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ECONOMICS 121B: INTERMEDIATE MICROECONOMICS  
Make Up Midterm  
2/13/12

This is a closed book exam, use paper and pen(cil) only. Please write legibly and document your arguments and calculations (useful for partial credit). The exam time is 75 minutes. Good luck!

1. (**25 min**) Consider the expenditure function

$$p_1x_1 + p_2x_2 \tag{1}$$

with the associated utility function:

$$u(x_1, x_2) = (x_1 + 1)(x_2), \quad x_1, x_2 \geq 0. \tag{2}$$

- (a) Fix a given utility level  $U > 0$  and find an explicit expression for the indifference curve defined by the utility level  $U > 0$ . Then, derive an explicit expression for the marginal rate of substitution between good 1 and good 2.
- (b) Draw the indifference curve (for this associated utility level  $U$ ) and carefully label the graph and all its elements.
- (c) Show that the expenditure function is strictly increasing (and hence monotone) in  $x_1$  and  $x_2$ .
- (d) Now formally state the expenditure minimization problem and briefly describe its content, in particular what constitutes the choice variables, and what constitutes the constraint variables (i.e. endogenous versus exogenous variables).
- (e) Provide an argument why in the present expenditure minimization problem, we can restrict attention without loss of generality to the case where utility constraint holds as an equality at the optimal solution, that is we can restrict attention to the equality constraint:

$$u(x_1^*, x_2^*) = U.$$

(The argument should not involve the explicit computation of the optimal choices.)

2. **(30 min)**. For the expenditure maximization problem given above by (1) and (2), we are now computing the optimal demands.
- (a) Formally, and separately, state the first order condition by using the method of substitution and the method of Lagrange. Be sure to briefly describe the optimality conditions and their composition.
  - (b) Graphically describe the optimality condition and carefully label the graph and all its elements.
  - (c) Explicitly compute the Hicksian demand functions by one of the above two methods.
  - (d) Define the notion of the own price elasticity and explicitly derive the own price elasticity of good 1 of the Hicksian demand function.
  - (e) Find the expenditure function and explicitly identify the marginal cost (expenditure) of utility.
3. **(20 min)** The compensated and the uncompensated demand, i.e. the Marshallian and Hicksian demand function, can be related through the Slutsky equation. The Slutsky equation can be derived from the equality

$$x_i^*(p, E(p, U)) = h_i^*(p, U), \quad (3)$$

where  $p$  is the vector of prices  $p = (p_1, \dots, p_n)$ .

- (a) State the Slutsky equation and briefly explain all the terms appearing in the Slutsky equation.
- (b) Formally derive the Slutsky equation from the equality (3).
- (c) Graphically illustrate the content of the Slutsky equation holds, carefully labelling all the objects that enter the graphic.