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Microeconomic Theory (501b)

Problem Set 5. Bayesian Games

3/4/14

This problem set is due on Tuesday, 3/25/14.

1. **(Market for Lemons)** Here I ask that you work out some of the details in perhaps the most famous of all information economics models. By contrast to discussion in class, we give a complete formulation of the game. A seller is privately informed of the value v of the good that she sells to a buyer. The buyer's prior belief on v is uniformly distributed on $[x, y]$ with $0 < x < y$. The good is worth $\frac{3}{2}v$ to the buyer.

   (a) Suppose the buyer proposes a price p and the seller either accepts or rejects p. If she accepts, the seller gets payoff $p - v$, and the buyer gets $\frac{3}{2}v - p$. If she rejects, the seller gets v, and the buyer gets nothing. Find the optimal offer that the buyer can make as a function of x and y.

   (b) Show that if the buyer and the seller are symmetrically informed (i.e. either both know v or neither party knows v), then trade takes place with probability 1.

   (c) Consider a simultaneous acceptance game in the model with private information as in part a, where a price p is announced and then the buyer and the seller simultaneously accept or reject trade at price p. The payoffs are as in part a. Find the p that maximizes the probability of trade.

2. **(Akerlof's Lemon in a General Equilibrium Model)** Suppose there is a continuum of buyers with a characteristic $v_b \in [0, 1]$ distributed according to a distribution function $G(v_b)$. There is also a continuum of sellers with mass one, all with the same characteristic $v_s \in (0, 1)$. Each seller owns an object of value $\theta \in [0, 1]$ which is distributed according to a distribution function $F(\theta)$ on the unit interval, independent across sellers. The value θ is known to the seller and unknown to the buyer. The valuations for the object are

$$u_s = \theta v_s$$

   and

$$u_b = \theta v_b.$$

(a) Determine the efficient volume of trade and the efficient mix of sellers (in terms of types of θ) and buyer (in terms of types of v_b).

(b) Suppose for the moment that the characteristic of each object is observable and hence that information is symmetric across sellers and buyers. What would be the equilibrium price for the object with value θ be and would trade be efficient. (Hint: The equilibrium allocation is going to involve a positive assortative matching of the form that buyers with higher willingness to pay, i.e. higher v_b will buy higher quality products, i.e. higher θ. The description of the equilibrium now includes a matching function $v_b(\theta)$ and a pricing function $p(\theta)$.

(c) Under asymmetric information, compute the demand and supply functions. Explain why the demand is not necessarily downward sloping (despite the absence of income effects.)

(d) Solve for the competitive equilibrium for f and g uniform on $[0,1]$. Verify that trade is suboptimal.

(e) Argue that, in general, there can exist multiple equilibria. Then assume that it is the case. Show that a higher price equilibrium Pareto dominates a lower price one. Compare with the first fundamental theorem of welfare economics.

(f) Show that a minimum quality standard $\theta_0 > 0$, if enforceable, may improve welfare.

3. Consider the following game of public good provision with private costs $c_i \geq 0$:

	C	D
Contribute	$1-c_1, 1-c_2$	$1-c_1, 1$
Don't Contribute	$1, 1-c_2$	$0, 0$

where the cost c_i is i.i.d. distributed with a uniform density on $[0,2]$.

(a) Define the notion of a mixed strategy in this Bayesian game and define the notion of a Bayesian equilibrium for this game.

(b) Compute a Bayesian Nash equilibrium of this game in pure strategies.

(c) Consider the same public good provision game, but now analyze it with the solution concept of iteratively eliminating strictly dominated strategies rather than the Bayesian equilibrium. Where does the process of iterative elimination of strictly dominated strategies lead to?

4. This question asks you to show the equivalence ex ante and interim definitions of (Bayesian) Nash equilibrium in static incomplete information games with a finite number of types. Consider a game consisting of a set of players $1, ..., I$; each player has a finite set of actions A_i; each player has a type $t_i \in T_i$. The profile of types $t \equiv (t_1, ..., t_I)$ is chosen according to a

probability distribution p with $p(t) > 0$ for all $t \in T$. Payoff function for player i is a function $g_i : A \times T \to \mathbb{R}$. An incomplete information game strategy for player i is a function $s_i : T_i \to A_i$. Write S_i for the set of such strategies, let $S = S_1 \times \cdots \times S_I$, and write $s(t) = (s_1(t_1), ..., s_I(t_I))$.

(a) Show that strategy profile s^* is an ex ante Bayesian Nash equilibrium of this game if and only if it is an interim Bayesian Nash equilibrium.

(b) Does the result still hold if $p(t)$ is only required to satisfy $p(t) \geq 0$? If not, which part of the result fails to hold?

5. Ex-post version of the Bayesian-Nash equilibrium.

   (a) Give a natural definition of an ex-post version of the Bayesian-Nash equilibrium, where ex post refers to the fact that the equilibrium conditions are evaluated conditional on knowing the entire type profile, and not only the own type as in the interim version of the Bayesian-Nash equilibrium.

   (b) Given your definition of an ex-post Bayes Nash equilibrium, what is the relationship with respect to the interim version of the Bayesian-Nash equilibrium. Either state results or give examples that demonstrate how these notions are different.

   (c) Give a definition of dominant strategy for Bayesian games. What is the relationship between ex post Bayes Nash equilibrium and an equilibrium in dominant strategies.

6. Consider the second price sealed bid auction with private values we discussed in class. We showed that truthful bidding is a Bayes Nash equilibrium for all distribution, in fact it is a Bayes-Nash equilibrium in dominant strategies. Do other Bayes Nash equilibria exist. Do other ex post Bayes-Nash equilibria exist?

Reading MWG: 8E. G: Chapter 3