

Auction Theory

An Introduction into Mechanism Design

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- game theory: take the rules as given, analyze outcomes
- mechanism design: what kind of rules should be employed
- abstract approach:
 - specify some objective
 - design institution to achieve objective when agents act in their own interest
- mechanism design: (reverse) "Engineering" economics

- one of the richest parts of microeconomic theory with important practical implications
- auctions for rights to natural resources (timber, oil, natural gas), government procurement, electricity markets, eBay: \$52 billion worth of goods traded in 2006
- US Treasury holds auctions for \$4 trillion in securities annually

Several Types of Auctions

- first price sealed bid auction
- second price sealed bid auction
- English (open ascending) auction
- Dutch (open descending) auction

First Price Sealed Bid Auction

- there is one asset to be auctioned.
- individuals submit sealed bids
- the asset goes to the highest bidder, who pays his own bid (actions=bids)
- payoffs, if highest bid:

$$v_i - b_i,$$

0 otherwise

- very popular format, but with one important defect:
- individuals have incentives to underreport their valuations

Second Price Sealed Bid Auction

- Vickrey's brilliant idea: there is one object to be auctioned.
- individuals simultaneously submit bids
- the asset goes to the highest bidder, who pays the second highest bid
- Theorem: Truth-telling is a dominant strategy under the second-price sealed bid auction.
- In 1996, William Vickrey was awarded the Nobel prize for this invention.

Second Price Sealed Bid or Vickrey Auction

- when a Vickrey auction is used, all bidders bid their true value the bidder with the highest value is sure to win, the auction allocates the object efficiently
- English auction: price is continuously raised and last bidder standing is the winner at the final price
- turns out to be strategically equivalent to a Vickrey auction
- Dutch auction: start with high price and decrease until someone accepts the seller's price) strategically equivalent to a first-price sealed bid auction

Internet Advertising (Google Search)

- n bidders, $m < n$ ordered slots on webpage
- each slot has click-through rate, α_s where $\alpha_1 > \alpha_2 > \dots > \alpha_m$
- each bidder i has private value v_i per click. If bidder i receives slot s at price p , her utility is:

$$u_i = \alpha_s v_i - p$$

Generalized First Price Auction

- in 1997, Overture (now part of Yahoo!) introduced the following Generalized First Price (GFP) auction for selling Internet advertising.
- each agent simultaneously bids for a slot
- the highest bidder receives the first slot at a price of her bid times the click-through rate of slot 1,
- the second highest bidder receives the second slot at a price of her bid times the click-through rate of slot 2, and so on
- auction format was adopted by major search engines including Yahoo! and MSN

Generalized Second Price Auction

- in February 2002, Google introduced its own pay-per-click system, AdWords Select, using the following Generalized Second Price auction.
- each agent simultaneously bids for a slot.
- the highest bidder receives the first slot at a price of the second highest bid times the click-through rate of slot 1,
- the second highest bidder receives the second slot at a price of the third highest bid times the click-through rate of slot 2, and so on.
- once Google introduced this new format, Yahoo!/Overture also switched to GSP.

- method of microeconomics: mathematical abstraction
- two common types of mathematical problems:
 - ① Constrained Optimization
 - ② Equilibrium: Mapping many individual constrained optimal choices into common outcome
- mappings from mathematics to economics
 - ① First and Second Order Conditions, Optimality and Diminishing Rates of Substitution
 - ② Prices as Lagrange Multipliers

- ① Single Optimizer Decisions: Consumer Theory, Choice under Uncertainty, Competitive Producer Theory, Monopoly Theory
- ② Purely Competitive Markets: General Equilibrium Analysis; Welfare Theorem
- ③ Fully Strategic Markets: Cournot, Bertrand Competition;

How to Embarrass Me ... and You

- what I would be embarrassed if you did not know when you take future economics courses and what you would be embarrassed not to know in the final exam!
- solve constrained (utility or profit) maximization problems in economic context, know how to use the calculus methods
- know how to use mathematical and economic reasoning to solve corner solutions, special cases.
- understand implications of rational behavior and how to derive them.
- understand the idea of general equilibrium both algebraically and graphically. Understand the key idea of why markets generate efficiency, as expressed in the welfare theorems.

Many Ways to Embarrass Me...and You

- understand the basics of game theory (Nash equilibrium in simultaneous move games, mixed strategies and how to use them in understanding imperfect competition.
- understand how uncertainty (with symmetric information) can be fully incorporated into existing theory.
- understand how asymmetric information interferes with the operation of markets: hidden action, hidden information.
- basic idea of moral hazard: insurance vs. incentives
- other market mechanism: auctions

- same format as two midterms, but with more time
- cumulative exam, with weight towards the last third of the course
 - 1 short answer question
 - 4 problem solving questions