Econ 121b: Intermediate Microeconomics

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Week of 4/15 - 4/21

1 Lecture 21: Second Degree Price Discrimination

In Section **??** we considered first and third degree price discrimination where the seller can identify the type of potential buyers. In contrast, second degree price discrimination occurs when the firm cannot observe to consumer's willingness to pay directly. Consequently they elicit these preferences by offering different quantities or qualities at different prices. The consumer's type is revealed through which option they choose. This is known as screening.

Suppose there are two types of consumers. One with high valuation of the good θ_h, and one with low valuation θ_l. θ is also called the buyers' marginal willingness to pay. It tells us how much a buyer what be willing to pay for an additional unit of the good. Each buyer's type is his private information. That means the seller does not know ex ante what type a buyer he is facing is. Let α denote the fraction of consumers who have the high valuation. Suppose that the firm can produce a product of quality q at cost $c(q)$ and assume that $c'(q) > 0$ and $c''(q) > 0$.

First, we consider the efficient or first best solution, i.e., the case where the firm can observe the buyers' types. If the firm knew the type of each consumer they could offer a different quality to each consumer. The condition for a consumer of type $i = h, l$ buying an object of quality q for price p voluntarily is

$$\theta_i q - p(q) \geq 0$$

and for the firm to participate in the trade we need

$$p(q) - c(q) \geq 0.$$

Hence maximizing joint payoff is equivalent to

$$\max_q \theta_i q - p(q) + p(q) - c(q)$$

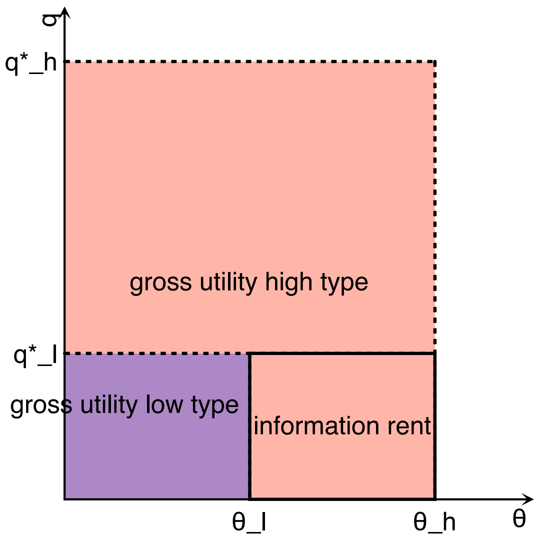

Figure 1: Price discrimination when types are known to the firm

or

$$\max_q \theta_i q - c(q).$$

The FOC for each quality level is

$$\theta_i - c'(q) = 0,$$

from which we can calculate the optimal level of quality for each type, $q^*(\theta_i)$. Since marginal cost is increasing by assumption we get that

$$q^*(\theta_l) < q^*(\theta_h),$$

i.e., the firm offers a higher quality to buyers who have a higher willingness to pay in the first best case. In the case of complete information we are back to first degree price discrimination and the firm sets the following prices to extract the entire gross utility from both types of buyers:

$$p_h^* = \theta_h q^*(\theta_h) \quad \text{and} \quad p_h^* = \theta_l q^*(\theta_l)$$

so that buyers' net utility is zero. In Figure 1, the buyers' gross utility, which is equal to the price charged, is indicated by the rectangles $\theta_i q_i^*$.

In many situations, the firm will not be able to observe the valuation/willingness to pay of the consumers. That is, the buyers' type is their private information. In such a situation the firm offers a schedule of price-quality pairs and lets the consumers self-select into contracts. Thereby, the consumers reveal their type. Since there are two types of consumers the firm will offer two different quality levels, one for the high valuation consumers and one for the low valuation consumers. Hence there will be a choice of two contracts (p_h, q_h) and (p_l, q_l) (also called a menu of choices). The firm wants high valuation consumers to buy the first contract and low valuation consumers to buy the second contract. Does buyers' private information matter, i.e., do buyers just buy the first best contract intended for them? High type buyers get zero net utility from buying the high quality contract, but positive net utility of $\theta_h q^*(\theta_l) - p_l > 0$. Hence, high type consumers have an incentive to pose as low quality consumers and buy the contract intended for the low type. This is indicated in Figure 1 as "information rent," i.e., an increase in high type buyers' net utility due to asymmetric information.

The firm, not knowing the consumers' type, however, can make the low quality bundle less attractive to high type buyers by decreasing q_l or make the high quality contract more attractive by increasing q_h or decreasing p_h. The firm's profit maximization problem now becomes

$$\max_{p_h, p_l, q_h, q_l} \alpha \left(p_h - c(q_h)\right) + (1-\alpha)\left(p_l - c(q_l)\right). \tag{1}$$

There are two type of constraints. The consumers have the option of walking away, so the firm cannot demand payment higher than the value of the object. That is, we must have

$$\begin{aligned} \theta_h q_h - p_h &\geq 0 & (2)\\ \theta_l q_l - p_l &\geq 0. & (3) \end{aligned}$$

These are known as the individual rational (IR) or participation constraints that guarantee that the consumers are willing to participate in the trade. The other type of constraints are the self-selection or incentive compatibility (IC) constraints

$$\begin{aligned} \theta_h q_h - p_h &\geq \theta_h q_l - p_l & (4)\\ \theta_l q_l - p_l &\geq \theta_l q_h - p_h, & (5) \end{aligned}$$

which state that each consumer type prefers the menu choice intended for him to the other contract. Not all of these four constraints can be binding, because that would determine the optimal solution of prices and quality levels. The IC for low type (5) will not binding because low types have no incentive to pretend to be high types: they would pay a high price for quality they do not value highly. On the other hand high type consumers' IR (2) will not be binding either because we argued above that the firm has to incentivize them to pick the high quality contract. This leaves constraints (3) and (4) as binding and we can solve for the optimal prices

$$p_l = \theta_l q_l$$

using constraint (3) and

$$p_h = \theta_h(q_h - q_l) + \theta_l q_l$$

using constraints (3) and (4). Substituting the prices into the profit function (1) yields

$$\max_{q_h, q_l} \alpha \left[\theta_h(q_h - q_l) + \theta_l q_l - c(q_h)\right] + (1-\alpha)\left(\theta_l q_l - c(q_l)\right).$$

The FOC for q_h is simply

$$\alpha\left(\theta_h - c'(q_h)\right) = 0,$$

which is identical to the FOC in the first best case. Hence, the firm offers the high type buyers their first best quality level $q_R^*(\theta_h) = q^*(\theta_h)$. The FOC for q_l is

$$\alpha(\theta_l - \theta_h) + (1-\alpha)\left(\theta_l - c'(q_l)\right) = 0,$$

which can be rewritten as

$$\theta_l - c'(q_l) - \frac{\alpha}{1-\alpha}(\theta_l - \theta_h) = 0.$$

The third term on the LHS, which is positive, is an additional cost that arises because the firm has to make the low quality contract less attractive for high type buyers. Because of this additional cost we get that $q_R^*(\theta_l) < q^*(\theta_l)$: the the quality level for low types is lower than in the first best situation. This is depicted in Figure . The low type consumers' gross utility and the high type buyers' information rent are decreased, but The optimal level of quality offered to low type buyers is decreasing in the fraction of high type consumer α:

$$\frac{dq_R^*(\theta_l)}{d\alpha} < 0$$

since the more high types there are the more the firm has to make the low quality contract unattractive to them.

This analysis indicates some important results about second degree price discrimination:

1. The low type receives no surplus.

2. The high type receives a positive surplus of $q_l(\theta_h - \theta_l)$. This is known as an information rent, that the consumer can extract because the seller does not know his type.

3. The firm should set the efficient quality for the high valuation type.

4. The firm will degrade the quality for the low type in order to lower the rents the high type consumers can extract.

2 Lecture 22: Auctions

Auctions are an important application of games of incomplete information. There are many markets where goods are allocated by auctions. Besides obvious examples such as auctions of antique furniture there are many recent application. A leading example is Google's sponsored search auctions. Google matches advertiser to readers of websites and auctions advertising space according to complicated rules.

Consider a standard auction with I bidders, and each bidder i from 1 to I has a valuation v_i for a single object which is sold by the seller or auctioneer. If the bidder wins the object at price p_i then he receives utility $v_i - p_i$. Losing bidders receive a payoff or zero. The valuation is often the bidder's private information so that we have to analyze the uncertainty inherent in such auctions. This uncertainty is captured by modelling the bidders' valuations as draws from a random distribution:

$$v_i \sim F(v_i).$$

We assume that bidders are symmetric, i.e., their valuations come from the same distribution, and we let b_i denote the bid of player i.

There are many possible rules for auctions. They can be either sealed bid or open bid. Examples of sealed bid auctions are the first price auction (where the winner is the bidder with the highest bid and they pay their bid), and the second price auction (where the bidder with the highest bid wins the object and pays the second highest bid as a price). Open bid auctions include English auctions (the auctioneer sets a low price and keeps increasing the price until all but one player has dropped out) and the Dutch auction (a high price is set and the price is gradually lowered until someone accepts the offered price). Another type of auction is the Japanese button auction, which resembles an open bid ascending auction, but every time the price is raised all bidders have to signal their willingness to increase their bid. Sometimes, bidders hold down a button as long as they want to increase their bid and release when they want to exit the auction.

Let's think about the optimal bidding strategy in a Japanese button auction, denotes by $b_i(v_i) = t_i$, where $t_i = p_i$ is the price the winning bidder pays for the good. At any time, the distribution of valuations, F, the number of remaining bidders are known to all players. As long as the price has not reached a bidder's valuation it is optimal for him to keep the button pressed because he gets a positive payoff if all other players exit before the price reaches his valuation. In particular, the bidder with the highest valuation will wait longest and therefore receive the good. He will only have to pay the second highest bidder's valuation, however, because he should release the button as soon as he is the only one left. At that time the price will have exactly reached the second highest valuation. Hence, it is optimal for all bidders to bid their true valuation. If the price exceeds v_i they release the button and get 0 and the highest valuation bidder gets a positive payoff. In other words, the optimal strategy is

$$b_i^*(v_i) = v_i.$$

What if the button auction is played as a descending auction instead? Then it is no longer optimal to bid one's own valuation. Instead, $b_i^*(v_i) < v_i$ because only waiting until the price reaches one's own valuation would mean that there might be a missed chance to get a strictly positive payoff.

In many situations (specifically when the other players' valuations does not affect your valuation) the optimal behavior in a second price auction is equivalent to an English auction, and the optimal behaviour in a first price auction is equivalent to a Dutch auction. This provides a motivation for considering the second price auction which is strategically very simple, since the English auction is commonly used. It's the mechanism used in the auction houses, and is a good first approximation how auctions are run on eBay.

How should people bid in a second price auction? Typically a given bidder will not know the bids/valuations of the other bidders. A nice feature of the second price auction is that the optimal strategy is very simple and does not depend on this information: each bidder should bid their true valuation.

Proposition 1. *In a second price auction it is a Nash Equilibrium for all players to bid their valuations. That is $b_i^* = v_i$ for all i is a Nash Equilibrium.*

Proof. Without loss of generality, we can assume that player 1 has the highest valuation. That is, we can assume $v_1 = \max_i\{v_i\}$. Similarly, we can assume without loss of generality that the second highest valuation is $v_2 = \max_{i>1}\{v_i\}$. Define

$$\mu_i(v_i, b_i, b_{-i}) = \left\{ \begin{array}{c} v_i - p_i, \text{ if } b_1 = \max_j\{b_j\} \\ 0, \text{ otherwise} \end{array} \right\}$$

to be the surplus generated from the auction for each player i. Then under the given strategies ($b = v$)

$$\mu_i(v_i, v_i, v_{-i}) = \left\{ \begin{array}{c} v_1 - v_2, \ i = 1 \\ 0, \text{ otherwise} \end{array} \right\}$$

So we want to show that no bidder has an incentive to deviate.

First we consider player 1. The payoff from bidding b_1 is

$$\mu_1(v_1, b_1, v_{-1}) = \left\{ \begin{array}{c} v_1 - v_2, \text{ if } b_1 > v_2 \\ 0, \text{ otherwise} \end{array} \right\} \leq v_1 - v_2 = \mu_1(v_1, v_1, v_{-1})$$

so player 1 cannot benefit from deviating.

Now consider any other player $i > 1$. They win the object only if they bid more than v_1 and would pay v_1. So the payoff from bidding b_1 is

$$\mu_i(v_i, b_i, v_{-i}) = \left\{ \begin{array}{c} v_i - v_1, \text{ if } b_i > v_1 \\ 0, \text{ otherwise} \end{array} \right\} \leq 0 = \mu_i(v_i, v_i, v_{-i})$$

since $v_i - v_1 \leq 0$. So player i has no incentive to deviate either.

We have thus verified that all players are choosing a best response, and so the strategies are a Nash Equilibrium. □

Note that this allocation is efficient. The bidder with the highest valuation gets the good.

Finally, we consider a first price sealed bid auction. There, we will see that it is optimal for bidders to bid below their valuation, $b_i^*(v_i) < v_i$, a strategy called bid shedding. Bidder i's expected payoff is

$$\max_{b_i} (v_i - b_i) \Pr(b_i > b_j \text{ for all } j \neq i) + 0 \Pr(b_i < \max\{b_j\} \text{ for all } j \neq i). \quad (6)$$

Consider the bidding strategy

$$b_i(v_i) = cv_i$$

i.e., bidders bid a fraction of their true valuation. Then, if all players play this strategy,

$$\Pr(b_i > b_j) = \Pr(b_i > cv_j) = \Pr\left(v_j < \frac{b_i}{c}\right). \quad (7)$$

With valuations having a uniform distribution on $[0, 1]$, (7) becomes

$$\Pr\left(v_j < \frac{b_i}{c}\right) = \frac{b_i}{c}$$

and (6) becomes

$$\max_{b_i} (v_i - b_i) \frac{b_i}{c} + 0$$

with FOC

$$\frac{v_i - 2b_i}{c} = 0$$

or

$$b_i^* = \frac{v_i}{2}.$$

Hence, we have verified that the optimal strategy is to bid a fraction of one's valuation, in particular, $c = \frac{1}{2}$.