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Economics 121b: Intermediate Microeconomics
Midterm
2/8/12

This is a closed book exam, use paper and pen(cil) only. Please write legibly and document your arguments and calculations (useful for partial credit). The exam time is 75 minutes. Good luck!

1. **(25 min)** Consider the utility function

$$u(x_1, x_2) = (x_1 + 2)(x_2 + 3), \quad x_1, x_2 \geq 0 \tag{1}$$

with the accompanying budget constraint:

$$p_1 x_1 + p_2 x_2 \leq I, \qquad p_1, p_2, I > 0. \tag{2}$$

(a) Fix a given utility level $U > 0$ and find an explicit expression for the indifference curve defined by the utility level $U > 0$. Then, derive an explicit expression for the marginal rate of substitution between good 1 and good 2.

(b) Draw the indifference curve (for this associated utility level U) and carefully label the graph and all its elements.

(c) Show that the utility function is strictly increasing (and hence monotone) in x_1 and x_2.

(d) Now formally state the utility maximization problem and briefly describe its content, in particular what constitutes the choice variables, and what constitutes the constraint variables (i.e. endogenous versus exogenous variables).

(e) Provide an argument why in the present utility maximization problem, we can restrict attention without loss of generality to the case where budget constraint holds as an equality at the optimal solution, that is we can restrict attention to the equality constraint:

$$p_1 x_1^* + p_2 x_2^* = I.$$

(The argument should not involve the explicit computation of the optimal choices.)

2. **(30 min).** For the utility maximization problem given above by (1) and (2), we are now computing the optimal demands.

   (a) Formally, and separately, state the first order condition by using the method of substitution and the method of Lagrange. Be sure to briefly describe the optimality conditions and their composition.

   (b) Graphically describe the optimality condition and carefully label the graph and all its elements.

   (c) Explicitly compute the Marshallian demand functions by one of the above two methods.

   (d) Define the notion of the own price elasticity and explicitly derive the own price elasticity of good 1 of the Marshallian demand function.

   (e) Find the indirect utility function and explicitly identify the marginal value of income.

3. **(20 min)** The compensated and the uncompensated demand, i.e. the Marshallian and Hicksian demand function, can be related through the Slutsky equation. The Slutsky equation can be derived from the equality

$$x_i^* \left(p, E\left(p, U\right)\right) = h_i^* \left(p, U\right),$$

   where p is the vector of prices $p = (p_1, ..., p_n)$.

   (a) State (or derive and state) the Slutsky equation and briefly explain all the terms appearing in the Slutsky equation.

   (b) Graphically illustrate the content of the Slutsky equation holds, carefully labelling all the objects that enter the graphic.

   (c) Briefly describe verbally two different ways in which the Slutsky equation can be used to analyze consumer behavior (as expressed in consumption data).