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Microeconomic Theory (501b)

Problem Set 10. Auctions and Moral Hazard

4/15/14

This problem set is due on Tuesday, 4/22/14.

1. **Budget Balanced Mechanism.** We say that the mechanism $(x(\cdot), t(\cdot))$ satisfies *ex-post budget balance* if for all θ,

$$\sum_{i=1}^{N} t_i(\theta) = 0.$$

For an environment with independently distributed types, we define the D'Aspremont Gerard-Varet mechanism or expected externality mechanism by $x^*(m)$, the social choice function that maximizes the sum of the utilityes, and transfers

$$t_i(m) = -\mathbb{E}_{\theta_{-i}}\left[\sum_{j\neq i} v_j(x^*(m_i, m_{-i}), m_j) + \tau_i(m_{-i})\right].$$

In this direct mechanism, m_i refers to the message and θ_i is the type of agent i. The social choice function $x^*(\cdot)$ maximizes the sum of the agents' welfare. The term $\tau_i(m_{-i})$ is function that depends only the messages of the other agents.

(a) Establish that $m_i = \theta_i$ is an optimal announcement in a Bayesian Nash equilibrium.

(b) We can define the expected externality of agent i with type m_i as:

$$\mathcal{E}_i(m_i) = \mathbb{E}_{\theta_{-i}} \sum_{j\neq i} v_j(x^*(m_i, \theta_{-i}), \theta_j),$$

and set

$$\tau_i(m_{-i}) = \frac{\sum_{j\neq i} \mathcal{E}_j(m_j)}{N-1}.$$

Show that this mechanism satisfies the balanced budget requirement ex post for every type profile realization.

(c) Is this mechanism also dominant strategy implementable ?

(d) Compute the transfers exactly for the single unit auction with two bidders and uniformly distributed values. Do the transfers guarantee the individiual rationality constraints for all the type profile of the agents?

2. **Nonlinear Pricing.** Consider the problem of an optimal menu when $v(\theta, q) = \theta\sqrt{q}$ and $c(q) = q$ and the distribution is given by the uniform distribution on the unit interval.

   (a) Compute the revenue maximizing direct mechanism, which associates to every reported type θ a pair $(q(\theta), t(\theta))$ of quantities $q(\theta)$ and prices $t(\theta)$, or $\theta \mapsto (q(\theta), t(\theta))$

   (b) Translate the direct mechanism into an indirect mechanism, in particular to a nonlinear pricing mechanism $(q, t(q))$ which associates to every q a $t(q)$, or $q \mapsto t(q)$.

   (c) What can you sat about $t(q)/q$, i.e. the price per quantity as the quantity increases.

   (d) Establish that the revenue maximizing direct mechanism could also be implemented by a menu of two part-tariffs, $(T(\theta), p(\theta))$, where $T(\theta)$ is the fixed fee and $p(\theta)$ is the price per unit (i.e. if a customer chooses a particular two-part tariff then he has to pay $T(\theta)$ independent of the quantity he purchase, but can then buy as many units as he likes at the price $p(\theta)$ per unit. (The concavity of $t(q)$ might be useful.)

3. Consider the regular moral hazard model with a risk-neutral principal and a risk averse agent. The agent can choose between two effort levels, $a_i \in \{\underline{a}, \overline{a}\}$ with associated cost $c_i \in \{\underline{c}, \overline{c}\} = \{0, c\}$, with $c > 0$. Each action generates stochastically one of two possible profit levels, $x_i \in \{\underline{x}, \overline{x}\}$ with $p(\underline{x}|\underline{a}) > p(\underline{x}|\overline{a})$. The utility function of the agent is

$$u(w, c_i) = \ln w - c_i.$$

   The value of the outside option is normalized to 0. (Risk-neutrality of the principal implies that his payoff function is $x - w$.)

   (a) Carefully describe the principal-agent problem when the principal wishes to implement the high effort level $\overline{a}$.

   (b) Solve explicitly for the optimal wage schedule to be offered to the agent which implements the high effort level $\overline{a}$.

   (c) **(Renegotiation 1)** Consider now the following extension to the moral hazard problem. **After** the principal has offered an (arbitrary) wage schedule and the agent has chosen and performed an (arbitrary) effort level, but **before** x is revealed, the principal has the possibility to offer a new contract to the agent. The agent can either accept or

reject the new offer. If he accepts the new contract, then it replaces the old contract, if he rejects the new contract, then the old one remains in place. Show that there is no subgame perfect equilibrium of the game where $\Pr(a = \overline{a}) = 1$. (Hint: Consider the optimal contract after a has been chosen but before x has been realized.)

(d) **(Renegotiation 2)** Consider now the following modification to the renegotiation problem above. Suppose now that the agent can make the new proposal and the principal can either accept or reject the new offer. Suppose further that the principal can observe the action at the time of the new proposal but that the contract can only depend on x and not on a. The timing is otherwise unchanged. Derive the subgame perfect equilibrium of the principal-agent problem. What can you say about the efficiency of the arrangement.

4. **(Moral Hazard in Teams, (Holmstrom 1982))** Consider the following moral hazard problem with many agents. Suppose output is one-dimensional, deterministic and concave in a_i and depends on the effort of the n agents in the team:

$$x = x(a_1, a_2, ..., a_n).$$

Each agent i has convex effort cost $c_i(a_i)$. Each agent observe his effort and the joint output x. A contract among the agents is a set of wages $\{w_i(x)\}_{i=1}^n$, which depend only on the publicly observable output x. The set of wages have to be budget balanced, or

$$\sum_{i=1}^{n} w_i(x) = x$$

for all x. (Think of the team as a cooperative or partnership). The utility function of each agent is $w_i(x) - c_i(a_i)$.

(a) Describe the first-best allocation policy a^*.

(b) Suppose (without loss of generality) that the team is restricted to using differentiable wages, or $w_i'(x)$ exists for all x and i. Show that there is no wage schedule which allows the team to realize the first best policy.

(c) Next, introduce an $n+1-th$, who does not deliver any effort to the team, but can be entitled to transfers (the "principal" or "budget-breaker"). Show that you can now design a wage schedule, not necessarily differentiable, such that

$$\sum_{i=1}^{n+1} w_i(x) = x$$

for all x. In fact, you can design the contract such that even

$$\sum_{i=1}^{n} w_i(x) = x$$

holds on the equilibrium path, but not off the equilibrium path.

5. (**First Order Stochastic Dominance versus Monotone Likelihood Ratio ((Milgrom 1981))).**

   (a) Define monotone likelihood ratio and first-order stochastic dominance.

   (b) Show that with two outcomes, the two notion are equivalent.

   (c) Give an example to show that the equivalence does not hold in general. Show that the monotone likelihood ratio property implies first order stochastic dominance.

Reading MWG: 23, S (=Salanie) 2 and 3

References

Holmstrom, B. (1982): "Moral Hazard in Teams," *Bell Journal of Economics*, 13, 224–240.

Milgrom, P. (1981): "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12, 380–391.