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Microeconomic Theory (501b)

Problem Set 7. Bayesian Games and Adverse Selection

3/25/14

This problem set is due on Tuesday, 4/1/14.

1. Consider the battle of the sexes game:

	Opera	Baseball
Opera	2,1	0,0
Baseball	0,0	1,2

   (a) Compute the pure and mixed strategy equilibria of this complete information game.

   (b) Consider now a perturbed version of the game where

	Opera	Baseball
Opera	$2+\delta\varepsilon_1, 1+\delta\varepsilon_2$	$0+\delta\varepsilon_1, 0$
Baseball	$0, 0+\delta\varepsilon_2$	$1, 2$

   where $\varepsilon_i \sim U[-1,1]$ and $0 < \delta < 1$ and ε_i is private information to agent i. Show that as $\delta \to 0$, *all* equilibria of the complete information game can be obtained as pure strategy limits of the Bayesian game with private information. (Hint: The pure strategy equilibria can be obtained immediately for δ small enough, the mixed strategy equilibrium requires the computation.)

2. We now consider a variation of the coordination game we discussed in class. A large, that is a continuum, population with unit mass (so you can index player $i \in [0,1]$), must choose an action, "invest" or "not invest". There is a cost y to investing and assume $0 < y < 1$. There is a benefit 1 of investing (investment "succeeds") if at least proportion $1-\theta$ of the population invests (i.e., at most θ do not invest), and as before state $\theta \in \mathbb{R}$ represents "fundamentals" of the economy.

   (a) Describe the complete information Nash equilibria as a function of the state θ which is assumed to be common knowledge among the agents.

(b) Suppose now that the players only have private and noisy information, that is we assume that

$$\theta \sim \mathcal{U}[-M, M]$$

for large $M > 0$, but each player only observe a signal x_i

$$x_i = \theta + \frac{1}{z}\varepsilon_i, \quad \varepsilon_i \sim \mathcal{U}\left[-\frac{1}{2}, \frac{1}{2}\right]$$

and so z represent the accuracy of the signal x_i. Suppose again that the agents pursue a threshold strategy of the form:

$$s_i(x_i) = \begin{cases} I & \text{if} \quad x_i > \widehat{x} \\ N & \text{if} \quad x_i < \widehat{x} \end{cases}.$$

i. Describe the proportion of agent that invest in the true state θ if the threshold is $\widehat{x}$ and the accuracy is z.

ii. Find the critical state $\widehat{\theta}$ where investment will succeed if the threshold is $\widehat{x}$ and the accuracy is z.

iii. Identify the best response condition where the marginal investor is indifferent between investing and not investing (and hence under the assumption of symmetry across the players) identify the Bayes Nash equilibrium strategy.

3. Local and Global Constraints with Spence Mirrlees Preferences.

(a) Exercise 2.1 in Salanie.

(b) Show that you can extend the argument to all preferences satisfying Spence-Mirrlees preferences, which are characterized by (supposing differentiability)

$$\frac{\partial u(\theta, q)}{\partial q} > 0 \text{ and } \frac{\partial^2 u(\theta, q)}{\partial q \partial \theta} > 0.$$

is sufficient to consider the case with finitely many types.

(c) Consider the finite model for the product case

$$u(\theta, q) = \theta \cdot q$$

and solve the optimal quality provision problem of the seller.

i. As a first intermediate step describe the net utility that each type gets after you replaced the transfers by appealing to the appropriate incentive and participation constraint

ii. As a second intermediate step describe the net revenue that the seller gets from the provision of quality q_k to type θ_k.

Reading MWG: 13 (except 13C), 14, S (=Salanie) 1,2 and 3