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Microeconomic Theory (501b)

Problem Set 9. Signalling - Adverse Selection

3/30/10

This problem set is due on Tuesday, 4/6/10.

1. **Gibbons 4.3 and 4.5 (Pooling and Separating Equilibrium)**
2. **Gibbons 4.7 (Hybrid Equilibrium)**
3. **Gibbons 4.14 (Pre-Trial Negotiation)**
4. **Signalling with Many Types.** Consider the signalling model we discussed in class with a modification regarding the ability type.

(a) Suppose there are now a finite number, k , of ability types

$$a \in \left\{ 1, 1 + \frac{1}{K}, \dots, 1 + \frac{K-1}{K}, 1 + \frac{K}{K} \right\},$$

with associated probabilities p_k .

- i. Identify the least cost separating equilibrium with the finite number of types. What is the education level provided by each type k .
 - ii. Can you construct semi-separating equilibria where the types $k = 1, \dots, l$ pool and all the types $k = l + 1, \dots, K$ separate from the pool and from each other.
- (b) Now consider the case of a continuum of types $a \in [1, 2]$ and a continuous density $f(a)$ about the interval $[1, 2]$. Construct the least cost separating equilibrium.

5. The owner of a small firm is contemplating selling all or part of his firm to outside investors. The profits from the firm are risky and the owner is risk averse. The owner's preferences over x , the fraction of the firm the owner retains, and p , the price "per share" paid by the outside investors, are given

by

$$u(x; \theta; p) = \theta x - x^2 + p(1 - x);$$

where θ is the value of the firm (i.e., expected profits). The quadratic term reflects the owner's risk aversion. The outside investors are risk neutral,

and so the payoff to an outside investor of paying p per share for $1 - x$ of the firm is then

$$\theta(1 - x) - p(1 - x).$$

There are at least two outside investors, and the price is determined by a first price sealed bid auction: The owner first chooses the fraction of the firm to sell, $1 - x$; the outside investors then bid, with the $1 - x$ fraction going to the highest bidder (ties are broken with a coin flip).

- (a) Suppose θ is public information. What fraction of the firm will the owner sell, and how much will he receive for it?
- (b) Suppose now θ is privately known by the owner. The outside investors have common beliefs, assigning probability $\alpha \in (0, 1)$ to $\theta = \theta_1 > 0$ and probability $1 - \alpha$ to $\theta = \theta_2 > \theta_1$. Characterize the separating perfect Bayesian equilibria. Are there any other perfect Bayesian equilibria?
- (c) Maintaining the assumption that θ is privately known by the owner, suppose now that the outside investors' beliefs over θ have support $[\theta_1, \theta_2]$, so that there is a continuum of possible values for θ . What is the initial value problem (differential equation plus initial condition) for characterizing separating perfect Bayesian equilibria?

Reading. Gibbons, Chapter 4. Salanie, Chapter 4.