

DIRK BERGEMANN  
DEPARTMENT OF ECONOMICS  
YALE UNIVERSITY

ECONOMICS 121B: INTERMEDIATE MICROECONOMICS  
Problem Set 5:  
Uncertainty  
2/29/12

This problem set is due on Wednesday, 3/21/12, in class. To receive full credit, provide a complete defense of your answer.

1. This homework first examines the issues involved in using the expected utility representation of preferences to model choices over risky alternatives. Consider the expected utility function

$$U(x_1, x_2) = \pi_1 \sqrt{x_1} + \pi_2 \sqrt{x_2},$$

where  $x_1$  and  $x_2$  are (monetary) consumption levels in states 1 and 2, respectively, which occur with probabilities  $\pi_1$  and  $\pi_2$ . Let  $\pi_1 = \frac{3}{8}$  and  $\pi_2 = \frac{5}{8}$ .

- (a) Calculate the expected utility of a lottery that pays nothing in state 1 and \$100 in state 2. Calculate the expected value of this lottery, and the utility of receiving this expected value with certainty, i.e., of receiving this expected value in both states of nature. Which is larger—the expected utility of the lottery, or the utility of the expected value of the lottery? What does this tell you about this person's attitude toward risk?
- (b) The *certainty equivalent* of the lottery is a payoff which, if received with certainty, would make the person indifferent between the lottery and receiving the certainty equivalent in each state. Calculate the certainty equivalent of the lottery in [a]. How does it compare to the expected value?
- (c) Show that, in general, if one is risk averse, then the certainty equivalent of a lottery falls short of its expected value. The difference between these two is often called the *risk premium* for the lottery.
- (d) Find the formula for an indifference curve, giving  $x_2$  as a function of  $x_1$ , identifying combinations of  $x_1$  and  $x_2$  that give the same expected utility.
- (e) Find the slope of this indifference curve when  $x_1 = x_2$ . Explain why it has this slope.
- (f) Show that the preferences described by this expected utility function are convex.

2. Suppose that  $f$  and  $u$  are increasing, strictly concave functions. Show that the function  $f(u(x_i))$  gives rise to a larger Arrow-Pratt measure of risk aversion than does  $u$ . Show that the same is not the case if  $f$  is a linear function. Hence, in the presence of uncertainty, increasing linear transformations preserve preferences, but increasing concave transformations in general do not.
3. We now examine some applications of our theory of choice under uncertainty. Continue with the utility function  $U(x_1, x_2) = \pi_1\sqrt{x_1} + \pi_2\sqrt{x_2}$ , and let  $x_1 = 36$ ,  $x_2 = 64$  and  $\pi_1 = \pi_2 = 1/2$ .
  - (a) Find the expected utility of this lottery.
  - (b) Suppose now there are two agents. Each faces this lottery, but the outcomes are independent. Hence, with probability  $\frac{1}{4}$  each receives 36, with probability  $\frac{1}{4}$ , each receives 64, and with probability  $\frac{1}{2}$ , one receives 36 and the other 64. The two agents make an agreement that if one receives 36 and the other 64, the fortunate agent transfers 14 to the unfortunate agent. Calculate their expected utility under this arrangement. Such an arrangement is the essence of an insurance policy—people facing uncorrelated risks can use the fact that they are unlikely to both experience losses to eliminate some of the risk they face.
  - (c) Suppose again that there are two agents, each facing this lottery, but with perfectly correlated lotteries. Hence, either both receive 36 or both receive 64. Can an arrangement like that of [b] give both agents a higher expected utility than that calculated in [a]?
  - (d) In light of your answers to [a]–[c], explain why it may be easier to insure houses against fires than floods. What you have just discovered is that if insurance is to be effective, it must involve uncorrelated rather than correlated risks. An insurance policy cannot help with risks that tend to either impose a loss on everyone, or on no one. What risks are likely to fall into this category?