Dirk Bergemann
Department of Economics
Yale University

Economics 521b: Advanced Microeconomic Theory
Problem Set 3: Incentive Compatibility and the Value of Information
2/20/13

This problem set is due 3/6/13.

1. (Roberts 1979) establishes that a deterministic allocation rule is implementable (in dominant strategies) if and only if it is a generalized utility maximizer. (Bergemann and Välimäki 2010) derive dynamic pivot mechanism in a dynamic environents and show that is period ex post incentive compatible. Extend the affine maximizer to dynamic environments as in (Bergemann and Välimäki 2010). (Hint: It suffices to consider the two period environment. In addition, the necessary part is likely to be difficult, so do only the sufficient part. The notes by (Mishra 2009) might be useful. In particular, it appears easy to establish the result with time varying weights. The interesting aspect is whether it can be done with time invarying weights. (This is an open problem and a complete argeument, necessary and sufficient part, would clearly be publishable.)

2. Consider the following two experiments

$$S: \begin{array}{ccccc} & & t_1 & t_2 & t_3 \\ & \theta_1 & \frac{1}{2} & \frac{1}{2} & 0 \\ & \theta_2 & 0 & \frac{1}{2} & \frac{1}{2} \end{array} \qquad S': \begin{array}{ccc} & t'_1 & t'_2 \\ \theta_1 & \alpha & 1-\alpha \\ \theta_2 & 1-\alpha & \alpha \end{array}$$

   (a) For what values of α is S sufficient for S'?

   (b) Suppose now that we replicate each experiment, so that the state space is $t_1t_1, t_1t_2, \ldots$. For what values of α is $(S)^2$ sufficient for $(S')^2$?

3. Theorem 12.2.2 in (Blackwell and Girshick 1954) states five equivalent conditions for the more informative ranking of Blackwell. Give a formal and verbal (and possibly graphic representation) of these different conditions. Try to be as precise as possible.

4. Consider the following multi-unit pricing problem. The valuation of the consumer for two good are uniformly, independently, and identically distributed on $[a, a+1]$, and hence the joint distribuion is on $[a, a+1] \times [a, a+1]$ with $a \geq 0$.

   (a) Consider pure item pricing, i.e. the seller only offers a price for each good separately. What is the optimal pure item pricing price (as a function of a)? For what value of a does the seller cease to exclude buyers. What are the profits of the seller?

(b) Consider pure bundling pricing, i.e. the seller only offers a single price at which the buyers acquire both items. For what value of a does the seller cease to exclude buyers. What are the profits of the seller? Do the profits of the seller with pure bundling ever exceed the profits of the pure item pricing.

(c) Consider finally mixed bundling pricing, where there are price for single items, $p_1 = p_2$, and prices for the bundle p_b. For what value of a does the seller cease to exclude buyers. What are the profits of the seller?

References

Bergemann, D., and J. Välimäki (2010): "The Dynamic Pivot Mechanism," *Econometrica*, 78, 771–790.

Blackwell, D., and M. Girshick (1954): *Theory of Games and Statistical Decisions*. John Wiley.

Mishra, D. (2009): "Monotonicity and Incentive Compatibility," .

Roberts, K. (1979): "The Characterization of Implementable Choice Rules," in *Aggregation and Revelation of Preferences*, ed. by J. Laffont. North-Holland, New York.