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ECONOMICS 121B: INTERMEDIATE MICROECONOMICS
Problem Set 8:
Game Theory
4/9/12

This problem set is due on Monday, 4/16/12, in class. To receive full credit, provide a complete defense of your answer.

1. **Dominated and Iteratively Dominated Strategies.** Consider the oligopoly model we discussed in class with $I = 2$ competitors and linear demand and cost functions:

$$p(q) = a - bq, \quad c_i(q_i) = c \cdot q_i$$

and the aggregate supply is:

$$q = q_1 + q_2.$$

In class, we defined a dominant and a dominated strategy. Now we try to apply the notion of domination repeatedly and iteratively.

(a) Thus suppose that each firm i is initally considering a quantity:

$$q_i \in \mathbb{R}_+,$$

and now suppose that each firm is eliminating all strategies, supply choices, that are strictly dominated by some other choices, and call the remaining set of undominated strategies

$$U_i^1 \subset \mathbb{R}_+.$$

Graphically describe the remaining set U_1^i of action/strategies for firm $i = 1, 2$.

(b) We can then refine and iterate the analysis by asking which strategies are dominated for firm i if firm j is know to only choose actions from U_j^1, and call the remaining strategies U_i^2. Graphically describe the remaining set of action/strategies for firm $i = 1, 2$. What do you observe?

(c) If we iterate the analysis for every k, then we can ask what is limit set of strategies that survives the iterative process of eliminating dominated strategies. Can you desribe

$$\lim_{k \to \infty} U_i^k.$$

2. **Mixed Strategy Nash Equilibrium.** Find the unique, mixed strategy equilibrium, of the matching pennies game:

		Bob	
		Head	Tail
Ann	Head	$1, -1$	$-1, 1$
	Tail	$-1, 1$	$1, -1$

   (a) First draw the best response function of Ann and Bob in a two-dimensional graph.

   (b) Then solve for the mixed strategy equilibrium algebraically.

3. **Mixed Strategy Nash Equilibrium.** Find the unique, mixed strategy equilibrium, of the Rock Paper Scissor game:

			Bob	
		R	P	S
	R	$0, 0$	$-1, 1$	$1, -1$
Ann	P	$1, -1$	$0, 0$	$-1, 1$
	S	$-1, 1$	$1, -1$	$0, 0$

   (a) First show that there cannot be an equilibrium strategy which only involves one or two strategies for any one player. Conclude that an equilibrium strategy must be completely mixed.

   (b) Then solve for the mixed strategy equilibrium algebraically.

4. **Pure and Mixed Strategy Nash Equilibrium.** Find all, pure and mixed strategy equilibria of the "Hawk-Dove" game:

	Defend	Attack
Defend	$3, 3$	$1, 4$
Attack	$4, 1$	$0, 0$

   (a) First draw the best response function of the row and the column player in a two-dimensional graph.

   (b) Then identify the pure and mixed strategy equilibria algebraically (guided by the geometric representation).

5. Two owners $i = 1, 2$ of a stand on the New Haven farmers' market sell apples. The effort that they put into marketing the apples is e_i. They can choose any effort between 0 and 1. The revenue that they make is an increasing function of both owners' effort: $R(e_1, e_2) = 2(e_1 + e_2)$. Each owner receives one half of this revenue. For each owner i the cost of effort e_i are $C_i(e_i) = \frac{1}{2}(e_i)^2$. Thus, owner i's net utility is:

$$u_i(e_1, e_2) = (e_1 + e_2) - \frac{1}{2}(e_i)^2.$$

(a) For each owner i write down the first order condition for the optimal choice of e_i given the other owner's choice e_j. Show that the second derivative of utility with respect to e_i is negative.

(b) Solve for the symmetric Nash equilibrium of the game. Denote the common equilibrium effort level by e^*. Substitute $e_1 = e_2 = e^*$ into the first order condition and solve for e^*.

(c) By contrast, suppose the two owners were to enter into a cooperative agreement and were to seek to maximize the sum of their net utility, i.e. they were to maximize

$$\max_{e_1, e_2} \left\{ 2\left(e_1 + e_2\right) - \frac{1}{2}e_1^2 - \frac{1}{2}e_2^2 \right\}.$$

Find the optimal solution of this problem, denote it by $e^{**} = (e_1^{**}, e_2^{**})$. How does it compare to $e^* = (e_1^*, e_2^*)$.

(d) The comparison above is an instance of the "tragedy of the commons". Briefly explain why.

Reading Assignment: NS Chapter 14, 15