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Economics 121b: Intermediate Microeconomics
Final Examination
5/1/12

This is a closed book exam, use paper and pen(cil) only. Please write legibly and document your arguments and calculations (useful for partial credit). The exam time is 180 minutes (and an additional 30 minutes review time). Good luck and enjoy your summer!

1. **(30 min)** Define and explain in a brief paragraph (about 80–90 words) the following two concepts, with an accompanying equation or diagram where appropriate, its significance in microeconomic theory:

    1. *moral hazard* and *adverse selection;*
    2. constrained optimization and first order conditions.

2. **(45 min)** Consider two firms, 1 and 2, who produce in a duopoly market. Firm 1 chooses its quantity of output x_1 and firm 2 chooses x_2. Given these outputs, the market price is given by

$$p = 1 - (x_1 + x_2).$$

There are zero costs of production.

1. Write the profit functions for firms 1 and 2. Find the optimal quantity of output x_2 for firm 2, as a function of x_1 (i.e. find firm 2's best response function).
2. Define the notion of Nash equilibrium in pure strategies for this game (i.e. which conditions in terms of the choices x_i and the profit function have to be satisfied.
3. Now find the Nash equilibrium outputs for firm 1 and firm 2 and the attendant profits for each firm.
4. Find the symmetric level of output $(x_1, x_2) = (x, x)$ that would maximize the sum of firm 1 and firm 2's profits if they each produce x . How does this compare to your answer from question [2.3], and why? In addition, explain clearly why producing x does not constitute a best response for either firm.
5. Now return to the setting of question [2.3]. In the Nash equilibrium, firm 2 chooses an output that is a best response to firm 1's output. Suppose that firm 2 is concerned that it might be wrong about how much output 1 is going to produce, and so hires an industrial spy who can (reliably) report firm 1's output to firm 2 before 2 makes a decision. Moreover, firm 1 knows that firm 2 has hired such a spy. As a result, firm 1 knows that whatever value of x_1 it produces, firm 2 will produce a best response. Hence, from firm 1's point of view, $x_2$is no longer to be viewed as fixed when 1 maximizes 1's profits, but rather x_2 is effectively a function of x_1, with this function given by firm 2's best response function from question [2.1].
   1. To analyze this situation, start with the profit function for firm 1 from question [2.1], replace x_2 in this profit function by firm 2's best response function, so that 1's profits are now entirely a function of x_1. Explicitly state the new profit function of firm 1.
   2. Now take the derivative of the profit function of firm 1 with respect to x_1 and solve to find the (new) equilibrium quantity x_1. Insert this optimal quantity into firm 2's best response function to find firm 2's equilibrium quantity.
   3. How do these compare to the quantities you found in question [2.3]? Explain the differences in your answers. Which firm benefits from firm 2's spy and which one loses?

3. **(45 min)** Farmer Bill and Farmer Jane grow wheat on their respective farms this year (period 1) and next year (period 2). Each farm generates an endowment of one unit wheat in each period. Both farmers like to consume wheat, but they like wheat this year better than wheat next year. The utility function for each farmer is:

$$u^B\left(x_1^B, x_2^B\right) = \ln x_1^B + \delta_B \ln x_2^B,$$

and

$$u^J\left(x_1^J, x_2^J\right) = \ln x_1^J + \delta_J \ln x_2^J,$$

where x_1^B is the consumption of wheat this year and x_2^B is the consumption of wheat next year, by Bill, and similarly for Jane. The parameter δ_B (and δ_J) is the discount factor, which diminishes the value of consumption next year relative to this year and we assume that $0 < \delta_B, \delta_J < 1$. Farmer Bill and Jane can trade wheat in period 1 against wheat in period 2 and the price of wheat in period 1 is normalized to $p_1 = 1$. Thus each farmer i faces a single (intertemporal) budget constraint:

$$p_1 x_1^i + p_2 x_2^i = p_1 e_1^i + p_2 e_2^i,$$

and given the information about endowments and price p_1, we can simplify it to

$$1 \cdot x_1^i + p_2 x_2^i = 1 \cdot 1 + p_2 \cdot 1.$$

(You can simply think about consumption this year and consumption next year as being two different goods in a single period, and the budget constraint linking the consumption across the two years.)

1. Describe graphically the consumption problem for Farmer Bill with his endowment and prices $p_1 = 1$ and arbitrary p_2 (in a single agent diagram).
2. Determine graphically how the solution to Farmer Bill's consumption problem changes with changes in his discount factor δ_B and the price p_2. Briefly describe the intuition behind the comparative static results (in a single agent diagram).
3. Determine the optimal demand for wheat by Farmer Bill analytically as a function of his discount factor and the price p_2. For which values of δ_B and p_2 does Farmer Bill wish to consume identical amounts of wheat in period 1 and period 2.
4. Graphically describe the trading environment for Farmer Bill and Jane in the Edgeworth box, identify the endowment point and label all axis and other objects you wish to introduce.
5. Suppose now that the discount factor for Bill and Jane are identical, or $\delta_B = \delta_J = \delta \in (0, 1)$. Find the competitive equilibrium and relate it to the endowment of the farmers and the discount factor.
6. Suppose now that the discount factor of Bill is lower than the discount factor of Jane, or $\delta_B < \delta_F$. What does this imply for the preferences of Bill in terms of consumption this year and next year relative to Jane's preferences? Intuitively, how would you think that the competitive equilibrium in this new environment changes to the earlier environment?
7. For $\delta_B < \delta_F$ derive the competitive equilibrium. What can you say about the trade pattern relative to the endowment and about the price of the second good, i.e. consumption of wheat tomorrow.

4. **(45 min)** A monopolist with a linear cost of production:

$$c(q) = c \cdot q, \quad c > 0,$$

faces a single consumer with a utility function:

$$u(q,t) = 2\theta\sqrt{q} - t,$$

where $q \in \mathbb{R}_+$ is the quantity consumed, $t \in \mathbb{R}_+$ is monetary transfer that the consumer pays to the monopolist for the entire purchase q, and $\theta \in \mathbb{R}_+$ is the preference intensity of the consumer. The profit function of the monopolist is:

$$\pi(t,q) = t - cq.$$

The consumer purchases q at t if and only if receives nonnegative utility:

$$u(q,t) = 2\theta\sqrt{q} - t \geq 0. \tag{0.1}$$

Thus the profit function and the utility function are linear in t.

1. For a given and known value of θ, describe the maximization program that the determines the Pareto efficient solution, i.e. the welfare maximizing solution $q^*(\theta)$ and then determine $q^*(\theta)$ explicitly.
2. For a given and known value of θ, describe the profit maximizing problem for the monopolist and then explicitly determine the bundle $\left(\overline{q}(\theta), \overline{t}(\theta)\right)$ that maximizes the profit of the monopolist given that the consumer is willing to participate, i.e. that (0.1) holds.
3. How does the solution of $\overline{q}(\theta)$ and $\overline{t}(\theta)$ change with the preference intensity θ? What is the per unit price paid, i.e. what is

$$\frac{\overline{t}(\theta)}{\overline{q}(\theta)}$$

   and how does it change with θ?
4. Now suppose that the monopolist cannot price the entire purchase q in terms of a single transfer t, but rather has to offer a *per unit price* p at which the consumer can buy any quantity s/he likes, so that the eventual transfer is $t = p \cdot q$, where $q \in \mathbb{R}_+$ is the choice variable of the consumer.
   1. Compute the optimal demand of the consumer for a given price p, i.e. find the demand function $q(\theta, p)$, which depends on the per unit price and his preference θ.
   2. Given $q(\theta, p)$ find the optimal price $p(\theta)$ to be charged by the monopolist.
   3. How does $p(\theta)$ vary with θ?
   4. How does $q(\theta, p(\theta))$ and $p(\theta)$ compare to the solution computed above in (4.2). Explain the reason for the divergence and describe the welfare properties of the revenue maximizing unit price policy by the monopolist.