Market Design with Limited Monetary Transfers

Yeon-Koo Che

Columbia University

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Yale University

Organization of this lecture

- Part 1: "Assigning Resources to Budget-Constrained Agents," by Che, Gale and Kim (2012).
  - Laying a foundation for suppressing monetary transfers in resource allocation
- Part 2: "Designing Random Allocation Mechanisms: Survey."
  - How do you design a market without money?

How should we allocate scarce resources to agents?

- Received Wisdom:
  - Coase theorem: How we assign resources doesn't matter. Unrestricted resale achieves Pareto efficiency, and Utilitarian efficiency if agents have quasilinear preferences.
  - Market principle: If the resale market doesn't function well, sell the resources via standard auctions. Pareto efficient by the first welfare theorem.
- Analysis of Coasian bargaining failure largely limited to asymmetric information on willingness to pay. $\Rightarrow$ largely supports the market principle.
- But the use of non-market assignment schemes abound: Education (e.g., public schools), immigration, human organs, low income housing, civic (e.g., military or jury) duty, etc.

$\Rightarrow$ *Their implications need to be understood.*

Oklahoma Land Race

Georgia Land Lottery

Military Draft Lottery

Budget Constraints and their Implications

- We explore the implications of another source of Coasian bargaining failure — agents' liquidity constraints.
- Liquidity constraints relevant when resources being allocated are significant relative to the cash holding and difficult to collateralize (education, immigration, spectrum licenses, housing, human organs, etc.) $\Rightarrow$ capital markets not functioning well
- Coase theorem does not work since limited ability to pay inhibits post-assignment bargaining: We find
  - Non-market assignment — such as random assignment of transferability property right — can be justified on Utilitarian efficiency ground, *although we do not claim to explain the emergence of non-market assignment*.
  - Favoring the poor in the assignment is desirable, justifying need-based programs.
  - Optimal mechanism contains both in-kind subsidy for the poor

Related Literature

- Mechanism Design with Budget-Constrained Agents: Che and Gale (1998, 2000, 2006), Pai and Vohra (2010), Kotowski (2010), Condorelli (2012)
- Resource Allocation with Budget Constraints in Large Markets: Fernandez and Gali (1999) and Esteban and Ray (2006)
- Benefits of Market Intervention: Weitzman (1997), Sah (1987).
- Efficient Redistribution of Benefits: Blackorby and Donaldson (1987), Besley and Coate (1991), Gahvari and Mattos (2007).

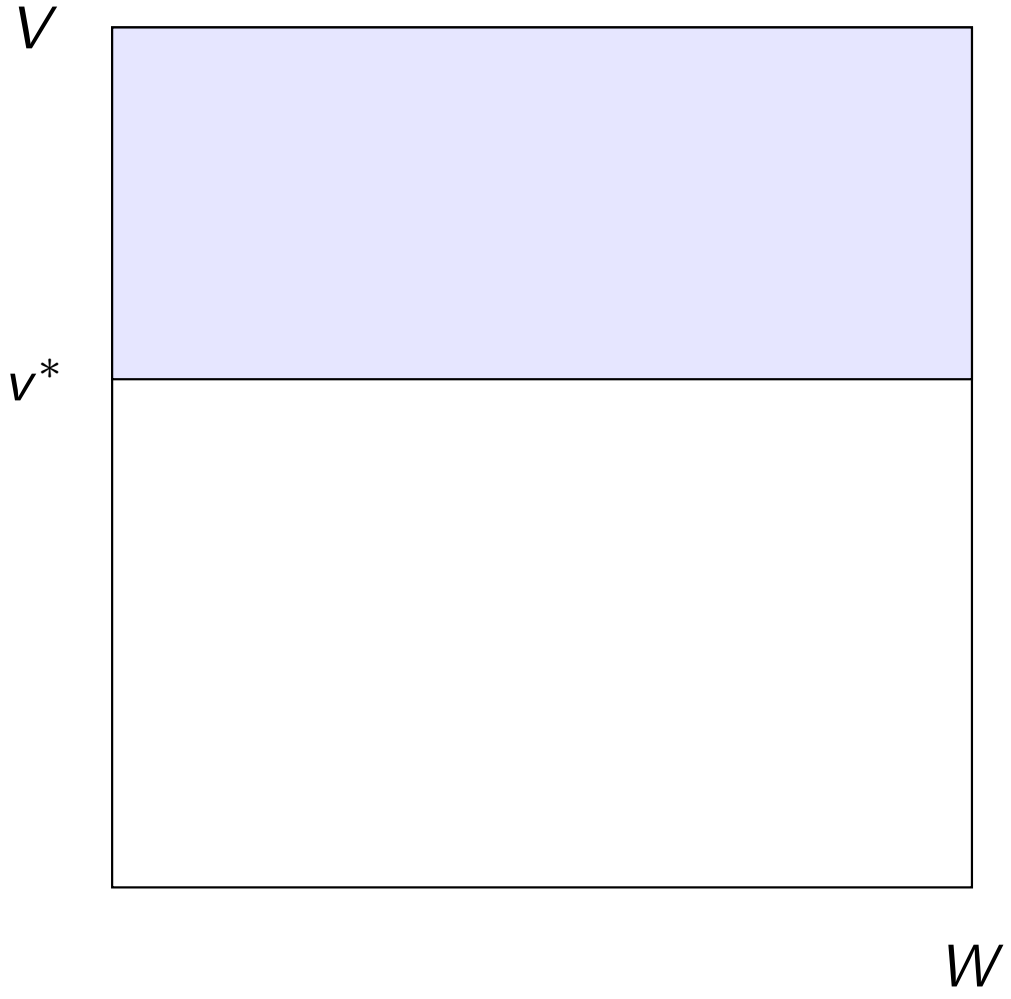
Model

- A planner has a mass $S < 1$ of indivisible good to allocate (e.g., spectrum licenses, housing...)
- A unit mass of risk-neutral buyers who each demand one unit.
- A buyer has a type, (w, v), unobservable to the planner
  - $w =$ wealth $\in [0, 1] \sim G(w)$
  - $v =$ valuation $\in [0, 1] \sim F(v)$ (profit or consumption value)
- Quasilinear preferences: a buyer with (w, v) enjoys $vx - t$ when obtaining the good with prob x and paying t in expectation, but can only spend w to pay.

Welfare Criterion

- Utilitarian efficiency: Total realized value (or average value realized per unit)
  - Without loss, if v is monetary return: Inefficiency arises from an imperfect capital market.
  - Impartial observer doctrine (Vickrey and Harsanyi): What would an individual choose should she have an equal chance of landing in the shoes of each member of the society?
- First-best benchmark: buyers with $v \geq v^*$ are served, where v^* satisfies $S = 1 - F(v*)$.
- Per-unit value realized $= \mathbb{E}[v|v \geq v*]$.

First-best Allocation



Three Mechanisms

- A competitive market: (resale right doesn't matter).
- Random assignment without resale: price is capped and a lottery assigns the good; not allowed to resell.
- Random assignment with resale: same as above except resale is permitted after assignment.
- A full-fledged mechanism design exercise comes later.

Examples

- Housing in Korea: Random assignment with resale.
- School choice: market (residence-based) and random assignment without resale (school chocie).
- Military recruitment under draft:
  - Random without resale (Vietnam);
  - Random with resale (Civil war).
- Government resources: all three used.
- Immigration visas: Random without resale.

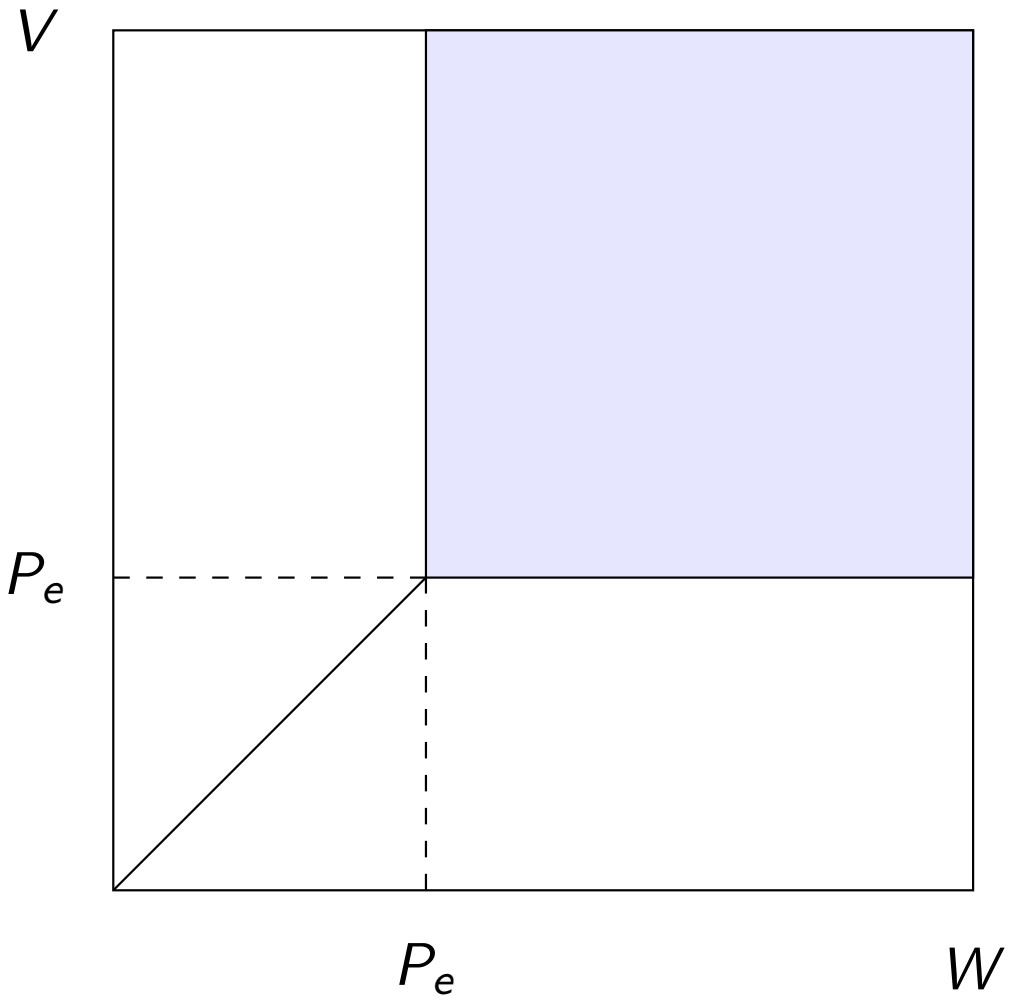
Competitive Market

- Demand = number of buyers willing and able to pay the price

$$D(p) = [1 - F(p)][1 - G(p)]$$

- Supply=S
- Equilibrium price: p_e satisfying $D(p_e) = S$.

Competitive Market Equilibrium

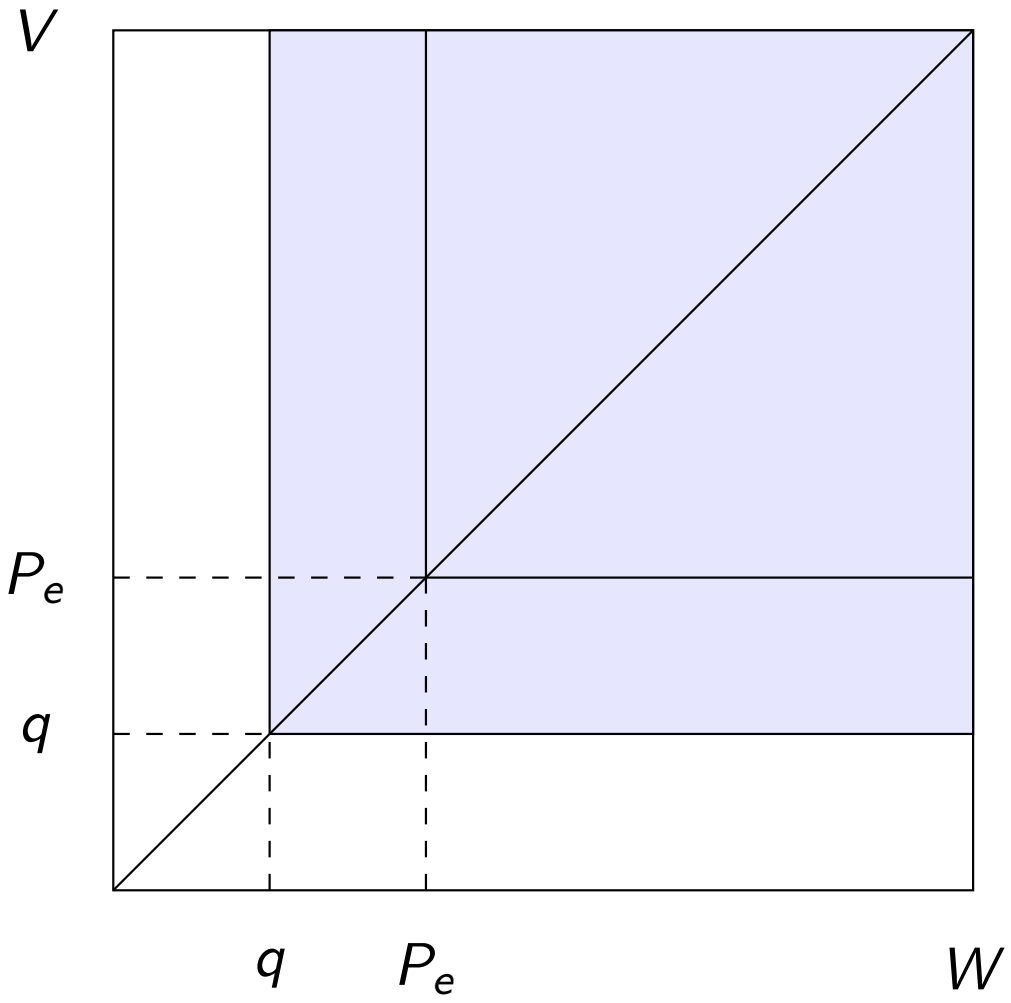


- Average value realized: $\mathbb{E}[v|v \geq p_e]$.

Random assignment without Resale

- The price is capped at $q < p_e$ and excess demand is assigned randomly (i.e., a lottery, with one entry per participating agent).
- Buyers with $(w, v) \geq (q, q)$ participate in the rationing and are successful with probability $\frac{S}{(1-F(q))(1-G(q))}$.

Random assignment without Resale



- Average value realized: $\mathbb{E}[v|v \geq q] < \mathbb{E}[v|v \geq p_e]$.

Random assignment without Resale

- Random assignment without resale is less efficient than the market.
- Intuition: Random assignment allows buyers with low wealth to consume, but also those with low valuations.

Random with Resale (RwR)

- Price is capped at $q < p_e$ and excess demand is rationed randomly; resale is permitted.
- Suppose the resale price, r, is higher than q (if not, there would not be rationing).
- All buyers with $w \geq q$ will participate in rationing.

Random with Resale (RwR)

- All buyers with $(w, v) > (q, 0)$ participate; each gets the good with probability
$$\rho(q) = \frac{S}{(1 - G(q))}$$
- Resale Market:
  - Demand side: Unsuccessful buyers purchase at the resale price, r, if $(w, v) \geq (r, r)$.
  - Supply side: Successful buyers/non-buyers with $v < r$ sell.
- Measure of buyers: $[1 - F(r)][1 - G(r)](1 - \rho(q))$.
- Measure of sellers: $S \cdot F(r)$.

Resale Market Equilibrium

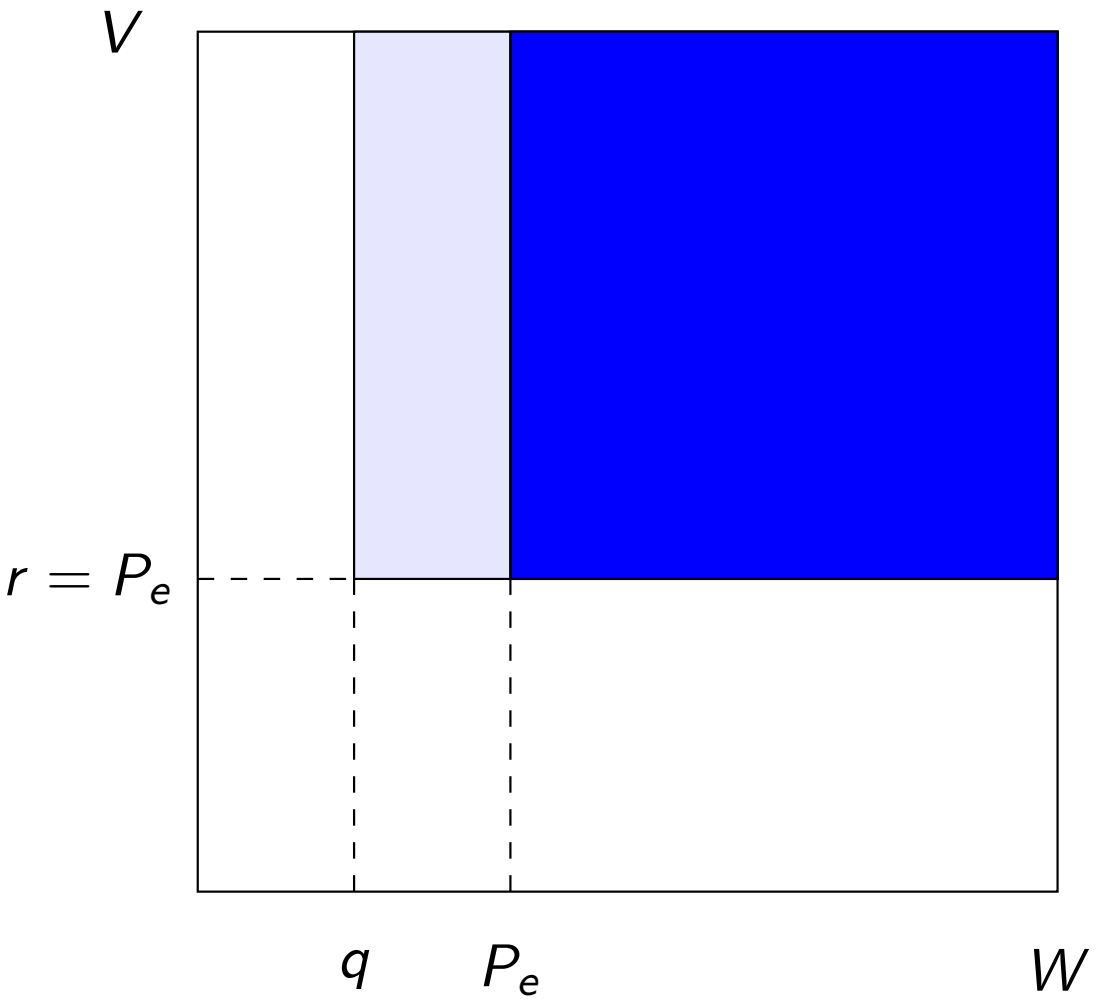
$$[1 - F(r)][1 - G(r)](1 - \rho(q)) = S \cdot F(r)$$

$$\Leftrightarrow D(r) = S - \rho(q)[1 - F(r)][G(r) - G(q)]$$

$$\Rightarrow \text{equilibrium resale price:} r^*(q) > p_e$$

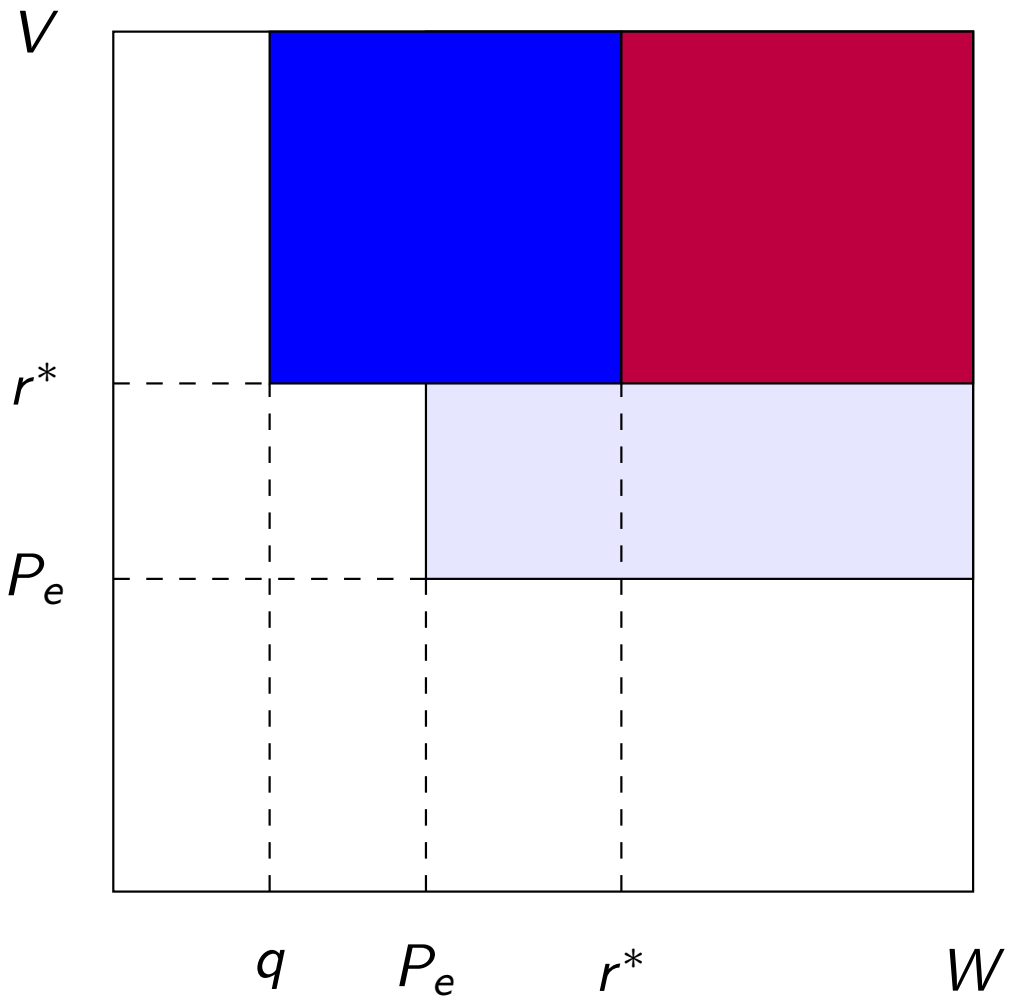
- Average value: $\mathbb{E}[v|v \geq r^*] > \mathbb{E}[v|v \geq p_e]$
- Lower q raises the average value realized.

RwR versus the Market



- There would be excess demand if $r^* = p_e \Rightarrow r^* > p_e$.

RwR versus the Market



Intuition

- Coase theorem doesn't apply if individuals are wealth constrained.
- Market, as a method of initial assignment, is biased toward those with high wealth
- Random assignment with resale does a better job than the market in allocating ownership to the poor.
- Can do even better than RwR: need-based programs justified based on Utilitarian efficiency.
- Generality?
  - RwR involves an in-kind subsidy. Can one do just as well with cash subsidy with competitive market?
  - The benefit of RwR limited by speculators. Can we prevent enrichment of the low-valuation agents?

  $\Rightarrow$ We study optimal mechanism.

General Mechanism

$$[P] \qquad \max_{x,t} \mathbb{E}[vx(w,v)]$$

subject to

$$(S) \qquad \mathbb{E}[x(w,v)] \leq S.$$

$$(BC) \qquad t(w,v) \leq wx(w,v),\ \forall w, v.$$

$$(IR) \qquad vx(w,v) - t(w,v) \geq 0,\ \forall w, v,$$

$$(IC) \qquad vx(w,v) - t(w,v) \geq vx(w',v') - t(w',v'),$$

$$\forall w', v' : t(w'.v') \leq x(w',v')w.$$

$$(BB) \qquad \mathbb{E}[t(w,v)] \geq 0.$$

Features of the Optimal Mechanism

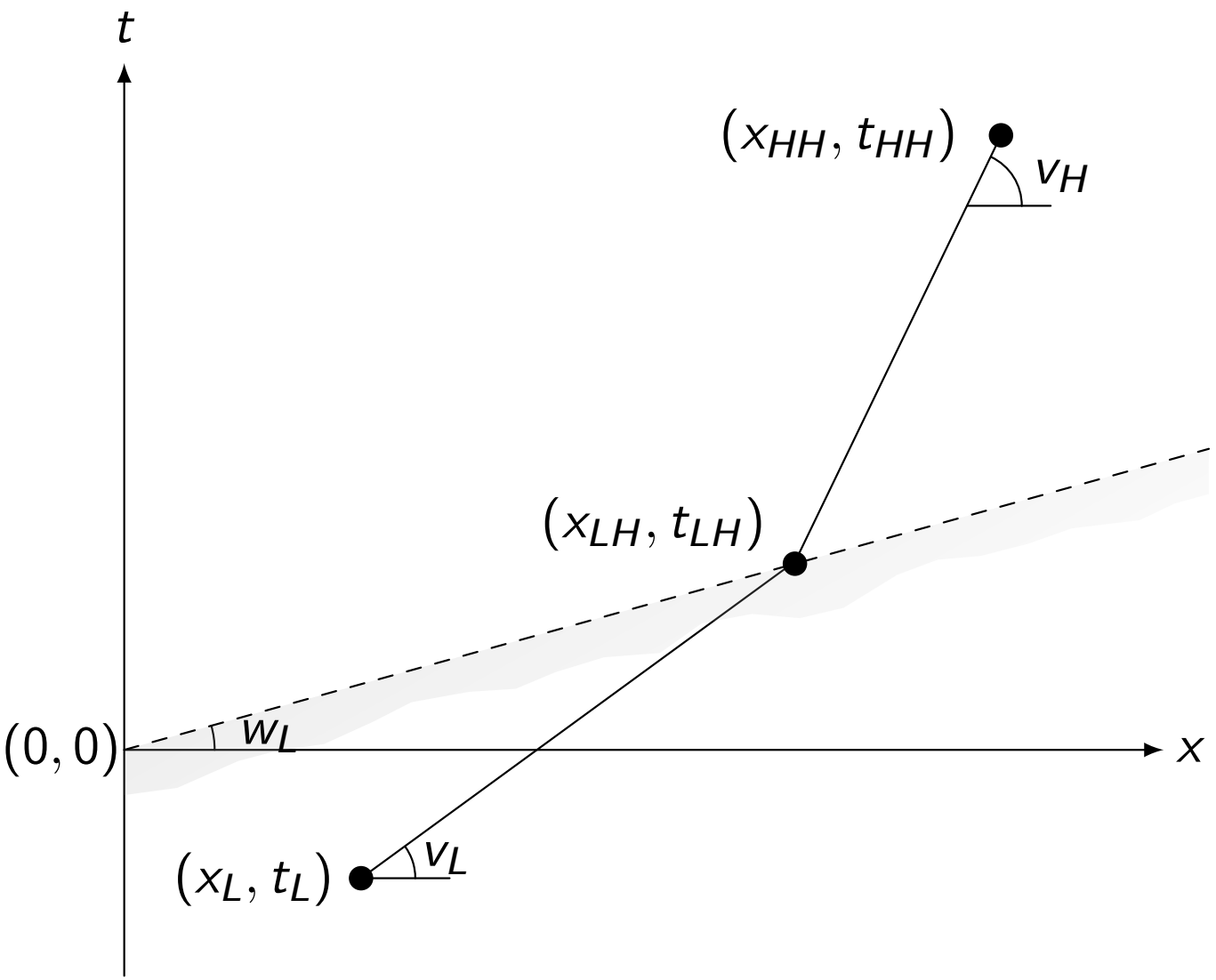
- Cash subsidy: Low valuation agents are paid cash not to consume. The enrichment of the low valuation agents is inevitable to enable consumption by high-valuation but cash poor agents. (NB: RwR confers a cash subsidy to low-valuation agents in the form of resale profit.)

- Random assignment (In-kind subsidy): necessary to discourage the agents with low valuation from consuming but also to extract payments from those with high wealth and high valuations, which are in turn used to finance cash subsidy.

2×2 types case:

$W = \{w_L, w_H\}$, $V := \{v_L, v_H\}$, with $w_L < v_L < v_H \leq w_H$.

For a low w_L, the optimal mechanism has a menu of 3 contracts.



Intuition

- The challenge is to maximize the quantity for the LH (low wealth high value) type.
- They can be at most charged w_L per unit; profitable for HL to mimic. $\Rightarrow$ Offer cash bribe to HL to discourage its consumption of the good.
- How to finance the cash? $\Rightarrow$ Charge HH high price, which requires $x_{LH} \downarrow$ from 1.
- Can be implemented by a menu of cash and in-kind (lottery), followed by regulated resale (taxed by $\tau = v_H - v_L$).

Optimal Need-Based Assignment

Suppose there is a signal $s \in \{\ell, h\}$ on wealth such that for $\rho \in (1/2, 1)$ the mass of each type (w_i, v_j) is given by

	v_L	v_H
w_L	$\frac{1}{4}\rho$	$\frac{1}{4}\rho$
w_H	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1-\rho)$

$s = \ell$: poor group

	v_L	v_H
w_L	$\frac{1}{4}(1-\rho)$	$\frac{1}{4}(1-\rho)$
w_H	$\frac{1}{4}\rho$	$\frac{1}{4}\rho$

$s = h$: rich group

We find

- Wealth perfectly observed ($\rho = 1$): The first-best attainable with RwR that assigns as much of the good as possible to the poor group $s = \ell$ in the initial assignment.
  - If $S \leq 1/2$, then the entire supply will be allocated (uniform-randomly) to the members of this group. Low value type resells to the high value with high wealth at $r = v_H$.
  - If $S > 1/2$, then every agent of the low-wealth group gets the good. The remainder is then allocated uniform-randomly to the members of the rich group $s = h$. The low value in the latter group sells to the high value in the group at $r = v_L$.
- More generally, the rich group cross subsidizes the poor group in the optimal mechanism:
  - Each type in the poor group consumes more than the same type in the rich group.
  - Each type in the poor group enjoys a higher payoff than the same type in the rich group.
- Intuition: Favoring the low-wealth high-value type in the poor group is cheaper since there are fewer pretenders to pay off.

Effect of Pure Speculation

- So far, we have assumed that the planner can control resale. What happens resale cannot be prevented?
- Suppose there is mass m of agents with zero valuation (and wealth distributed according to G).
- If the recipients can't be prevented from reselling, the outcome of any mechanism converges to that of CM as $m \to \infty$ (cf Coase theorem?).
- Suggests that restricting resale can be desirable.

Conclusion

- Both in-kind subsidy (making the good available at below-market price) and cash subsidy valuable; cash subsidy valuable not for fungibility but for incentive compatibility
- Need based assignment desirable
- Non-market assignment of transferable ownership rights is a simple way of harnessing these features (transferable school vouchers, transferable immigration rights?).