mu}$

$\tilde{\mu}$ induces the square-preferred stable matching in stage 2

Multiplicity (Continued)

- With enough “alignment” of preferences across squares and rounds, no such examples
- In general, dynamic mechanisms with irreversible matchings at certain periods are prone to these issues

Frictions Make Things Even Worse (Hand-waving)

- Suppose squares and rounds know their own utilities from matching with others, but have incomplete information about everyone else's preferences

Frictions Make Things Even Worse (Hand-waving)

- Suppose squares and rounds know their own utilities from matching with others, but have incomplete information about everyone else's preferences
- Incomplete information is likely in large markets (but not much is known, even in the static world)

Frictions Make Things Even Worse (Hand-waving)

- Suppose squares and rounds know their own utilities from matching with others, but have incomplete information about everyone else's preferences
- Incomplete information is likely in large markets (but not much is known, even in the static world)
- To make things easy, suppose we know that in each realization of preferences/utilities, there is a unique stable matching, can we implement it?

Frictions Make Things Even Worse (Hand-waving)

- Suppose squares and rounds know their own utilities from matching with others, but have incomplete information about everyone else's preferences
- Incomplete information is likely in large markets (but not much is known, even in the static world)
- To make things easy, suppose we know that in each realization of preferences/utilities, there is a unique stable matching, can we implement it?
- **First guess:** Yes, squares and rounds can simply emulate the deferred acceptance strategies

Simply Use Gale-Shapley?

Suppose there is some discounting (frictions)

Simply Use Gale-Shapley?

Suppose there is some discounting (frictions)

Example: complete information

$$U_1 = \begin{array}{|c|c|} \hline \mathbf{4} & 1 \\ \hline 3 & \mathbf{2} \\ \hline \end{array}$$

Simply Use Gale-Shapley?

Suppose there is some discounting (frictions)

Example: complete information

$$U_1 = \begin{array}{|c|c|}\hline \mathbf{4} & 1 \\ \hline 3 & \mathbf{2} \\ \hline \end{array}$$

- s_2 knows r_2 will accept an offer immediately
- s_2 will not make an offer to r_1

Simply Use Gale-Shapley?

Suppose there is some discounting (frictions)

Example: complete information

$$U_1 = \begin{array}{|c|c|} \hline \mathbf{4} & 1 \\ \hline 3 & \mathbf{2} \\ \hline \end{array}$$

- s_2 knows r_2 will accept an offer immediately
- s_2 will not make an offer to r_1

Simply Use Gale-Shapley?

- Generally not an equilibrium
- Agents can deduce that some on their preference list are inconceivable partners – will prefer to pass over

Simply Use Gale-Shapley?

- Generally not an equilibrium
- Agents can deduce that some on their preference list are inconceivable partners – will prefer to pass over
- Can we just modify deferred-acceptance strategies to skip over inconceivable partners?

Simply Use Gale-Shapley?

- Generally not an equilibrium
- Agents can deduce that some on their preference list are inconceivable partners – will prefer to pass over
- Can we just modify deferred-acceptance strategies to skip over inconceivable partners?
- Yes, but still tricky in equilibrium when there is incomplete information (timing of actions is informative and manipulable...)

Simply Use Gale-Shapley?

- Generally not an equilibrium
- Agents can deduce that some on their preference list are inconceivable partners – will prefer to pass over
- Can we just modify deferred-acceptance strategies to skip over inconceivable partners?
- Yes, but still tricky in equilibrium when there is incomplete information (timing of actions is informative and manipulable...)
- Getting a stable matching implemented in equilibrium requires harsh restrictions on preferences

Search Frictions and Matching

- **Original references**: Burdett and Coles (1997), Eeckhout (1999), Shimer and Smith (2000)

Search Frictions and Matching

- **Original references**: Burdett and Coles (1997), Eeckhout (1999), Shimer and Smith (2000)
- Distribution of squares and rounds in each period stationary
- Squares and rounds have "pizzazz" parameters on their foreheads (Example 3)

Search Frictions and Matching

- **Original references**: Burdett and Coles (1997), Eeckhout (1999), Shimer and Smith (2000)
- Distribution of squares and rounds in each period stationary
- Squares and rounds have "pizzazz" parameters on their foreheads (Example 3)
- Squares and rounds are paired randomly in each period and decide whether to accept one another and leave, or continue searching, with common discount factor

Search Frictions and Matching

- **Original references**: Burdett and Coles (1997), Eeckhout (1999), Shimer and Smith (2000)
- Distribution of squares and rounds in each period stationary
- Squares and rounds have "pizzazz" parameters on their foreheads (Example 3)
- Squares and rounds are paired randomly in each period and decide whether to accept one another and leave, or continue searching, with common discount factor
- Each participant solves an *option value problem*

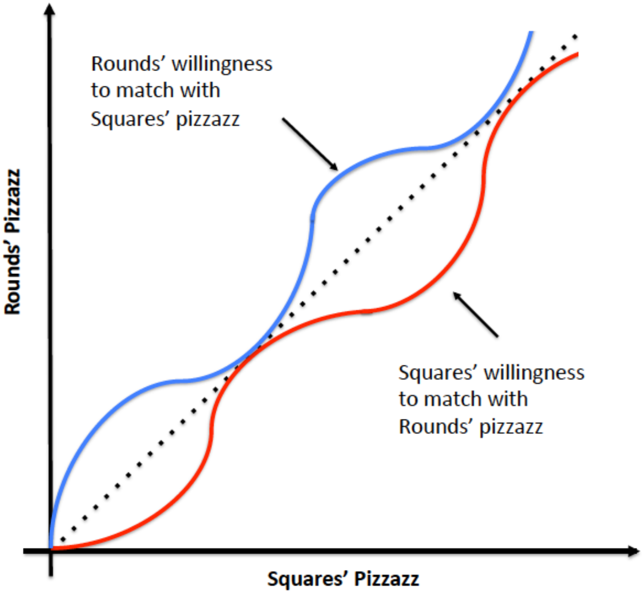
Search Frictions and Matching

- **Original references**: Burdett and Coles (1997), Eeckhout (1999), Shimer and Smith (2000)
- Distribution of squares and rounds in each period stationary
- Squares and rounds have “pizzazz” parameters on their foreheads (Example 3)
- Squares and rounds are paired randomly in each period and decide whether to accept one another and leave, or continue searching, with common discount factor
- Each participant solves an *option value problem*
- Recall Becker’s prediction of *assortative matching*

Search Frictions and Matching

- Each square of pizzazz P_s is willing to match immediately with rounds of pizzazz of at least $P_r^*(s)$
- Each round of pizzazz P_r is willing to match immediately with squares of pizzazz of at least $P_s^*(r)$

Search Frictions and Matching



Search Frictions and Matching

- In general, "almost" stable/assortative matchings
- Achieve assortative matching as discount factor $\to 0$

THE END