Market Design with Limited Monetary Transfers

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Organization of this lecture

- Part 1: "Assigning Resources to Budget-Constrained Agents," by Che, Gale and Kim (2012).
  - Laying a foundation for suppressing monetary transfers in resource allocation
- Part 2: "Designing Random Allocation Mechanisms: Survey."
  - How do you design a market without money?

Part 2: Designing Random Allocation Mechanisms

- Monetary transfers are limited in many resource allocation problems, e.g., school choice, housing, office, parking spaces, course allocations, jury duty,...
- Further the goods are often indivisible $\Rightarrow$ Classical markets cannot be relied upon to achieve a desirable allocation,
- Mechanism must essentially rely on "cheap" talk messages, and
- Random allocation (designing lotteries) plays an important role for achieving
  - Fairness
  - "Divisibilization:" probability units can act as divisible currency, work like "transfers" in mechanism design.

Scope of Talk

- "House Allocation" Environment: n agents to be assigned n objects/goods, one for each. (We consider some slight variations.) Assume each agent has strict preferences.
- Two Types of Mechanisms:
  - **ordinal** mechanisms: map ordinal preference lists to a random allocation
  - **cardinal** mechanisms: map vNM values to a random allocation

Some Desirable Properties and Impossibility

1. **Efficiency**: Pareto undomination in lotteries (cardinal); Stochastic undomination (ordinal)
2. **Symmetry**: equal treatment of equals
3. **Strategy-proofness**: weak dominance of truth-telling.

- **Ordinal Impossibility** (Bogomolnaia and Moulin): *For $n \geq 4$, there is no (ordinal) mechanism that achieves efficiency, symmetry, and strategy-proofness all in ordinal sense.*
- **Cardinal Impossibility** (Zhou): *For $n \geq 3$, there is no mechanism that achieves ex ante Pareto efficiency, symmetry, and strategy-proofness (in cardinal preferences).*

Ordinal Mechanisms: Random Priority (RP) mechanism

- Uniform-randomly order the agents
- The first agent receives her most preferred good, the next agent his most preferred good among the remaining ones, and so on.

  $\Rightarrow$ Symmetric, strategy-proof and ex post efficient.

  $\Rightarrow$ Ordinally inefficient. [$\exists$ another allocation stochastically dominating the allocation.]

RP is Ordinally Inefficient

- goods $O = \{a, b\}$ with one copy each and agents $N = \{1, 2, 3, 4\}$,

$$\begin{array}{lll} \text{1 and 2 like} & a, b, \emptyset & \text{(in this order)} \\ \text{3 and 4 like} & b, a, \emptyset & \end{array}$$

- The random assignments under RP

	Good a	**Good** b	**Good** $\emptyset$
Agents 1 and 2	$5/12$	$1/12$	$1/2$
Agents 3 and 4	$1/12$	$5/12$	$1/2$

- Everyone prefers

	Good a	**Good** b	**Good** $\emptyset$
Agents 1 and 2	$1/2$	0	$1/2$
Agents 3 and 4	0	$1/2$	$1/2$

Probabilistic Serial Mechanism

- Bogomolnaia and Moulin (2001) define PS based on an "eating algorithm":
  - Imagine each good is a divisible good of "probability shares."
  - Imagine there is a time interval $[0, 1]$.
  - Each agent "eats" the best good with speed one at every time (among goods that have not been completely eaten away).
  - At time $t = 1$, each agent is endowed with probability shares.
  - PS assignment is the resulting profile of shares.
- The resulting profile of shares are "feasible" in the sense of Birkhoff-von Neumann theorem.
- PS is symmetric (in fact envy free), ordinally efficient relative to stated preferences, but it is not strategy-proof.

Ordinal efficiency of PS: Example (Bogomolnaia and Moulin)

- The same example as before: $O = \{a, b\}$, $N = \{1, 2, 3, 4\}$,

$$\begin{array}{lll} \text{1 and 2 like} & a, b, \emptyset & \text{(in this order)} \\ \text{3 and 4 like} & b, a, \emptyset & \end{array}$$

- Compute the PS assignment:
  - $t = 0$: Agents 1 and 2 start eating a, and agents 3 and 4 start eating b.
  - $t = 1/2$: goods a and b are eaten away. No (real) goods remain.
  - The resulting assignment

	Good a	**Good b**	**Good $\emptyset$**
Agents 1 and 2	1/2	0	1/2
Agents 3 and 4	0	1/2	1/2

is ordinally efficient.

Non-strategyproofness of PS: Example

- The same example as before: $\tilde{O} = \{a, b, \emptyset\}$, $M \geq 2$ copies of each object. Three types of agent,

Agent i likes	$a, b, \emptyset$	(in this order)
M agents prefer	$a, \emptyset, b$	
$M+1$ agents prefer	$b, \emptyset, a$	

- If agent i ranks truthfully, then he consumes a from $t = 0$ till $t = \frac{M}{M+1}$ and $\emptyset$ thereafter
- If he lies and reports $b \succ a \succ \emptyset$, then he consumes b on $[0, \frac{M}{M+2}]$ and a during $[\frac{M}{M+2}, \frac{M(M+3)}{(M+1)(M+2)}]$ and $\emptyset$ thereafter.
- The latter is better if b is not too worse than a for i.
- But notice, as $M \to \infty$, reporting truthfully becomes dominant.

Large Market Comparison of RP and PS

- Imagine a market/problem becoming large in the sense of the number of copies for each object and the number agents of each preference type grows to infinity; relevant for school choice application.
- Kojima and Manea (2010) show that PS becomes strategy-proof if the economy gets large.

  $\Rightarrow$ PS is better?

- Che and Kojima (2010) show that RP and PS become identical asymptotically as the economy gets large.

  $\Rightarrow$ RP becomes ordinally efficient asymptotically, perhpas better given its simplicity.

Example

- Consider q-fold replica economies of the previous example: The probability of obtaining less preferred good approaches zero as $q \to \infty$.

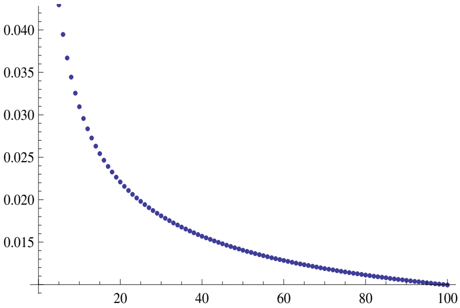

Abbildung: Horizontal axis: Market size q. Vertical axis: Mis-allocation probability.

Cardinal Mechanisms: Motivation

- Ordinal mechanisms such as RP are not responsive to agents' preference intensities, which entails ex ante welfare loss.
- Example: 3 agents, $N = \{1, 2, 3\}$, and 3 goods, $O = \{a, b, c\}$, each with one copy.
- All agents have the same ordinal preferences: $a \succ b \succ c$, but have different cardinal preferences, given by vNM values:

	v_j^1	v_j^2	v_j^3
$j = a$	4	4	3
$j = b$	1	1	2
$j = c$	0	0	0

- *Every assignment is ex post Pareto efficient and ordinally efficient; so no distinction possible based on these criteria.*

- Under RP (same as DA with random type breaking) or PS, all three submit true (ordinal) preferences, so the agents are assigned the schools with equal probability.

  $\Rightarrow EU_1 = EU_2 = EU_3 = \frac{5}{3}$

- Pareto-dominated by the following assignment:

	1	2	3
a	$\frac{1}{2}$	$\frac{1}{2}$	0
b	0	0	1
c	$\frac{1}{2}$	$\frac{1}{2}$	0

  $\Rightarrow EU_1' = EU_2' = EU_3' = 2 > \frac{5}{3}$

Pseudo-Market Mechanism: Hylland and Zeckhauser (1979)

- Each agent submits vNM values of the goods, then the mechanism computes agents' probability shares by "simulating" the competitive markets:
  - Each agent is endowed with the same budget in "fictitious" currency (e.g., 100 tokens).
  - For a profile of prices *per unit probability of obtaining alternative goods*, each agent buys optimal probability shares of alternative goods
  - The prices are chosen to clear the markets, and pin down the equilibrium lotteries of goods, one for each agent.

- The feasibility of lotteries is ensured by Birkhoff-von Neumann.
- *The equilibrium is symmetric (in fact, envy free) and ex ante Pareto efficient (by the first welfare theorem).*
- Example: With budget $= 100$, $(p_a, p_b, p_c) = (200, 100, 0)$, the "good" allocation implemented.

Boston mechanism

- Assignment is prioritized based on the agents' ordinal preferences. Seats of a school (copies of an object) are assigned first to those who top-ranked the school (with random tie-breaking if necessary); those who ranked it below top (and rejected by their first choice) are assigned the remaining seats if there are any left, and so on.

- Not strategy-proof!
- But may enable agents to express cardinal preferences.
- In the example, the good allocation implemented (1 and 2 top-rank a; 3 top-ranks b). More generally,

Theorem:

(Abdulkadiroglu-Che-Yasuda, 2011) *If the agents with the common ordinal preferences draw vNM values, any symmetric BNE of the Boston mechanism Pareto dominates the outcome of RP.*

Is Non-Strategyproofness Necessarily Bad?

- In the run-up to the celebrated redesign of school choice program in the BPS, parents argued:

  > *"I'm troubled that you're considering a system that takes away the little power that parents have to prioritize... what you call this strategizing as if strategizing is a dirty word..." (BPS Hearing, 2005)*

  > *"... if I understand the impact of Gale Shapley, ... I thought I understood that in fact the random number in fact [has] preference over your choices..." (BPS Hearing, 2005)*

Choice-Augmented Deferred Acceptance (CADA) (Abdulkadiroglu-Che-Yasuda)

- Motivation: How can we allow the agents to express their cardinal preferences with much of the benefits from DA preserved? Simple modification of DA to allow for signaling of preference intensities.
- Agents submit preference rankings of goods, plus "name" of a target good.
- Run agent-proposing deferred acceptance. But instead of random tie-breaking (in which case DA coincides with RP), agents who targeted a good (school) favored in tie at that good (school).

Properties of CADA (Abdulkadiroglu-Che-Yasuda)

- Strategy-proof with ordinal preferences;
- Can imbed priorities (e.g., school choice); if priorities are strict, then the mechanism reduces to the standard DA so the the outcome is SOSM.
- If agents have the same ordinal preferences (and there are no priorities with objects), then the outcome Pareto-dominates the outcome of DA. In the example, the better allocation is implemented (1 and 2 target a; 3 targets b).
- In the large market, ex ante efficiency is achieved within the set of "overdemanded" objects (schools).

Large Economy Model

- There are $n \geq 2$ schools, $O = \{1, ..., n\}$, each with a unit mass of seats to fill.
- There are mass n of students who are indexed by vNM values $\mathbf{v} = (v_1, ..., v_n) \in \mathcal{V} = [0, 1]^n$, with a measure μ that admits strictly positive density in the interior of $\mathcal{V}$.
- An **allocation** is a mapping from $\mathcal{V}$ to lotteries over O satisfying feasibility.
- "Scope of Efficiency": For a subset $K \subset O$ of schools, an allocation is **within-K efficient** if it is not Pareto-dominated by another allocation that simply reallocates probability shares of the schools in K.

Ex Ante Welfare Properties of CADA

Theorem:

(i) The equilibrium allocation is ordinally efficient, which implies efficiency within every pair of schools.

(ii) The allocation is ex ante efficient within set K of "oversubscribed" schools (those whose capacity does not exceed the measure of all who target the schools).

(iii) If all but one schools are oversubscribed, then the equilibrium allocation is fully ex ante efficient.

Intuition: "Targeting" effectively activate competitive markets in popular schools; a degree of congestion at a school serves to price that school efficiently.

Ex Ante Welfare Properties of CADA

Definition: A school *a* is **popular** if the size of the students prefer *a* most is as large as its capacity.

Proposition:

A popular school is over-subscribed in equilibrium.

Corollary:

The CADA allocation is efficient within the set of popular schools; if all but one schools are popular, then the CADA allocation is fully efficient.

Theorem:

By contrast, generically, the RP is not efficient within a set of no more than two schools.

Further development

- Generalizing the framework (controlled choice, multiple-unit demand, nonlinear preferences etc.)
  - BM and HZ's method, focusing on lotteries for agents as primitive, is largely limited to the house allocation environment. Generalizing them to incorporate various real world features (group-specific quotas, flexible capacities, and multi-unit demand with nonlinear preferences) requires generalizing the Birkhoff von-Neumann: Budish, Che, Kojima and Milgrom (2011).
- Incorporating priorities on the object side (e.g., "school choice" environment): If the priorities are coarse (as in school choice), then there is a room for random assingment.
  - Fractional DA: Kesten and Unver (2010)
  - CADA: Abdulkadiroğlu, Che, and Yasuda (2011)

Beyond the Matching Environment

- Random allocation is an important part of mechanism design when the use of monetary transfers is limited.
  - Mechanism design with financially-constrained agents: Che and Gale (1999), Pai and Vohra (2010), Che, Kim and Gale (2011)
  - Communication mechanism: Che, Dessein and Kartik (AER, forth.), Ben Porath, Dekel and Lipman (2011).