Econ 121b: Intermediate Microeconomics

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Week of 4/1 - 4/7

1 Lecture 17: Oligopoly

Now, we consider the case with I competitors. The inverse demand function (setting $a = b = 1$) is

$$p(q) = 1 - \sum_{i=1}^{I} q_i$$

and firm i's profit function is

$$\pi(q_i, q_{-i}) = \left(1 - \sum_{i=1}^{I} q_i - c\right) q_i, \tag{1}$$

where the vector q_{-i} is defined as $q_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_I)$, i.e., all quantities excluding q_i.

Again, we can define an equilibrium in this market as follows:

Definition 1. A *Nash Equilibrium in the oligopoly game* is a vector $q^* = (q_1^*, \ldots, q_I^*)$ such that for all i

$$\pi_i(q_i^*, q_{-i}^*) \geq \pi_i(q_i, q_{-i}^*) \text{ for all } q_i.$$

We simply replaced the quantity q_j by the vector q_{-i}.

Definition 2. A Nash equilibrium is called *symmetric* if $q_i^* = q_j^*$ for all i and j.

The FOC for maximizing the profit function (1) is

$$1 - \sum_{j \neq i} q_j - 2q_i - c = 0$$

and the best response function for all i is

$$q_i = \frac{1 - \sum_{j \neq i} q_j - c}{2}. \tag{2}$$

Here, only the aggregate supply of firm i's competitors matters, but not the specific amount single firms supply. It would be difficult to solve for I separate values of q_i, but due to symmetry of the profit function we get that $q_i^* = q_j^*$ for all i and j so that equation (2) simplifies to

$$q_i^* = \frac{1-(I-1)q_i^* - c}{2},$$

which leads to the solution

$$q_i^* = \frac{1-c}{I+1}.$$

As I increases (more firms), the market becomes more competitive. Market supply is equal to

$$\sum_{i=1}^{I} q_i^* = I q_i^* = \frac{I}{I+1}(1-c).$$

As the number of firms becomes larger, $I \longrightarrow \infty$, $q_i^* \longrightarrow 0$ and

$$\sum_{i=1}^{I} q_i^* \longrightarrow 1-c,$$

which is the supply in a competitive market. Consequently,

$$p^* \longrightarrow c$$

As each player plays a less important strategical role in the market, the oligopoly outcome converges to the competitive market outcome.

Note that we used symmetry in deriving the market outcome from the firms' best response function. We cannot invoke symmetry when deriving the FOC. One might think that instead of writing the profit function as (1) one could simplify it to

$$\pi(q_i, q_{-i}) = (1 - Iq_i - c)\, q_i.$$

This is wrong, however, because it implies that firm i controls the entire market supply (acts as a monopolist). Instead, in an oligopoly market, firm i takes the other firms' output as given.

2 Lecture 18: Game Theory

2.1 Basics

Game theory is the study of behavior of individuals in a strategic scenario, where a strategic scenario is defined as one where the actions of one individual affects the payoff or utility of other individuals. In the previous section we introduced game theory in the context of firm competition. In this section, we will generalize the methods used above and introduce some specific language. The specification of (static) game consists of three elements:

1. The players, indexed by $i = 1, \ldots, I$. In the duopoly games, for example, the players were the two firms.

2. The strategies available: each player chooses strategy a_i from the available strategy set A_i. We can write $a_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_I)$ to represent the strategies of the other $I-1$ players. Then, a strategy profile of all players is defined by $a = (a_1, \ldots, a_I) = (a_i, a_{-i})$. In the Cournot game, the player's strategies were the quantities chose, hence $A_i = \mathbb{R}_+$.

3. The payoffs for each player as a function of the strategies of the players. We use game theory to analyze situations where there is strategic interaction so the payoff function will typically depend on the strategies of other players as well. We write the payoff function for player i as $u_i(a_i, a_{-i})$. The payoff function is the mapping

$$u_i : A_1 \times \cdots \times A_I \longrightarrow \mathbb{R}.$$

Therefore we can define a game the following way:

Definition 3. A game (in normal form) is a triple,

$$\Gamma = \Big\{ \{1, 2, \ldots, I\}, \{A_i\}_{i=1}^{I}, \{u_i(\cdot)\}_{i=1}^{I} \Big\}$$

We now define the concept of best response, i.e. the action for a player i which is best for him (in the sense of maximizing the payoff function). But since we are studying a strategic scenario, what is best for player i potentially depends on what others are playing, or what player i believes others might be playing.

Definition 4. An action a_i is a **best response** for player i against a profile of actions of others a_{-i} if

$$u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \quad \forall\, a_i' \in A_i$$

We say that,

$$a_i \in BR_i(a_{-i})$$

Now we define the concept of Nash Equilibrium for a general game.

Definition 5. An action profile

$$a^* = (a_1^*, a_2^*, \dots, a_I^*)$$

is a **Nash equilibrium** if,

$$\text{for all } i, \quad u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \quad \forall\, a_i \in A_i$$

or, stated otherwise,

$$\text{for all } i, \quad a_i^* \in BR_i(a_{-i}^*)$$

We know that

$$BR_i \colon \times_{j \neq i} A_j \to A_i$$

Now let's define the following function

$$BR \colon \times_{i=1}^{I} A_i \to \times_{i=1}^{I} A_i$$

as

$$BR = (BR_1, BR_2, \dots, BR_I)$$

Then we can redefine Nash equilibrium as follows:

Definition 6. An action profile

$$a^* = (a_1^*, a_2^*, \dots, a_I^*)$$

is a **Nash equilibrium** if,

$$a^* \in BR(a^*)$$

2.2 Pure Strategies

We can represent games (at least those with a finite choice set) in normal form. A normal form game consists of the matrix of payoffs for each player from each possible strategy. If there are two players, 1 and 2, then the normal form game consists of a matrix where the (i, j)th entry consists of the tuple (player 1's payoff, player2's payoff) when player 1 plays their ith strategy and player 2 plays their jth strategy. We will now consider the most famous examples of games.

Example 1. (Prisoner's Dilemma) Suppose two suspects, Bob and Rob are arrested for a crime and questioned separately. The police can prove the committed a minor crime, and suspect they have committed a more serious crime but can't prove it. The police offer each suspect that they will let them off for the minor crime if they confess and testify against their partner for the more serious crime. Of course, if the other criminal also confesses the police won't need his testimony but will give him a slightly reduced sentence for cooperating. Each player then has two possible strategies: Stay Quiet (Q) or Confess (C) and they decide simultaneously. We can represent the game with the following payoff matrix:

		Rob	
		Q	C
Bob	Q	3, 3	−1, 4
	C	4, −1	0, 0

Each entry represents (Bob, Rob)'s payoff from each of the two strategies. For example, if Rob stays quiet while Bob confesses Bob's payoff is 4 and Rob's is -1. Notice that both players have what is known as a dominant strategy; they should confess regardless of what the other player has done. If we consider Bob, if Rob is Quiet then confessing gives payoff $4 > 3$, the payoff from staying quiet. If Rob confesses, then Bob should confess since $0 > -1$. The analysis is the same for Rob. So the only stable outcome is for both players to confess. So the only Nash Equilibrium is (Confess, Confess). Notice that, from the perspective of the prisoners this is a bad outcome. In fact it is Pareto dominated by both players staying quiet, which is not a Nash equilibrium.

The above example has a dominant strategy equilibrium, where both players have a unique dominant strategy.

Definition 7. A strategy is a_i is **dominant** if

$$u_i(a_i, a_{-i}) > u_i(a_i', a_{-i}) \text{ for all } a_i' \in A_i, a_{-i} \in A_{-i}.$$

If each player has a dominant strategy, then the only rational thing for them to do is to play that strategy no matter what the other players do. Hence, if a dominant strategy equilibrium exists it is a relatively uncontroversial prediction of what will happen in the game. However, it is rare that a dominant strategy will exist in most strategic situations. Consequently, the most commonly used solution concept is Nash Equilibrium, which does not require dominant strategies.

Note the difference between Definitions **??** and 7: A Nash Equilibrium is only defined for the best response of the other players, s_{-i}^*, whereas dominant strategies have to hold for strategies $s_{-i} \in S_{-i}$. A strategy profile is a Nash Equilibrium if each player is playing a best response to the other players' strategies. So a Nash Equilibrium is a stable outcome where no player could profitably deviate. Clearly when dominant strategies exist it is a Nash Equilibrium for all players to play a dominant strategy. However, as we see from the Prisoner's Dilemma example the outcome is not necessarily efficient. The next example shows that the Nash Equilibrium may not be unique.

Example 2. (Coordination Game) We could represent a coordination game where Bob and Ann are two researcher both of whose input is necessary for a project. They decide simultaneously whether to do research (R) or not (N).

		Bob	
		R	N
Ann	R	3, 3	−1, 0
	N	0, −1	1, 1

Here (R,R) and (N,N) are both equilibria. Notice that the equilibria in this game are Pareto ranked with both players preferring to coordinate on doing research. Both players not doing research is also an equilibrium, since if both players think the other will play N they will play N as well.

A famous example of a coordination game is from traffic control. It doesn't really matter if everyone drives on the left or right, as long as everyone drives on the same side.

Example 3. Another example of a game is a "beauty contest." Everyone in the class picks a number on the interval $[1, 100]$. The goal is to guess as close as possible to $\frac{2}{3}$ the class average. An equilibrium of this game is for everyone to guess 1. This is in fact the only equilibrium. Since no one can guess more than 100, $\frac{2}{3}$ of the mean cannot be higher than $66\frac{2}{3}$, so all guesses above this are dominated. But since no one will guess more than $66\frac{2}{3}$ the mean cannot be higher than $\frac{2}{3}(66\frac{2}{3}) = 44\frac{4}{9}$, so no one should guess higher than $44\frac{4}{9}$. Repeating this n times no one should guess higher than $\left(\frac{2}{3}\right)^n 100$ and taking $n \longrightarrow \infty$ all players should guess 1. Of course, this isn't necessarily what will happen in practice if people solve the game incorrectly or expect others too. Running this experiment in class the average guess was approximately 12.