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ECONOMICS 121B: INTERMEDIATE MICROECONOMICS
Problem Set 6:
Monopoly
3/21/12

This problem set is due on Monday, 4/2/12, in class. To receive full credit, provide a complete defense of your answer.

1. Consider a perfectly competitive firm that hires quantities x_1 and x_2 of inputs 1 and 2, at prices p_1 and p_2 , and sells the resulting output in a perfectly competitive market at price p . The production function is given by $f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$.
 - (a) Formulate the cost minimization problem for this firm and find the associated first-order conditions for any given output y .
 - (b) Find the input demand functions for x_1 and x_2 for the cost-minimizing quantity of output y .
 - (c) Show that the production function $f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$ displays constant returns to scale.
 - (d) Given the constant returns to scale technology, argue that there must be a specific link between the input prices p_1, p_2 and the output price p , or else all the firms would either not produce at all or strictly prefer to increase ever larger quantities. Discuss how this link is related to the constant returns to scale assumption.
2. First, we examine a monopolistic firm. The firm faces a market described by the demand function $p = A - By$, where p is the price the firm receives if it sells quantity y of output. The firm's cost function is given by $C(y) = \frac{1}{2}y^2$.
 - (a) Find the profit-maximizing quantity of output and the corresponding profit for this firm.
 - (b) Show that if the firm could sell more output at the (constant) equilibrium price, it would do so. (To do this, take a derivative of the firm's payoff under the assumption that price is constant rather than a function of output, and then evaluate this derivative at the equilibrium price and quantity from part 2a.)
 - (c) Let p^* be the equilibrium price and y^* the equilibrium quantity, so that $p^* = A - By^*$. Now suppose the demand in the market shifts, so that the new demand curve is $p = A' - B'y$, with $A' > A$ and $B' > B$,

but with it still being the case that $p^* = A' - B'y^*$. Hence, if the firm does not change its quantity, its price will also not change. Is the quantity of output y^* still optimal, given this new demand curve? If not, will the firm decide to produce more or less? Formulate your first-order condition from part 2a in terms of the elasticity of demand, and use this to explain your result in this part.

3. Now suppose you must design a tax on the monopoly. Let the demand function be $p = A - By$.

- (a) First suppose you consider a sales tax of 10% on this market. Suppose that the firm has to pay the tax, as is typically the case with sales taxes. Hence, if the firm produces quantity y of output, the price paid by consumers is $p = A - By$, but the price received by the firm is $.9p = .9(A - By)$, where the .9 appears because the firm gets to keep only ninety percent of the purchase price, paying the remaining ten percent to the government in taxes. Find the profit maximizing quantity of output for the firm, the price paid by consumers, the price received by the firm, and the firm's profit. Explain how these answers compare to those of part 2a. In particular, do consumers pay more as a result of the tax? Does the price received by the firm fall? Does the firm's profit fall?
- (b) Instead of a sales tax, the government considers a profits tax. Hence, if the firm chooses quantity of output y and charges price $p = A - By$, the government collects $t[py - \frac{1}{2}y^2] = t[(A - By)y - \frac{1}{2}y^2]$ in tax revenue, leaving $(1 - t)[py - \frac{1}{2}y^2] = (1 - t)[(A - By)y - \frac{1}{2}y^2]$ as after-tax profit for the firm. Once again, find the firm's profit-maximizing quantity of output and the resulting price. What effect does the profits tax have on the quantity of output and price? What effect does it have on the firm's profit? In light of your answers, given that a fixed amount of revenue is to be raised, which tax would consumers prefer—a sales tax or profits tax? Why?
- (c) Now suppose the cost function is given by $\frac{1}{2}x^2 + F$, where F is a fixed cost. For example, F may be the cost of conducting environmental impact studies or acquiring the licenses needed to produce. What effect does F have on the firm's optimal quantity of output, price, and profits? Many communications firms are monopolies because the government gives them exclusive rights to use certain bands of air-wave lengths. Sometimes the government simply gives the firms this exclusive right, while other times it sells the right to the highest bidder. The latter method has been criticized on the grounds that it will drive up the firms' costs and hence the prices charged to consumers. What do you think of this argument?

Reading Assignment: NS Chapter 14, 15