

DIRK BERGEMANN  
DEPARTMENT OF ECONOMICS  
YALE UNIVERSITY

ECONOMICS 121B: INTERMEDIATE MICROECONOMICS  
Problem Set 1  
1/11/12

This problem set is due on Monday, 1/23/12, in class. (All future problem sets will also be due on Monday.) To receive full credit, provide a complete defense of your answer.

1. True - False - Uncertain:

1. A person who gives money away to people on the street does not have preferences that can be represented by a utility function.
2. If Patrick's utility function is  $U(x, y) = xy$  and Julie's utility function is  $U(x, y) = \sqrt{xy}$ , then Patrick will always derive more happiness than Julie does from any combination of  $x$  and  $y$ .
3. A monotonic transformation of a utility function does not change the marginal rate of substitution at any point.
4. If Lori has downward-sloping indifference curves, then this is the same thing as saying that she prefers averages to extremes.
5. If Steve is maximizing  $U(x, y)$  subject to a budget constraint, then at the maximal point, the marginal utilities of  $x$  and  $y$  will be equal.

2. Consider the real-valued function  $f$  defined over the interval  $[-3, 3]$  by

$$f(x) = \begin{cases} 4 - (x + 2)^2, & \text{if } x \leq -1 \\ x^2(2 - x), & \text{if } x \geq -1 \end{cases}$$

1. Sketch a rough graph of the function.
2. Find all critical points (i.e., points with zero first derivative) of the function. Identify the interior local maxima and minima.
3. Is the function discontinuous anywhere? Is the function non-differentiable anywhere? If your answer to either of these questions is yes, is there a local maximum or minimum at these points?
4. Does the function have any local maxima or minima at its end-points?

5. Find its global maximum and minimum.
3. In each of the following questions,  $x$  and  $y$  must be non-negative numbers:
  1. Maximize  $xy$  subject to  $x + 2y = 12$  by substitution;
  2. Minimize  $x + 2y$  subject to  $xy = 18$  by substitution.
4. You have two midterm exams upcoming, and have to decide how to allocate your time. After eating, sleeping, exercising, and maintaining some human contact, you will have 10 hours each day in which to study for your exams. You have figured out that your grade point average ( $G$ ) from your two courses, Introduction into Astronomy and Introduction into Sociology, takes the form

$$G = \frac{4}{7}(2\sqrt{A} + \sqrt{S}),$$

where  $A$  is the number of hours per day spent studying for Astronomy and  $S$  is the number of hours per day spent studying for Sociology (these are to be regarded as continuous variables). You only care about your GPA.

1. What is your optimal allocation of study time?
  2. If you follow this optimal strategy, what will be your GPA?
  3. What will be the shadow value, measured in GPA units, of the study time?
5. Consider the consumption maximization problem of young John D. Rockefeller in 1855. He chooses to work hours  $L$  for wage  $w$ , since he is only a clerk in the merchant house of Hewitt & Tuttle. He must choose how to spend his income on two goods: clothing  $C$  and housing  $H$ , from which he derives utility via utility function  $U = C^\alpha + H^\alpha - L$  where  $\alpha \in (0, 1)$ . Suppose that prices of one year of rent for housing and on year's worth of clothing are  $r$  and  $p$ , respectively.
  1. Solve for Rockefeller's optimal choice of  $L_1, C_1$ , and  $H_1$ .
  2. It is now 1872, and John D. Rockefeller has just finished consolidating his competitors in oil refining to form Standard Oil. His hourly wage has now jumped to  $\phi \gg w$ . He still lives in the same small house in Cleveland as he did in 1855, though, since he still has time left on his lease. Solve for his optimal choice of  $C_2$  and  $L_2$ , as well as his indirect utility as a function of model parameters.
  3. At the end of the year, Rockefeller's lease is finally up, so he can choose housing freely. Solve for his new choices of  $L_3, C_3$ , and  $H_3$ , as well as his indirect utility. Show formally that the utility function is convex in the price of housing  $r$ .

4. In which of the time periods does Rockefeller work the most? The least?  
What is the intuition for these effects? Would the result that  $L$  is increasing in the wage change if we changed the utility function?

**Reading Assignments: NS:** Chapters 2,3,4.