Econ 121b: Intermediate Microeconomics

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1 Lecture 10: Welfare Measurement

In order to do welfare comparison of different price situations it is important that we move out of the utility space and deal with money as then we would have an objective measure that we can compare across individual unlike utility. Let the initial price vector be given by,

$$p^0 = (p_1^0, p_2^0, \ldots, p_n^0)$$

and the new price vector be,

$$p^1 = (p_1^1, p_2^1, \ldots, p_n^1)$$

1.1 Compensating Variation

The notion of compensating variation asks how much additional amounts of income is required to maintain the initial level of utility under new prices.

$$CV = E(p^1, u_0) - E(p^0, u_0)$$

where $E(p^0, u_0)$ is the expenditure function evaluated at price p^0 and utility level u_0. This gives us a measure of loss or gain of welfare of one individual in terms of money due to change in prices.

1.2 Equivalent Variation

The notion of equivalent variation asks how much additional amounts of income is required to raise the level of utility from the initial level to a specified new level given the same prices.

$$EV = E(p^0, u_1) - E(p^0, u_0)$$

Let the change in price from p^0 to p^1 is only through the change in price of commodity 1. Let $p_1^1 > p_1^0$ and $p_i^1 = p_i^0$ for all other $i = 2, 3, \ldots, n$. Then we can write,

$$\begin{aligned} CV &= E(p^1, u_0) - E(p^0, u_0) \\ &= \int_{p_1^0}^{p_1^1} \frac{\partial E(p_1, u_0)}{\partial p_1} dp_1 \\ &= \int_{p_1^0}^{p_1^1} h_1^*(p_1, u_0) dp_1 \end{aligned}$$

1.3 Introduction of New Product

Let's think of a scenario where a new product is introduced. Let that be commodity k. This can be thought of as a reduction of the price of that product from $p_k = \infty$ to $p_k = \bar{p}$ where $\bar{p}$ is the price of the new product. Then one can measure the welfare gain of introducing a new product by calculating the CV with the change in price of the new product from infinity to $\bar{p}$.

$$CV = -\int_{\bar{p}}^{\infty} h_k^*(p_k, u_0) dp_k$$

1.4 Inflation Measurement

Let the reference consumption bundle be denoted by,

$$x^0 = (x_1^0, x_2^0, \ldots, x_n^0)$$

and the reference price be,

$$p^0 = (p_1^0, p_2^0, \ldots, p_n^0)$$

Then one measure of inflation is the Laspayers Price Index,

$$I_L = \frac{p^1 \cdot x^0}{p^1 \cdot x^0}$$

The other measure is Paasche Price Index,

$$I_P = \frac{p^1 \cdot x^1}{p^1 \cdot x^1}$$

where x^1 is the consumption bundle purchased at the new price p^1. Here the reference bundle x^0 is the optimal bundle under the price situation p^0. Therefore we can say,

$$p^0 \cdot x^0 = E(p^0, u_0)$$

where u_0 is the utility level achieved with price p^0 and income $p^0 \cdot x^0$. Now given the new price situation p^1 we know that,

$$p^1 \cdot x^0 \geq E(p^1, u_0)$$

$$\Rightarrow I_L = \frac{p^1 \cdot x^0}{p^1 \cdot x^0} \geq \frac{E(p^1, u_0)}{E(p^0, u_0)}$$

Hence we see that the Laspayers Price Index is an overestimation of price change.

2 Lecture 11: Pareto Efficiency and Competitive Equilibrium

We now consider a model with many agents where we make prices endogenous (initially) and later incomes as well. Let there be I individuals, each denoted by i,

$$i = 1, 2, \ldots, I$$

K commodities, each denoted by k,

$$k = 1, 2, \ldots., K$$

a consumption bundle of agent i be denoted by x^i,

$$x^i = (x_1^i, x_2^i, \ldots, x_K^i)$$

and the utility function of individual i be denoted by,

$$u^i \colon \mathbb{R}_+^K \to \mathbb{R}$$

and society has endowment of commodities denoted by e,

$$e = (e_1, e_2, \ldots, e_K)$$

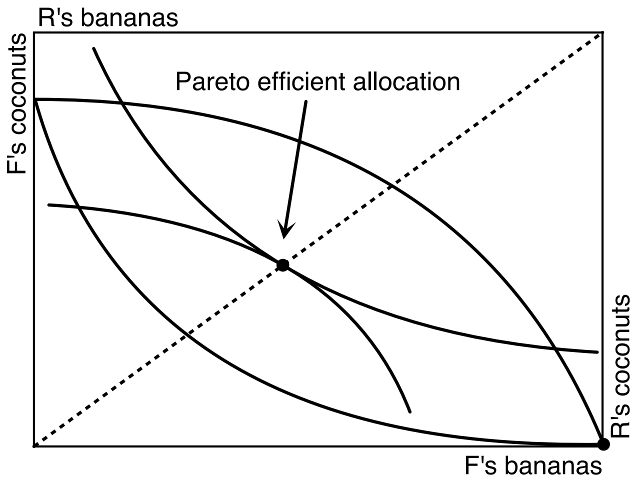
A social allocation is a vector of consumption bundles for all the individuals,

$$x = (x^1, x^2, \ldots, , x^i, \ldots, x^I)$$

The total consumption of commodity k by all the individuals can not exceed the endowment of that commodity, which is referred to as the feasibility constraint. We say a social allocation is feasible if,

$$\sum_{i=1}^{I} x_k^i \leq e_k \;\; \forall \; k = 1, 2, \ldots, K$$

which represent the K feasibility constraints.



Definition 1. An allocation x is **Pareto efficient** if it is feasible and there exists no other feasible allocation y such that nobody is worse off and at least one individual is strictly better off, i.e. there is no y such that for all i:

$$u^i(y^i) \geq u^i(x^i)$$

and at for some i':

$$u^{i'}(y^{i'}) > u^{i'}(x^{i'})$$

We say that an allocation y is **Pareto superior** to another allocation x if for all i:

$$u^i(y^i) \geq u^i(x^i)$$

and at for some i':

$$u^{i'}(y^{i'}) > u^{i'}(x^{i'})$$

and we say that y Pareto dominates x if, for all i:

$$u^i(y^i) > u^i(x^i)$$

In a 2 agent (say, Robinson and Friday), 2 goods (say, coconuts and bananas) economy we can represent the allocations in an Edgeworth box. Note that we have a total of four axes in the Edgeworth box. The origin for Friday is in the south-west corner and the amount of bananas he consumes is measured along the lower horizontal axis whereas his amount of coconuts is measured along the left vertical axis. For Robinson, the origin is in the north-east corner, the upper horizontal axis depicts Robinson's banana consumption, and the right vertical axis measures his coconut consumption. The height and width of the Edgeworth box are one each since there are one banana and one coconut in this economy. Hence, the endowment bundle is the south-east corner where the amount of Friday's bananas and Robinson's coconuts are both equal to one. This also implies that Friday's utility increases as he moves up and right in the Edgeworth box, whereas Robinson is better off the further down and left he gets. Any point inside the the two ICs is an allocation that gives both Robinson and Friday higher utility. Hence any point inside is a Pareto superior allocation than the initial one. The point where the two ICs are tangent to each other is a Pareto efficient point as starting from that point or allocation, it is not possible to raise one individual's utility without reducing other's. Hence the set of Pareto efficient allocations in this economy is the set of points in the Edgeworth box where the two ICs are tangent to each other. This is depicted as the dotted line in the box. It is evident from the picture that there can be many Pareto efficient allocations. Specifically, allocations that give all the endowment of the society to either Robinson or Friday are also Pareto efficient as as any other allocation would reduce that person's utility.

2.1 Competitive Equilibrium

A competitive equilibrium is the pair (p, x), where p is the price vector for the K commodities:

$$p = (p_1, \dots, p_k, \dots, p_K)$$

and x is the allocation:

$$x = (x^1, x^2, \dots, , x^i, \dots, x^I),$$

such that markets clear for all commodities k:

$$\sum_{i=1}^{I} x_k^i \leq e_k,$$

allocation is affordable for each individual i:

$$p \cdot x^i \leq p \cdot e^i,$$

and for each individual i there is no y^i such that

$$p \cdot y^i \leq p \cdot e^i$$

and

$$u^i(y^i) > u^i(x^i)$$