ENTRY AND VERTICAL DIFFERENTIATION

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Entry and Vertical Differentiation[1]

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This paper analyzes the entry of new products into vertically differentiated markets where an entrant and an incumbent compete in quantities. The value of the new product is initially uncertain and new information is generated through purchases in the market. We derive the (unique) Markov perfect equilibrium of the infinite horizon game under the strong long run average payoff criterion.The qualitative features of the optimal entry strategy are shown to depend exclusively on the relative ranking of established and new products based on current beliefs. Superior products are launched relatively slowly and at high initial prices whereas substitutes for existing products are launched aggressively at low initial prices.The robustness of these results with respect to different model specifications is discussed. *Journal of Economic Literature* Classification Numbers: C72, C73, D43, D83.

Key Words: entry; duopoly; quantity competition; vertical differentiation; Bayesian learning; Markov perfect equilibrium; experimentation; experience goods.

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1. INTRODUCTION

1.1. *Motivation*

In this paper, we analyze the optimal entry strategies for different types of experience goods in a dynamic Cournot duopoly with vertically differentiated buyers. Our main goal is to obtain a characterization of the features of the new product that lead to qualitatively different entry strategies. We show that a new product that represents a certain improvement to an existing product is launched in the market at prices above the static equilibrium level and sales quantities below the static level. A new product that has a positive probability of being the leading brand in the market, but also a positive probability of being revealed to be inferior to the current product, is launched with a more aggressive strategy where the initial prices are low and initial sales exceed the static equilibrium quantities.

The firms compete in a continuous time model with an infinite horizon. The uncertainty about the new product is common to all buyers in the market. Additional information about the quality of the new product is generated only through experiments, i.e., through purchases in the market. The information generated is assumed to be public, and while the exact mechanism of information transmission is left unmodelled it is motivated by considerations such as word of mouth communication between the buyers and consumer report services. As a consequence all buyers have identical beliefs about the new product, and we can represent the stage game as a vertically differentiated quantity game parametrized by the common belief about the new product. Examples of markets where the assumptions of common value (aside from the aspect of vertical differentiation) and common information may be valid include markets for transportation or communications services. To take a precise example, suppose buyers evaluate the services of an airline carrier on the basis of the probability of on-time departure and arrival and/or the probability of lost or misplaced baggage. The uncertainty about the quality of the service is then common to all customers of the airline, provided that the uncertainties are not route specific. The (expected) performance or reliability of the new service is then best predicted by aggregate and publicly available statistics such as the percentage of on-time performances by an airline. In particular, all idiosyncratic experiences are of equal value in providing information and can therefore be replaced by sufficient aggregate statistics. Our model only requires that all consumers rank reliability of the airlines or congestion in the provision of internet services according to the same scale, yet they can differ in their willingness to pay for different service qualities. Hence the model displays vertical but not horizontal differentiation.

We have chosen a model of quantity competition as the stage game. With this choice, we extend the scope of viable new products. In particular, quantity competition allows for the possibility of launching an innovation which brings the two competitors closer to each other without a change in the leadership. In a model of price competition, such innovations would never be profitable, and as a result improved substitutes would never be observed. In those models, the profits of both firms vanish as the substitutability of the two products increases, and as a result the static profit functions of the two firms are nonmonotonic in the level of differentiation. We believe that a model where each firm's profit is increasing in its own quality is better suited for a dynamic investigation of a market with vertical differentiation.

In order to simplify the analysis, we assume that there is no discounting. As we want to stay close to the model with small discounting, we use the strong long-run average criterion as defined in Dutta [10] as the intertemporal evaluation criterion. This criterion can be justified as the limit of models where the discount rate is tending to zero, and it retains the recursive formulation of standard discounted dynamic programming. Under the assumptions of no discounting and quantity competition, it is surprisingly simple to examine the Markov perfect equilibria of the model. In Section 5, we show that for quite general demand structures the comparisons between static and dynamic equilibrium policies can be based exclusively on information about static payoff functions. It is hoped that the simplicity of the technique of undiscounted dynamic programming as used here will prove useful in other applications beyond the scope of this paper.

In Section 4, we assume that the underlying stage game is the standard linear model used in the literature on vertical differentiation. This allows us to interpret the dynamic equilibria in an economically intuitive manner. Using the curvature properties of the stage game profit functions, we show that aggressive entry corresponds to relatively low (current) expected quality of the entrant's product while cautious entry corresponds to higher expected quality. In the linear model, we can also solve the dynamic equilibrium policies explicitly and, as a result, we get a set of empirically testable predictions for the model.

The paper proceeds as follows. Section 2 introduces the basic model and the learning environment. Section 3 derives the benchmark results of the static duopoly game. The main results are then presented for the standard linear demand specification in Section 4 where we derive the Markov perfect equilibrium of the intertemporal game. In Section 5, we extend the model beyond the linear specification and show that the qualitative conclusions extend to much more general demand structures. All the proofs are relegated to an appendix.

Initially all market participants have a common prior belief α_0 that the new product has a high valuation, or

$$\alpha_0 = \Pr(\mu = \mu_H) .$$

The expected value given a belief $\alpha(t)$ in period t is denoted by $\mu(\alpha(t))$, where

$$\mu(\alpha(t)) \triangleq \alpha(t)\, \mu_H + (1-\alpha(t))\, \mu_L.$$

Since the buyers are nonatomic they have no individual effect on prices and quantities, and as a result they choose according to their myopic preferences at each stage.[4] To complete the description of the stage game payoffs we need to specify the profit functions for the two firms. The flow profits resulting from a vector of quantities $(q_I(t), q_E(t))$ are given by $p_i(t)\, q_i(t)$ for $i = I, E$, where the $p_i(t)$ are obtained from static market clearing conditions.

The uncertainty about the new product can only be resolved over time by experience with the new product. We assume that the evolution of the belief about the quality of the new product is governed by the diffusion process

$$d\alpha(t) = \sqrt{\frac{q_E(t)\, \alpha(t)(1-\alpha(t))(\mu_H - \mu_L)}{\sigma^2}}\, dB(t), \tag{1}$$

where $B(t)$ is the standard Wiener process. In the appendix, we provide a microfoundation for this particular form of the evolution of the beliefs. There we derive the diffusion process $\alpha(t)$ from a discrete time model with a finite number of buyers, where each buyer is sampling from a normal distribution with known variance σ^2 and unknown mean μ, which is either μ_L or μ_H.

Observe that, being a posterior belief, $\alpha(t)$ follows a martingale; i.e., has a zero drift. The variance of the process is at its largest when $\alpha(t)$ is away from its boundaries as the marginal impact of new information is at its largest when the posterior is relatively imprecise. The economic assumption behind the form of this particular process is that the variance in the posterior belief is linear in the quantity of sales by the entrant and thus the informativeness of the market experiment grows linearly in the sales of the entrant. The remaining term in the expression, $(\mu_H - \mu_L)/\sigma^2$, is sometimes referred to as the signal-to-noise ratio as it measures the strength of the

[4] We are implicitly assuming that the firms' information sets consist of all past market observations, i.e., all past prices and quantities.

signal $\mu_H - \mu_L$ compared to the inherent noise in the observation structure, σ^2. Define

$$\Sigma(\alpha(t)) \equiv \sqrt{\frac{\alpha(t)(1-\alpha(t))(\mu_H - \mu_L)}{\sigma^2}}.$$

From (1), we see that as long as $q_E(t)$ is bounded away from 0 for all t, $\alpha(t)$ converges to $\alpha^* \in \{0, 1\}$ almost surely. In fact, the convergence is fast enough to make the limit

$$\lim_{T \to \infty} \mathbb{E}_{\alpha_0}\left[\int_0^T [\phi(\alpha(t)) - \phi(\alpha^*)]\, dt\right],$$

where $\phi(\cdot)$ is an arbitrary continuous and piecewise smooth function of α, finite almost everywhere. This result allows us to use the strong long-run average as the intertemporal evaluation criterion in our model.

3. STATIC EQUILIBRIUM

In this section, we derive some of the basic equilibrium properties in the static model for the case where $f(\theta)$ is the uniform density. The safe product is worth s and the new product is worth $\mu(\alpha)$ for a given α. In the description of the equilibrium conditions we shall assume that $\mu(\alpha) \leqslant s$. The corresponding results for $\mu(\alpha) > s$ are symmetric and stated in the relevant proposition as well. Define α_m as the belief at which the expected value of the new product is equal to the established one:

$$\mu(\alpha_m) = s \Leftrightarrow \alpha_m = \frac{s - \mu_L}{\mu_H - \mu_L}.$$

The static prices and quantities are denoted by P_E, P_I, Q_E, and Q_I for the entrant and the incumbent respectively. The *equilibrium* prices and quantities are denoted by $P_i(\alpha)$ and $Q_i(\alpha)$ as we are interested in the comparative static behavior of the equilibrium variables as a function of the belief α. The equilibrium conditions are given by the profit maximization conditions of the firms and the indifference conditions of the marginal buyers. The latter can be stated as

$$(1 - Q_I)\, s - P_I = (1 - Q_I)\, \mu(\alpha) - P_E$$

and

$$(1 - Q_I - Q_E)\, \mu(\alpha) - P_E = 0.$$

The first indifference condition implies that at the equilibrium prices buyers with valuations $\theta \in [1-Q_I, 1]$ prefer the incumbent. The second indifference condition implies that buyers with valuations $\theta \in [1-Q_I-Q_E, 1-Q_I)]$ prefer the entrant. It also follows that all buyers get a nonnegative expected utility from their purchases, but the segment with the lowest valuations may not buy at all. The market clearing prices for given quantities $\{Q_E, Q_I\}$ are

$$P_E = \mu(\alpha)(1-Q_I-Q_E)$$

and

$$P_I = s(1-Q_I) - \mu(\alpha)\, Q_E.$$

Since the derivation of the static equilibrium is completely standard, the derivation of the results is relegated to the appendix.

PROPOSITION 1 (Static Policies).

1. *$P_E(\alpha)$, $Q_E(\alpha)$, and $P_E(\alpha)\, Q_E(\alpha)$ are increasing in α.*
2. *$P_I(\alpha)$, $Q_I(\alpha)$, and $P_I(\alpha)\, Q_I(\alpha)$ are decreasing in α.*

Proof. See the appendix. ∎

As expected, the quantity and the price of the entrant are increasing in α. The entrant can increase his sales as well as his margins as the quality is improved. The incumbent responds to an increase in the value of the competing product by lowering his sales as well as his margins. It is worthwhile to point out that the monotonicity result extends over the entire range of posterior beliefs and holds also around the point α_m where the leadership between the two firms is changing. This is one instance where the model with quantity competition behaves in a more regular manner than the one with price competition, which displays nonmonotonicities in the prices and quantities around the switching point α_m.

PROPOSITION 2 (Curvatures).

1. *For $\mu(\alpha) < s$,*
   (a) *$P_E(\alpha)$, $Q_E(\alpha)$, and $P_E(\alpha)\, Q_E(\alpha)$ are convex in α;*
   (b) *$P_I(\alpha)$, $Q_I(\alpha)$, and $P_I(\alpha)\, Q_I(\alpha)$ are concave in α.*
2. *For $\mu(\alpha) > s$,*
   (a) *$P_E(\alpha)$, $Q_E(\alpha)$, and $P_E(\alpha)\, Q_E(\alpha)$ are concave in α;*
   (b) *$P_I(\alpha)$, $Q_I(\alpha)$, and $P_I(\alpha)\, Q_I(\alpha)$ are convex in α.*

Proof. See the appendix. ∎

An intuition for the curvature as well as for the change in the curvature of the policies and revenues can be given as follows. For low posterior beliefs where $\mu(\alpha) < s$, a marginal increase in α increases the profit of the entrant through two channels. First, it increases the price for fixed quantities. This direct effect is the same at all levels of α as long as the quantities supplied are unchanged. There is also the indirect effect from a stronger competitive position of the entrant and the corresponding reduction in the quantity of the incumbent. This effect is strongest when α is close to α_m, and vanishes for very low values of α. The combination of these two effects leads to a convex profit function as long as $\mu(\alpha) < s$. As α increases beyond α_m, the position of the entrant resembles increasingly that of a monopolist. The indirect effect then becomes weaker and it is only the ability of the new firm to increase its prices which increases its profits.

4. DYNAMIC EQUILIBRIUM

In Subsection 4.1 we consider the dynamic optimization problems of the firms, and we also introduce the model without discounting using the strong long-run average criterion for evaluating payoffs. In Subsection 4.2 we then characterize the unique equilibrium and the associated equilibrium policies.

4.1. *Dynamic Optimization*

In a discounted model the entrant's value function is the solution to the Hamilton–Jacobi–Bellman equations[5]

$$rV_E(\alpha) = \max_{q_E(\alpha)} \{p_E(\alpha)\, q_E(\alpha) + \tfrac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha)\}. \tag{2}$$

If we tried to solve this equation jointly with the corresponding one for the incumbent, we would obtain a nonlinear system of second-order differential equations. An analytical solution of such systems is, in general, impossible. For this reason we consider the limiting case as the discount rate vanishes or $r \to 0$ and then derive the equilibrium policies under the strong long-run average criterion.[6]

[5] See Dixit and Pindyck [9] or Harrison [17] for a complete derivation of the dynamic programming equation in continuous time when uncertainty is represented by a Brownian motion.

[6] See Dutta [10] for a detailed analysis of the link between optimality criteria under discounting and no discounting.

The *strong* long-run average criterion has the important property that the optimal policies under this criterion are the unique limits to the associated policies under discounting. The equilibrium policies to be derived therefore maintain all the qualitative properties of the equilibrium with small, but positive, discount rates $r > 0$. In particular, they preserve the intertemporal tradeoff of the experimentation policies under discounting.

We start by fixing the policies of all other players to a set of arbitrary (Markovian) policies and consider the decision problem of the entrant. In the next subsection, we return to the full equilibrium problem. The reformulation for the incumbent only requires the obvious substitutions. The long run average payoff for the entrant under an initial belief α_0 is given by

$$v_E(\alpha_0) = \sup_{q_E(\alpha(t))} \lim_{T \to \infty} \frac{1}{T} \mathbb{E}_{\alpha_0} \left[\int_0^T q_E(\alpha(t))\, p_E(\alpha(t))\, dt \right].$$

Since $\alpha(t)$ converges almost surely to zero or one, the long-run average starting at any arbitrary belief α_0 is given by[7]

$$v_E(\alpha_0) = (1-\alpha_0)\, v_E(0) + \alpha_0 v_E(1)\,.$$

As $v_E(0)$ and $v_E(1)$ are simply the full information payoffs associated with the static payoffs at $\alpha = 0$ or $\alpha = 1$, the long-run average can be computed exclusively on the basis of the static problem. In contrast, the *strong* long-run average is defined through the optimization problem:

$$V_E(\alpha_0) = \sup_{q_E(\alpha(t))} \lim_{T \to \infty} \mathbb{E}_{\alpha_0} \left[\int_0^T (q_E(\alpha(t))\, p_E(\alpha(t)) - v_E(\alpha(t)))\, dt \right]. \tag{3}$$

Thus the strong long-run average criterion maximizes the expected return net of the long-run average. The limit as $T \to \infty$ in (3) is well-defined and finite. The strong long-run average hence discriminates between policies based on finite time intervals as well. The infinite horizon problem (3) can be represented by a dynamic programming equation,[8]

$$v_E(\alpha) = \max_{q_E(\alpha)} \{ q_E(\alpha)\, p_E(\alpha) + \tfrac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha) \}. \tag{4}$$

The difference between the dynamic programming equation under discounting (2) and under no discounting (4) is simply that the flow payoff, $rV_E(\alpha)$, is replaced by the long-run average payoff, $v_E(\alpha)$, whereas the

[7] It is easy to see that the sales of both firms are bounded away from zero at all points in time.

[8] For the full details on the strong long-run average in continuous-time dynamic programming models see Krylov [21], and for a derivation of Bellman's equation for the problem in a related application see Bolton and Harris [6].

right-hand side of the equation remains identical. However, as the long-run average is independent of the current policy $q_E(\alpha)$, we can rewrite (4) to read

$$0 = \max_{q_E(\alpha)} \{q_E(\alpha)\, p_E(\alpha) - v_E(\alpha) + \tfrac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha)\}.$$

After dividing the entire expression through $q_E(\alpha)$ (assuming that $q_E(\alpha) > 0$ can be guaranteed), the optimality equation can be rewritten as

$$0 = \max_{q_E(\alpha)} \left\{ p_E(\alpha) - \frac{v_E(\alpha)}{q_E(\alpha)} \right\} + \frac{1}{2} \Sigma^2(\alpha)\, V_E''(\alpha). \tag{5}$$

This last expression demonstrates the advantage of analyzing the undiscounted program rather than the discounted one. The first-order conditions do not involve the second derivative of the value function any more. The only modification relative to the static program is the introduction of the long-run average, but as we saw above it can be computed on the basis of the static equilibrium as well.

4.2. *Equilibrium Analysis*

Consider now the entire set of equilibrium conditions under no discounting. The dynamic programming equation for the entrant is

$$0 = \max_{q_E(\alpha)} \{p_E(\alpha)\, q_E(\alpha) - v_E(\alpha) + \tfrac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha)\}, \tag{6}$$

and for the incumbent the equation is by extension

$$0 = \max_{q_I(\alpha)} \{p_I(\alpha)\, q_I(\alpha) - v_I(\alpha) + \tfrac{1}{2} q_E(\alpha)\, \Sigma^2(\alpha)\, V_I''(\alpha)\}, \tag{7}$$

where $v_I(\alpha)$ and $v_E(\alpha)$ are the long-run average payoffs of the sellers.

Since each buyer is of negligible size, her decision doesn't influence the market experiment and hence her value of information is independent of her decision. The purchase decision of each buyer is therefore exclusively determined by the current payoff offered by the various alternatives. In consequence, the sorting of buyers in the intertemporal equilibrium will display the same structure as in the static equilibrium: for $\mu(\alpha) \leqslant s$,

$$\begin{aligned} p_E(\alpha) &= \mu(\alpha)(1 - q_I(\alpha) - q_E(\alpha)), \\ p_I(\alpha) &= s(1 - q_I(\alpha)) - \mu(\alpha)\, q_E(\alpha); \end{aligned} \tag{8}$$

and symmetrically for $\mu(\alpha) > s$,

$$\begin{aligned} p_E(\alpha) &= \mu(\alpha)(1-q_E(\alpha))-sq_I(\alpha), \\ p_I(\alpha) &= s(1-q_I(\alpha)-q_E(\alpha)). \end{aligned} \tag{9}$$

DEFINITION 1 (Markov Perfect Equilibrium). A Markov perfect equilibrium is a pair of functions $\{q_E(\alpha), q_I(\alpha)\}$ such that the equations (6)–(9) are satisfied for all $\alpha \in [0, 1]$.

By substituting the prices into the value functions, we obtain the value functions of the firms as a function of the quantities $\{q_E(\alpha), q_I(\alpha)\}$,

$$0 = \max_{q_E(\alpha)} \{(\mu(\alpha)(1-q_E(\alpha))-m(\alpha)\, q_I(\alpha))\, q_E(\alpha)-v_E(\alpha)+\tfrac{1}{2}\, q_E(\alpha)\, \Sigma^2(\alpha)\, V_E''(\alpha)\}, \tag{10}$$

and

$$0 = \max_{q_I(\alpha)} \{(s(1-q_I(\alpha))-m(\alpha)\, q_E(\alpha))\, q_I(\alpha)-v_I(\alpha)+\tfrac{1}{2}\, q_E(\alpha)\, \Sigma^2(\alpha)\, V_I''(\alpha)\}. \tag{11}$$

We can then solve for the unique equilibrium quantities by the methods from the previous subsection to get

$$q_E(\alpha) = \sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}} \tag{12}$$

and

$$q_I(\alpha) = \frac{1}{2} - \frac{1}{2}\frac{m(\alpha)}{s}\sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}, \tag{13}$$

where $m(\alpha) \triangleq \min\{s, \mu(\alpha)\}$. It can be verified that the quantity $q_I(\alpha)$ is continuous at $\alpha = \alpha_m$, but not differentiable. The equilibrium prices $p_E(\alpha)$ and $p_I(\alpha)$ follow from the indifference conditions (8) and (9) of the marginal buyers. The monotonicity properties of quantities and prices as a function of the posterior belief α, which we observed in the static equilibrium (as a comparative static result), are preserved in the dynamic model.

PROPOSITION 3 (Prices and Quantities).

1. *$p_E(\alpha)$, $q_E(\alpha)$, and $p_E(\alpha)\, q_E(\alpha)$ are increasing in α.*
2. *$p_I(\alpha)$, $q_I(\alpha)$, and $p_I(\alpha)\, q_I(\alpha)$ are decreasing in α.*

Proof. See the appendix. ∎

Next we want to contrast the dynamic entry policies with the static policies. To this end, suppose that the value of information to the entrant is zero at some critical posterior belief α_c, or $V''_E(\alpha_c) = 0$. From (10), his dynamic best-response function at α_c is identical to the static one. As intertemporal considerations in terms of $V''_E(\alpha)$ or $V''_I(\alpha)$ enter the best response function of the incumbent only indirectly through the choices of the entrant (see (11)), it follows that if $V''_E(\alpha_c) = 0$, then necessarily $q_i(\alpha_c) = Q_i(\alpha_c)$ and $p_i(\alpha_c) = P_i(\alpha_c)$ for all $i \in \{E, I\}$. Moreover, since the dynamic programming equation (10) has to hold it follows that at α_c the *flow revenues* (static or intertemporal) of the entrant have to be equal to his *long-run average* $v_E(\alpha_c)$.

Recall that the long-run average $v_E(\alpha)$ at α_c is the expected value of the static equilibrium revenues at $\alpha = 0$ and $\alpha = 1$ weighted with $1-\alpha_c$ and α_c respectively. Thus, even if we don't know $V_E(\alpha)$ or $V_I(\alpha)$ we can find the points where static and intertemporal values coincide through a comparison of the static values with the long-run average. Conversely, at all points where static revenues and the long-run average diverge we can expect to see discrepancies between static and intertemporal policies.

PROPOSITION 4 (Single Crossing).

1. *The difference* $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$ *crosses zero at most once and only from below.*
2. *The critical point* α_c *satisfies* $\alpha_c > \alpha_m$.
3. *A necessary condition for crossing to occur is* $\mu_L < s < \mu_H$.
4. *A necessary and sufficient condition for crossing to occur is*

$$[P_E(0)\, Q_E(0)]' - v'_E(0) < 0, \quad \text{and} \quad [P_E(1)\, Q_E(1)]' - v'_E(1) < 0.$$

Proof. See the appendix. ∎

The proof proceeds by establishing the above properties first for the static revenues $P_E(\alpha)\, Q_E(\alpha)$ and then extending them to the intertemporal flow revenues $p_E(\alpha)\, q_E(\alpha)$. Thus there is at most one critical point where the value of information for the entrant is zero. As the equilibrium policies we derived earlier as well as the long-run average are continuous, it follows that the preference of the entrant toward information represented by $V''_E(\alpha)$ changes signs at most once. As the sign of the term $V''_E(\alpha)$ determines the bias in the intertemporal policy relative to the static policy, the proposition shows that this bias changes sign at most once, and in fact a necessary condition for the change is that there is uncertainty about the ranking of the alternatives, or $\mu_L < s < \mu_H$.

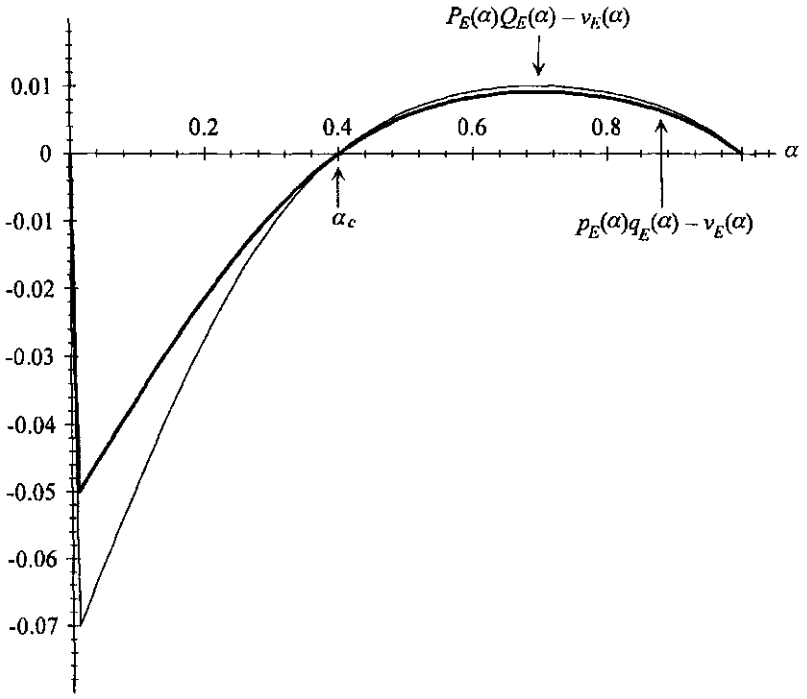

FIG. 1. Equilibrium revenue minus long-run average for entrant for $\mu_L = 99/100$, $s = 1$, and $\mu_H = 2$.

Observe that the necessary and sufficient condition for a change of sign is given entirely in terms of the static profit functions. The long-run average $v_E(\alpha)$ is linear in α and satisfies $v_E(\alpha) = P_E(\alpha)\, Q_E(\alpha)$ for $\alpha \in \{0, 1\}$. We showed earlier (in Proposition 2), that the static profit function of the entrant is convex whenever $\mu(\alpha) < s$ and concave whenever $\mu(\alpha) > s$. Hence it is sufficient to compare the local behavior of the static profit to the long-run average around the endpoints. The equilibrium revenue and long-run average for the entrant are displayed in Fig. 1 for the case that the necessary and sufficient condition is satisfied.

The value of information for the entrant is represented by the second derivative of the value function $V_E''(\alpha)$. The dynamic programming equation (10) immediately shows that the value of information has the opposite sign of $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$. This allows us to establish directly how the presence of market learning affects the equilibrium policies of the firms on either side of the critical value α_c.

PROPOSITION 5 (Static vs. Dynamic Strategies).

1. *For* $\alpha < \alpha_c$,

(a) $q_E(\alpha) > Q_E(\alpha)$ *and* $p_E(\alpha) < P_E(\alpha)$;

(b) $q_I(\alpha) < Q_I(\alpha)$ *and* $p_I(\alpha) < P_I(\alpha)$.

2. *For* $\alpha > \alpha_c$,

(a) $q_E(\alpha) < Q_E(\alpha)$ *and* $p_E(\alpha) > P_E(\alpha)$;

(b) $q_I(\alpha) > Q_I(\alpha)$ *and* $p_I(\alpha) > P_I(\alpha)$.

Proof. See the appendix. ∎

The behavior of the equilibrium prices and the quantities is displayed (in their differences) in Fig. 2 for the same environment as that in Fig. 2.

The curvature properties of the equilibrium policies provide us with valuable additional information about the intertemporal properties as the curvature properties can be directly translated into a time series profile by exploiting the fact that α is a martingale.

PROPOSITION 6.

1. $q_E(\alpha)$ *is concave in* α,
2. $p_E(\alpha)$ *is convex if* $\mu(\alpha) \leqslant s$ *and concave if* $\mu(\alpha) > s$,
3. $q_I(\alpha)$ *is convex in* α, *and*
4. $p_I(\alpha)$ *is convex in* α.

Proof. See the appendix. ∎

It is an immediate consequence of the previous proposition that $q_E(\alpha)$ is a submartingale, whereas $q_I(\alpha)$ and $p_I(\alpha)$ are supermartingales. Hence the expected sales and the expected prices of the incumbent rise over time whereas the expected sales of the entrant fall over time.

When we combine the time series behavior of the equilibrium with the properties of the equilibrium policies relative to their static counterparts, a rather complete picture regarding the entrance and deterrence behavior emerges. As the policies depend essentially on the current position of the firms in the quality spectrum, it is useful to consider the two polar cases relative to the intermediate case where $\mu_L < s < \mu_H$. If $\mu_L < \mu_H \leqslant s$ we refer to the new product as a *substitute*, and if $s \leqslant \mu_L < \mu_H$ then we refer to it as an *improvement*. A substitute is at best equal to the established product, whereas an improvement is at least as good as the established product. The first scenario may represent the introduction of a generic pharmaceutical or a no-name product, whereas the second may represent a new version of a current product with additional features whose (positive) contribution is yet uncertain.

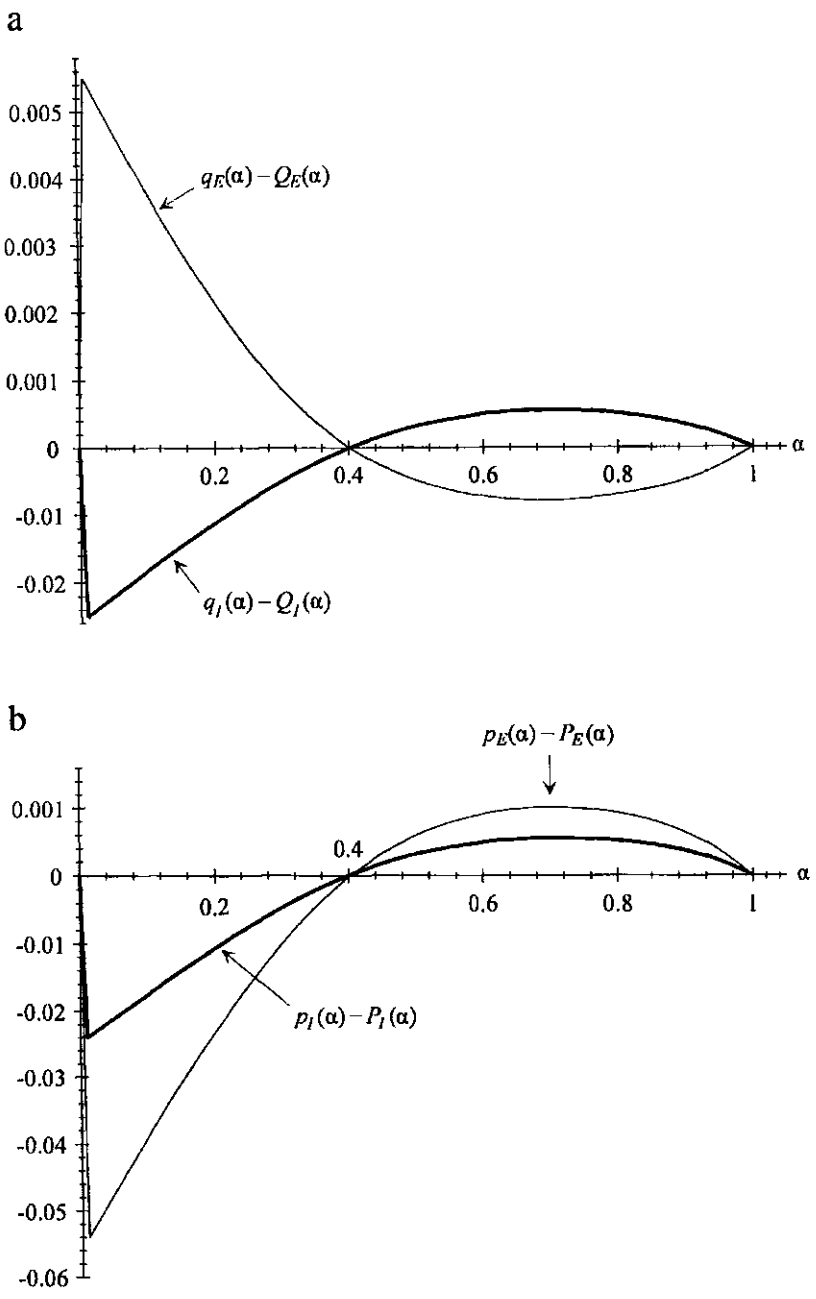

FIG. 2. Static and dynamic equilibrium policies for $\mu_L = 99/100$, $s = 1$, and $\mu_H = 2$: (a) $q_E(\alpha) - Q_E(\alpha)$, $q_I(\alpha) - Q_I(\alpha)$, (b) $p_E(\alpha) - P_E(\alpha)$, $p_I(\alpha) - P_I(\alpha)$.

With a substitute entry is aggressive and the equilibrium price of the entrant is below the static price. Over time, the expected equilibrium price of the entrant is increasing and the expected supply is decreasing as the entrant becomes more established and less aggressive. The effect of entry with uncertain valuations on the incumbent is that sales as well as prices are uniformly lower for the incumbent. But the submartingale property of both equilibrium variables then shows that sales and prices are expected to increase over time.

The entry strategy with an improved product is substantially different. The supply is at all times lower than with a static equilibrium, as the new firm will lose more through a (gradual) decrease in the posterior than through a (gradual) increase. In consequence, the new firm will start with lower than myopic quantities and will be essentially cream-skimming. Over time, its expected price is decreasing and the expected sales and revenues of the incumbent are increasing. Thus the aggressiveness of the strategy is almost entirely predicated by the position of the new firm relative to the established firm.

Finally, we may ask why the value of the information of the entrant has different signs for a substitute and for an improvement. The intuition behind this result can be obtained by considering the strategic incentives in the static game. With uncertainty, sales by the entrant lead to the release of more information to the market participants. This release of information can be thought of as inducing a zero mean lottery over posterior beliefs. From Proposition 2, the entrant's static revenue is convex in α with a substitute and concave in α with an improvement. Thus, if there were a single possibility to acquire additional information, the entrant would prefer more information as represented by more variance in the posterior belief with a convex static equilibrium profit function and less information with a concave one. The results above show that this preference for additional information in a single experiment also holds in the general dynamical model where information is acquired at all instants.

5. ROBUSTNESS

In this section we discuss in some detail how robust our equilibrium results are to different modeling assumptions. In Subsection 5.1 we remove the assumption of a uniform distribution on θ and extend the analysis to more general inverse demand functions. In Subsection 5.2 we discuss how our qualitative results would be changed by considering price competition.

5.1. *Quantity Competition and General Distributions*

Consider a general distribution $F(\theta)$ over the unit interval. Associated with any $F(\theta)$ and an initial belief α is a static profit function $\pi_i(Q_E, Q_I \mid \alpha)$

for firm i. In addition, denote by $\pi_E(Q_E \mid \alpha)$ the profit function of the entrant when he faces a competitive fringe with quality s rather than a single competitor. We make the following three assumptions about the behavior of the static profit functions for the remainder of this section:

1. $\pi_i(Q_E, Q_I \mid \alpha)$ is concave in Q_i for all i and all α.
2. $\pi_E(Q_E \mid \alpha)$ is concave in Q_E for all α.
3. The static best response functions satisfy the stability condition: $-1 < Q_i'(Q_j) < 0$, $\forall i$.

As our main interest is in the dynamic aspects of the model, we do not attempt to present the most general conditions on $F(\theta)$ which would guarantee that the above fairly standard assumptions on the static profit functions are met. Yet it can be verified that a sufficient condition for all three assumptions jointly is that the distribution function $F(\theta)$ is convex, which includes the uniform density model analyzed so far.

We proceed to show that the qualitative properties of the entry and deterrence behavior can be derived in this general setting based exclusively on the interaction between static profit functions and long-run average values.

As before, dynamic programming equations characterize the Markov perfect equilibria; i.e.,

$$0 = \max_{q_E} \{\pi_E(q_E, q_I \mid \alpha) - v_E(\alpha) + \tfrac{1}{2} q_E \Sigma^2(\alpha) V_E''(\alpha)\},$$

and

$$0 = \max_{q_I} \{\pi_I(q_E, q_I \mid \alpha) - v_I(\alpha) + \tfrac{1}{2} q_E \Sigma^2(\alpha) V_I''(\alpha)\},$$

where $v_E(\alpha)$ and $v_I(\alpha)$ are the long-run average revenues under the static profit functions $\pi_E(Q_E, Q_I \mid \alpha)$ and $\pi_I(Q_E, Q_I \mid \alpha)$. For $q_E > 0$, we may divide the above equations by q_E to obtain:

$$0 = \max_{q_E} \left\{ \frac{\pi_E(q_E, q_I \mid \alpha) - v_E(\alpha)}{q_E} \right\} + \frac{1}{2} \Sigma^2(\alpha) V_E''(\alpha), \tag{14}$$

$$0 = \max_{q_I} \left\{ \frac{\pi_I(q_E, q_I \mid \alpha) - v_I(\alpha)}{q_E} \right\} + \frac{1}{2} \Sigma^2(\alpha) V_I''(\alpha). \tag{15}$$

In order to facilitate the comparison with the static equilibrium which is a solution to

$$\max_{q_i} \{\pi_i(Q_E, Q_I \mid \alpha)\}, \qquad \forall i,$$

we consider the first order conditions to the dynamic programming equations (14) and (15):

$$q_E \frac{\partial}{\partial q_E} \pi_E(q_E, q_I \mid \alpha) = \pi_E(q_E, q_I \mid \alpha) - v_E(\alpha) \tag{16}$$

and

$$\frac{\partial}{\partial q_I} \pi_I(q_E, q_I \mid \alpha) = 0. \tag{17}$$

We observe that the first-order condition of the incumbent leads to the same best response function as his static one. Moreover, if the right-hand side in (16) vanishes, then the equations (16) and (17) reduce to the static equilibrium conditions. Hence we know that the dynamic equilibrium conditions are satisfied at the static equilibrium values of $\{Q_E(\alpha), Q_I(\alpha)\}$ if and only if $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) = v_E(\alpha)$. Thus the coincidence of static and dynamic equilibrium policies is in general linked to the equality of the static equilibrium profit and the long-run average for the *entrant*.

Denote by $Q_i(q_j)$ the myopic best response of firm i to firm j's quantity, where we omit the dependence of Q_i on α for notational simplicity. As indicated by Eq. (17), the static and the dynamic best response are identical for the incumbent, or $q_I(q_E) = Q_I(q_E)$. All dynamic equilibria must therefore lie on the reaction curve of the incumbent, $\{q_E, q_I(q_E)\}$. Assumptions 1 and 3 guarantee that there is a single stable static equilibrium, and thus we know that for all $q_E > Q_E(\alpha)$, $q_E > Q_E(Q_I(q_E))$ and hence, by the strict concavity of $\pi_E(q_E, q_I)$ in q_E,

$$\frac{\partial \pi_E(q_E, q_I(q_E) \mid \alpha)}{\partial q_E} < 0, \tag{18}$$

for all $q_E > Q_E(\alpha)$. A similar argument can be made for $q_E < Q_E(\alpha)$ to show that

$$\frac{\partial \pi_E(q_E, q_I(q_E) \mid \alpha)}{\partial q_E} > 0. \tag{19}$$

As the first-order condition of the entrant in the dynamic equilibrium requires that

$$\operatorname{sgn}\left(\frac{\partial \pi_E(q_E, q_I(q_E) \mid \alpha)}{\partial q_E}\right) = \operatorname{sgn}(\pi_E(q_E, q_I(q_E) \mid \alpha) - v_E(\alpha)), \tag{20}$$

a local argument around the static equilibrium quantities $\{Q_E(\alpha), Q_I(\alpha)\}$ based on (18) and (19) suggests the direction in which dynamic quantities

deviate from static ones. In fact, the argument is easy for the case that $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) < v_E(\alpha)$. More care is required in the case where $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) > v_E(\alpha)$ if we want to guarantee that *all* equilibria have the desired property. Therefore assume initially the following relation between the static equilibrium profits and the long-run average:

$$\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) < v_E(\alpha).$$

To determine the location of the dynamic equilibrium we must determine $\operatorname{sgn}(\pi_E(q_E, q_I(q_E) \mid \alpha) - v_E(\alpha))$ on the locus $\{q_E, q_I(q_E)\}$. The claim is that every quantity q_E which satisfies the dynamic equilibrium conditions must imply that $q_E > Q_E(\alpha)$. To see this we observe that we have either

$$\pi_E(q_E, q_I(q_E) \mid \alpha) < v_E(\alpha) \qquad \text{for all} \quad q_E \tag{21}$$

or

$$\pi_E(q_E, q_I(q_E) \mid \alpha) \geq v_E(\alpha) \Rightarrow q_E > Q_E(\alpha)\,. \tag{22}$$

In the first case, the static profit function remains below the long-run average for all pairs $\{q_E, q_I(q_E)\}$, and the first-order condition (20) together with condition (18) implies that $q_E(\alpha) > Q_E(\alpha)$. Consider next the case of (22). If there exist values $\{q_E, q_I(q_E)\}$ such that the static profit exceeds the long-run average, then condition (18) shows that (20) cannot possibly hold at $q_E > Q_E(\alpha)$. Hence we can conclude that whenever $\pi_E(Q_E, Q_I \mid \alpha) < v_E(\alpha)$, the dynamic equilibrium quantity sold by the new firm exceeds the static equilibrium quantity, or $q_E(\alpha) > Q_E(\alpha)$. To establish this argument we only used the stability condition of the static best response function. The complementary results for

$$\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) > v_E(\alpha),$$

are proved in the appendix under the additional concavity assumptions.

PROPOSITION 7. *Suppose that assumptions* 1–3 *hold. Then,*

1. $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) < v_E(\alpha) \Rightarrow q_E(\alpha) > Q_E(\alpha)$,
2. $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) > v_E(\alpha) \Rightarrow q_E(\alpha) < Q_E(\alpha)$.

Proof. See the appendix. ∎

Under Assumptions 1–3, the predictions for the dynamic model are then straightforward. To determine whether the equilibrium quantities of the new firm exceed or fall short of the myopic quantities, all we need to do is to compare the myopic equilibrium profits to the long-run average profits. If the static equilibrium profits are below the long-run average profits, then

the new firm will adopt an aggressive sales policy and by the property of the best response function the incumbent will adopt a more defensive stance. In contrast, if the static equilibrium profits are above the long-run average revenues, the entrant will proceed cautiously with the introduction of the new product and the incumbent will increase his supply to the market. The dynamic programming equations also inform us that the entry strategies are always associated with $V_E''(\alpha) > 0$ and $V_E''(\alpha) < 0$, respectively. The change in the entry strategy can therefore generally be located as in the uniform model analyzed earlier at the intersection $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha)$ $v_E(\alpha)$, where all the necessary data can be computed on the basis of the static profit function alone.

5.2. *Price Competition*

Finally, we sketch how the qualitative results would be affected by a model of price competition within the linear specification. We show that despite some fundamental differences in the static equilibrium profit functions, the dynamic equilibria of the two models share very similar properties.

The most important change in terms of the static equilibria of the two models is that the equilibrium profits are no longer monotone in the quality of the new product. As emphasized in the literature on vertical differentiation, the competitor with a lower quality product doesn't want to increase the quality of his product if this brings the inferior product too close to the superior product. In consequence, the equilibrium prices and revenues are not monotone in α either, rather they display a global minimum at $\alpha = \alpha_m$. At the point α_m, price competition with identical products leads to the Bertrand outcome with marginal cost pricing. The static equilibrium profit functions display a kink at α_m, but on the intervals $[0, \alpha_m)$ and (α_m, ∞) they are concave for the entrant as well as for the incumbent.

In the dynamic model, the strategic interaction is more complex with price competition. With quantity competition, the only variable which affects the evolution of future states, i.e., the level of sales by the entrant, is directly a decision variable of the entrant. In obtaining the dynamic best response of the incumbent, we can therefore ignore the impact of his current decision on future states. But this implies that the best response of the incumbent to any output decision by the entrant is the same in the static and the dynamic models. As a result, all comparisons can be carried out by analyzing the shifts in the best response function of the entrant. In a model with price competition, the price decisions by the firms *jointly* determine the sales level of the entrant. In consequence, we have to analyze the joint effects of changes in the two best response functions on the dynamic equilibrium. To see how this interaction is resolved in equilibrium, we check how the static policies are modified by intertemporal consider-

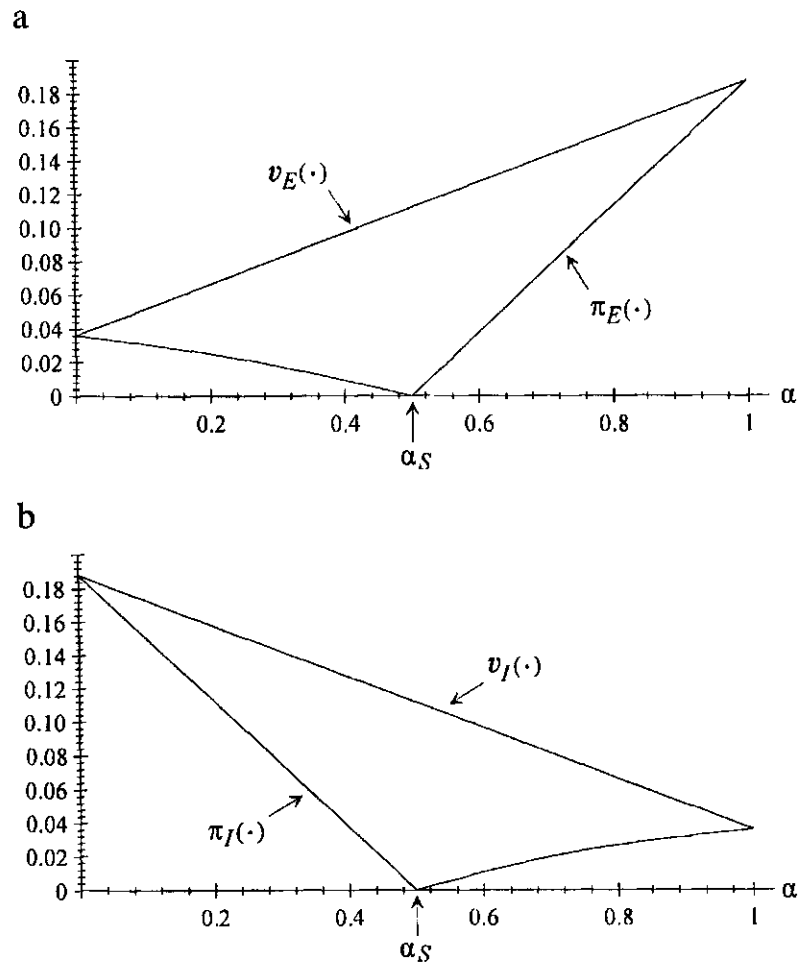

FIG. 3. Static equilibrium profits and long-run average (I), with $\mu_L = 3/2$, $s = 2$, and $\mu_H = 5/2$: (a) $\pi_E(Q_E(\alpha), Q_I(\alpha))$, and $v_E(\alpha)$; (b) $\pi_I(Q_E(\alpha), Q_I(\alpha))$ and $v_I(\alpha)$.

ations. Figure 3 illustrates the static equilibrium revenues as well as the long run average revenues for the case that the value of the new product can either be lower or higher than the established product. Due to the local minimum at $\alpha = \alpha_m$, the long-run average is always above the static revenues. Thus if the static policies were in fact the dynamic equilibrium policies, then the respective Bellman equations would indicate that $V_E''(\alpha) > 0$ as well as $V_I''(\alpha) > 0$. But this would imply that both firms would like to see more sales by the entrant relative to the static equilibrium. We can therefore conjecture that the entrant will lower and the incumbent will raise its price relative to the static equilibrium price. In consequence, sales by the

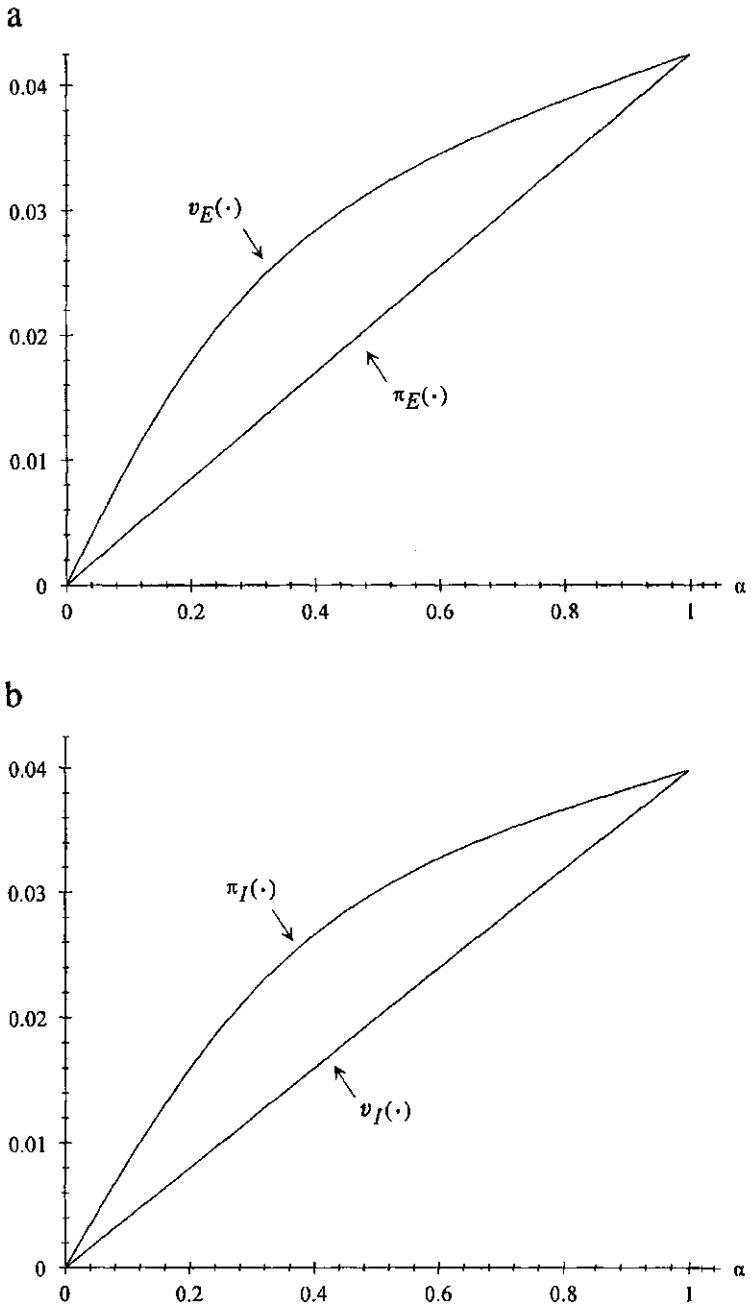

FIG. 4. Static equilibrium profits and long-run average (II), with $s = \mu_L$ and $\mu_H = 2$: (a) $\pi_E(Q_E(\alpha), Q_I(\alpha))$ and $v_E(\alpha)$; (b) $\pi_I(Q_E(\alpha), Q_I(\alpha))$ and $v_I(\alpha)$.

entrant must be larger (and the incumbent's sales must be lower) than in the static equilibrium. If, on the other hand, the product is an improvement and $s < \mu_L < \mu_H$, then the long-run average revenue is lower than the static equilibrium revenue, as Fig. 4 illustrates.

By the same intuition as that above we can then infer from the value functions that if the static policies were indeed equilibrium policies in the dynamic model, then it would have to be that $V_E''(\alpha) < 0$ as well as $V_I''(\alpha) < 0$. But this implies that both firms perceive sales by the entrant as carrying a negative value of information. The strategic response relative to the static solution for the new firm is to raise its price, and for the incumbent it is to decrease its price. This leads to lower quantities for the entrant and higher quantities for the incumbent. Thus the qualitative behavior of entrant and incumbent are similar in a model for quantity competition.

The only difference between the two models arises when $\mu_L < \mu_H < s$. By the concavity of the static profit function, the static revenues of the new firm are always below the long-run average. Observe, however, that a marginal improvement actually brings the new firm closer to its competitor in the quality spectrum and this leads to lower profits. If we interpret the random product quality as reflecting the uncertain value of some new features in the product, it would be unlikely that these features would be included in the product if $\mu_L < \mu_H < s$.

Our earlier paper, [4], analyzed a model of horizontal differentiation with price competition. In that model, an increase in the entrant's expected quality leads to a lower equilibrium price for the incumbent. This clearly has an adverse effect on the entrant's profit, but as the expected quality increases the entrant's profits depend to a lesser extent on the incumbent's decisions. As a result, the entrant's static profit increases in α at an increasing rate or, in other words, the static profit function is convex in α. A similar analysis applies to the incumbent, and as a result both the incumbent and the entrant have a positive value for information. In the current model with purely vertical differentiation, the incumbent's price increases with the entrant's expected quality in the static equilibrium in the region where $\mu(\alpha) > s$. As a consequence, the entrant's static profit function is concave in that region. Hence the differences in the two papers depend crucially on the economic distinction between the two models (i.e., the type of differentiation) rather than on the more arbitrary decision of price vs quantity competition.

6. CONCLUSION

This paper analyzed the entry game in a model with vertical differentiation. The precise location of the new product relative to the existing

product was initially uncertain and was learned over time through experience. We derived the optimal entry and deterrence strategies for the competitors. It was shown that their qualitative properties depend on the current position of the new product in the quality spectrum. This allowed particularly sharp characterization results for the polar cases of a substitute or an improvement, respectively. By focusing on the Markov perfect equilibrium of the game, we derived a set of time series implications which may be amenable to empirical tests.

The current analysis faced some restrictions by the very nature of the model. First, we assumed that the value *and* the uncertainty about the new product were common to all buyers, after controlling for the element of vertical differentiation. It may be interesting to pursue how the equilibrium strategies would be affected if the experience by the buyers would contain an idiosyncratic element (see Milgrom and Roberts [25] for a simple monopoly model). The second limitation is the "once and for all" nature of the innovation presented by the new product. This was reflected in the model by the fact the posterior beliefs converged to either of the absorbing states $\alpha \in \{0, 1\}$ almost surely.

The techniques employed in this paper, however, generalize beyond the present model. The use of the undiscounted optimization criterion, and in particular the notion of the long-run average, allowed us to make a series of predictions based almost exclusively on the static equilibrium behavior. While the long-run average here was computed on the basis of the absorbing and mutually exclusive posterior beliefs, the technique extends naturally to ergodic distributions of the state variables. This should make the methodology used in this paper an attractive candidate for a much richer class of strategic models such as investment games and models of industry evolution, for which there are very few explicit solutions currently known (e.g., Ericson and Pakes [12]). In particular, it would seem feasible to combine dynamic competition models such as the one analyzed here with an ongoing process of innovation.

APPENDIX

We first present a derivation of the Bayesian filtering equation (1) based on a discrete time model with a finite number of buyers. The limiting behavior of the discrete learning model will lead to the Brownian motion depicted in (1) as the number of buyers becomes large and the time elapsed between any two periods converges to zero. Suppose therefore in an economy with N buyers, that each individual experiment with the new product by buyer i is an independent and identically distributed random variable $\tilde{x}_i$ with a normal distribution of unknown mean ($\mu_N = \mu/N$) and

known variance ($\sigma_N^2 = \sigma^2/N$). The parameter μ can take on the value μ_L or μ_H. Note that the mean as well as the variance of the individual experiment is scaled with respect to the total number of buyers N. The utility for buyer i is then given by $\theta_i x_i$, where x_i is a sample realization of $\tilde{x}_i$. Based on the individual experiences of all buyers, we can describe the aggregate or market experience in every period. As the informational content in every realization x_i is independent of the willingness to pay θ_i of individual i, we take the market experience to be the sum of the individual random variables while omitting the weights θ_i; i.e.,

$$\tilde{x}(N) = \sum_{i=1}^{N} \tilde{x}_i.$$

As the mean and variance of the random variable $\tilde{x}_i$ are normalized by the number of buyers in the market, the aggregate mean and aggregate variance of the market experiment $\tilde{x}(N)$ are independent of the number N of buyers and are given by (μ, σ^2). If only a number k of buyers experiment with the new product, where $k \leqslant N$, then the aggregate experiment is given by the random variable

$$\tilde{x}(k) = \sum_{i=1}^{k} \tilde{x}_i,$$

which is again normally distributed with mean $\frac{k}{N}\mu$ and variance $\frac{k}{N}\sigma^2$. If we take the limit as N goes to infinity, the distribution of an aggregate experiment with a fraction n of the buyers, where

$$n = \frac{k}{N},$$

is given by

$$\tilde{x}(n) \sim N(n\mu, n\sigma^2)\,.$$

Next we take the limit as the time between any two periods converges to zero. In the continuous time limit the market experiment then becomes a Brownian motion which can be described by the stochastic differential equation

$$dx(n(t)) = n(t)\,\mu dt + \sigma\sqrt{n(t)}\,dB(t), \qquad t \in [0, \infty).$$

The flow realization in period t is given by the true mean μ weighted by the fraction of buyers participating in the experiment and the random term of the standard Brownian motion $dB(t)$ weighted by the standard deviation $\sigma\sqrt{n(t)}$.

Based on the evolution of the market experiment the market can update the prior belief α_0 to the posterior belief $\alpha(t)$. Based on the standard result for Bayesian updating in continuous time, it can be shown that the posterior belief $\alpha(t)$ also evolves as a Brownian motion.[9] It can be represented by

$$d\alpha(t) = \sqrt{\frac{n(t)\,\alpha(t)(1-\alpha(t))(\mu_H-\mu_L)}{\sigma^2}}\,dB(t),$$

and as in equilibrium $n(t) = q_E(t)$, Eq. (1) follows.

Proof of Proposition 1. The static Nash equilibrium of the duopoly is obtained by solving simultaneously for profit maximizing $\{Q_E(\alpha), Q_I(\alpha)\}$.[10] By solving the best response functions simultaneously, we get

$$Q_E(\alpha) = \frac{\mu(\alpha)+M(\alpha)-m(\alpha)}{4M(\alpha)-m(\alpha)} \quad \text{and} \quad Q_I(\alpha) = \frac{s+M(\alpha)-m(\alpha)}{4M(\alpha)-m(\alpha)}, \tag{23}$$

where $m(\alpha)$ and $M(\alpha)$ are defined as follows:

$$m(\alpha) \triangleq \min\{s, \mu(\alpha)\}, \qquad M(\alpha) \triangleq \max\{s, \mu(\alpha)\}.$$

The equilibrium prices follow from the market clearing conditions and the monotonicity properties follow directly from the relevant derivatives. ∎

Proof of Proposition 2. The curvature properties follow directly from the second derivatives of the equilibrium (23) and prices. ∎

The next lemma records the construction of the long-run averages for the firms.

LEMMA 1 (Long-Run Averages). *The long-run averages are given by*

$$v_E(\alpha) = (1-\alpha)\frac{\mu_L(\mu_L+M(0)-m(0))^2}{(4M(0)-m(0))^2} + \alpha\frac{\mu_H(\mu_H+M(1)-m(1))^2}{(4M(1)-m(1))^2} \tag{24}$$

and

$$v_I(\alpha) = (1-\alpha)\frac{s(s+M(0)-m(0))^2}{(4M(0)-m(0))^2} + \alpha\frac{s(s+M(1)-m(1))^2}{(4M(1)-m(1))^2}.$$

Proof. The long-run average values $v_i(\alpha)$ are equal to the expected full-information payoffs

$$v_i(\alpha) = (1-\alpha)\,v_i(0) + \alpha v_i(1),$$

[9] See Lipster and Shiryayev [22, Chap. 9] for the derivation of the filtering equation for the continuous time, Brownian motion model.

[10] With the linear demand specification, the profit function of each firm is concave in its own quantity, and therefore first-order conditions are also sufficient for optimality.

if $q_E(\alpha)$ is bounded away from zero for all α. It can be verified from (12) that this indeed guaranteed in equilibrium. As $v_I(0)$ and $v_I(1)$ are simply the values of the full information static equilibrium problems with $\alpha \in \{0, 1\}$, the composite values follow. ∎

Next we record without proof some properties of ratios and products of $\mu(\alpha)$ and $v_E(\alpha)$.

LEMMA 2.

1. *The ratios* $v_E(\alpha)/\mu(\alpha)$ *and* $\sqrt{v_E(\alpha)/\mu(\alpha)}$ *are increasing and concave in* α.
2. *The product* $v_E(\alpha)\,\mu(\alpha)$ *is increasing and convex in* α.
3. *The product* $\sqrt{v_E(\alpha)\,\mu(\alpha)}$ *is increasing and concave in* α.

Proof of Proposition 3. The first-order conditions associated with (10) and (11) deliver the solutions for $q_E(\alpha)$ and $q_I(\alpha)$ given in (12) and (13). The market clearing conditions (8) and (9) lead to the equilibrium prices:

$$p_E(\alpha) = \mu(\alpha)\left(1-\sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}\right)-m(\alpha)\left(\frac{1}{2}-\frac{1}{2}\frac{m(\alpha)}{s}\sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}\right)$$

and

$$p_I(\alpha) = \frac{s}{2}-\frac{m(\alpha)}{2}\sqrt{\frac{v_E(\alpha)}{\mu(\alpha)}}.$$

Next we prove the monotonicity properties. Consider first $\mu(\alpha) \geqslant s$ or $m(\alpha) = s$. A necessary and sufficient condition for $q_E(\alpha)$ to be increasing is that $v_E(1)\,\mu(0) \geqslant v_E(0)\,\mu(1)$, which is equivalent to

$$\frac{\mu_L + M(0) - m(0)}{4M(0) - m(0)} \leqslant \frac{\mu_H + M(1) - m(1)}{4M(1) - m(1)},$$

which holds for all values of μ_L, μ_H, and s. It follows directly that $q_I(\alpha)$ and $p_I(\alpha)$ are decreasing in α. It remains to show that $p_E(\alpha)$ is increasing. Suppose initially that $\mu_L, \mu_H \geqslant s$. It is sufficient to show that $\mu(\alpha) - \sqrt{\mu(\alpha)\,v_E(\alpha)}$ is increasing in α. As $\mu(\alpha) > v_E(\alpha)$ for all α, it suffices to show that

$$\frac{\mu'(\alpha)}{v_E'(\alpha)} \geqslant \sqrt{\frac{\mu(\alpha)}{v_E(\alpha)}}.$$

By Lemma 2, the rhs is convex and decreasing, and evaluating the inequality at $\alpha = 0$ is sufficient as $\mu'(\alpha)$ and $v'_E(\alpha)$ are constant. We then obtain

$$\frac{\mu_H - \mu_L}{\dfrac{\mu_H(2\mu_H - s)^2}{(4\mu_H - s)^2} - \dfrac{\mu_L(2\mu_L - s)^2}{(4\mu_L - s)^2}} \geqslant \frac{4\mu_L - s}{2\mu_L - s}. \tag{25}$$

As the lhs is increasing in μ_H, it is sufficient to evaluate it as $\mu_H \downarrow \mu_L$, and (25) reads as

$$(4\mu_L - s)^2 \geqslant 8\mu_L^2 - 2\mu_L s + s^2,$$

which is satisfied by the hypothesis of $\mu_L \geqslant s$. Suppose next that $\mu_L < s < \mu_H$. Then with (25) the argument changes only slightly as $v_E(\alpha)$ has a different form, or

$$\frac{\mu_H - \mu_L}{\dfrac{\mu_H(2\mu_H - s)^2}{(4\mu_H - s)^2} - \dfrac{\mu_L(s)^2}{(4s - \mu_L)^2}} \geqslant \frac{4s - \mu_L}{s}.$$

As the lhs is now decreasing in μ_H, it is sufficient to evaluate it in the limit as $\mu_H \to \infty$, where the inequality is satisfied as it reads

$$4 \geqslant \frac{4s - \mu_L}{s}.$$

Consider next $\mu(\alpha) \leqslant s$. The price is then given by

$$p_E(\alpha) = \frac{1}{2}\mu(\alpha) - \sqrt{\mu(\alpha)\, v_E(\alpha)} + \frac{\mu(\alpha)}{2s}\sqrt{\mu(\alpha)\, v_E(\alpha)}\,.$$

Suppose initially that $\mu_L, \mu_H < s$. It is now sufficient to show that

$$\frac{1}{2}\mu(\alpha) - \sqrt{\mu(\alpha)\, v_E(\alpha)} \tag{26}$$

is increasing in α. By the multiplication rule this is equivalent to showing that

$$\mu'(\alpha)\sqrt{\mu(\alpha)\, v_E(\alpha)} \geqslant \mu'(\alpha)\, v_E(\alpha) + \mu(\alpha)\, v'_E(\alpha).$$

As the term in (26) is concave in α, it remains to show that the inequality holds at $\alpha = 0$ or that

$$\frac{\mu_H - \mu_L}{4s - \mu_L} \geqslant \frac{(\mu_H - \mu_L)\, s}{(4s - \mu_L)^2} + \frac{\mu_H s}{(4s - \mu_H)^2} - \frac{\mu_L s}{(4s - \mu_L)^2}.$$

As the rhs term is increasing faster in μ_H than the lhs, it is sufficient to evaluate it at $\mu_H = s$, or

$$\frac{s-\mu_L}{4s-\mu_L} \geqslant \frac{(s-\mu_L)\, s}{(4s-\mu_L)^2} + \frac{1}{9} - \frac{\mu_L s}{(4s-\mu_L)^2},$$

which is satisfied for all $\mu_L \leqslant s$. Suppose now that $\mu_L < s < \mu_H$; it is then sufficient to show that $p'_E(\alpha) > 0$ at $\alpha = 0$ by Lemma 2, which is equivalent to showing that at $\alpha = 0$

$$\mu'(\alpha) \sqrt{\mu(\alpha)\, v_E(\alpha)} + \frac{1}{2s}\left(3\mu(\alpha)\, \mu'(\alpha)\, v_E(\alpha) + (\mu(\alpha))^2\, v'_E(\alpha)\right)$$

$$\geqslant \mu'(\alpha)\, v_E(\alpha) + \mu(\alpha)\, v'_E(\alpha).$$

Since the lhs is increasing faster in μ_H than the rhs it is sufficient to evaluate the inequality at $\mu_H = s$, and again it can be verified that the inequality holds for all $\mu_L \leqslant s$. ∎

The proof of Proposition 4 relies on the following two lemmas. The first states that the difference $P_E(\alpha)\, Q_E(\alpha) - v_E(\alpha)$ satisfies the same single-crossing properties as $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$ does and the second shows that the crossing points of the two differences coincide.

Denote by A_c the crossing point for the static revenue function.

LEMMA 3. 1. *The difference* $P_E(\alpha)\, Q_E(\alpha) - v_E(\alpha)$ *crosses zero at most once and only from below.*

2. *The critical point* A_c *satisfies* $A_c > \alpha_m$.

3. *A necessary condition for crossing is* $\mu_L < s < \mu_H$.

4. *A necessary and sufficient condition for crossing to occur is:*

$$[P_E(0)\, Q_E(0)]' - v'_E(0) < 0 \qquad \textit{and} \qquad [P_E(1)\, Q_E(1)]' - v'_E(1) < 0.$$

Proof. (1) Observe initially that $P_E(0)\, Q_E(0) - v_E(0) = 0$ and $P_E(1)\, Q_E(1) - v_E(1) = 0$. We first show that if $\mu_L < \mu_H \leqslant s$, or $s \leqslant \mu_L < \mu_H$, then $P_E(\alpha)\, Q_E(\alpha) - v_E(\alpha)$ never crosses at any $\alpha \in (0, 1)$. By Lemma 1, $v_E(\alpha)$ is linear in α, and by Proposition 2, $P_E(\alpha)\, Q_E(\alpha)$ is either convex or concave, respectively. This together with the behavior at the end points excludes an interior crossing point. Consider next $\mu_L < s < \mu_H$, then the revenue function $P_E(\alpha)\, Q_E(\alpha)$ changes curvature behavior exactly once at $\alpha = \alpha_m$. As the curvature changes from convex to concave, the boundary behavior then implies that $P_E(\alpha)\, Q_E(\alpha) - v_E(\alpha)$ has to cross from below and can cross zero at most once.

(2) It is easily verified that at $\alpha = \alpha_m$, $P_E(\alpha_m)\, Q_E(\alpha_m) - v_E(\alpha_m) < 0$.

(3) The necessary condition follows from the arguments given for (1).

(4) The necessary and sufficient conditions follow from the curvature and boundary behavior of the static and long-run revenue functions. ▮

LEMMA 4. $\alpha_c = A_c$.

Proof. As $p_E(\alpha)\, q_E(\alpha)$ and $v_E(\alpha)$ are continuous, a change in sign for $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$ requires a point $\alpha = \alpha_c$ at which

$$p_E(\alpha_c)\, q_E(\alpha_c) - v_E(\alpha_c) = 0. \tag{27}$$

At such a point α_c, either $q_E(\alpha_c) = Q_E(\alpha_c)$ or $q_E(\alpha_c) \neq Q_E(\alpha_c)$. Suppose first that $q_E(\alpha_c) = Q_E(\alpha_c)$ were to hold, then it follows by the equilibrium conditions (10) and (8)–(9) that $p_E(\alpha_c) = P_E(\alpha_c)$ as well. But then it is has to be the case that $\alpha_c = A_c$. Suppose to the contrary that $q_E(\alpha_c) \neq Q_E(\alpha_c)$ would hold, then we show that (27) can't hold. Since $q_E(\alpha_c) \neq Q_E(\alpha_c)$, it has to be the case that $V_E''(\alpha_c) \neq 0$, by the first-order conditions from the Bellman equation (10). But then the hypothetical policies at α_c don't satisfy the Bellman equation and hence cannot be equilibrium conditions. Thus if $\alpha_c \in (0, 1)$ it has to be that $\alpha_c = A_c$. It remains to show that if $P_E(\alpha)\, Q_E(\alpha) - v_E(\alpha)$ changes sign, then $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$ necessarily changes signs as well. This is established easily as at α_c, $q_E(\alpha_c) = Q_E(\alpha_c)$ is a solution to the first order condition (10), and as the solution is unique the claim follows. ▮

Proof of Proposition 4. (1–3) By Lemma 3, the difference $p_E(\alpha)\, q_E(\alpha) - v_E(\alpha)$ shares the single-crossing behavior with the difference $P_E(\alpha)\, Q_E(\alpha) - v_E(\alpha)$. By Lemma 4, they also share the crossing point.

(4) As the myopic and intertemporal policies are identical at the endpoints, or $q_E(\alpha) = Q_E(\alpha)$ and $p_E(\alpha) = P_E(\alpha)$ for $\alpha \in \{0, 1\}$, it follows that the gradient of the flow revenues at the endpoints are necessary and sufficient conditions as well. ▮

Proof of Proposition 5. The asymmetry in the relationship between myopic and intertemporal quantities for the sellers follows directly from the best response function based on (10). It is therefore sufficient to consider the relationship between $q_E(\alpha)$ and $Q_E(\alpha)$. It follows from the first order condition of the entrant that $q_E(\alpha) > Q_E(\alpha)$ if and only if $V_E''(\alpha) > 0$. Likewise $q_E(\alpha) < Q_E(\alpha)$ if and only if $V_E''(\alpha) < 0$. The results concerning the equilibrium quantities follow then directly from Proposition 4.

For the equilibrium prices consider first the interval $\alpha \in [0, \alpha_m]$. As the inequality $q_E(\alpha) > Q_E(\alpha)$ leads to $q_I(\alpha) < Q_I(\alpha)$, the best response function

based on (10) implies together with the market clearing condition (8) that $p_E(\alpha) < P_E(\alpha)$, which in turn leads to $p_I(\alpha) < P_I(\alpha)$. Consider next the interval $\alpha \in [\alpha_m, \alpha_c]$. The inequality $q_E(\alpha) > Q_E(\alpha)$ leads to $q_I(\alpha) < Q_I(\alpha)$. The best response function based on (10) together with the market clearing condition (9) implies that $p_I(\alpha) < P_I(\alpha)$, which in turn leads to $p_E(\alpha) < P_E(\alpha)$. In the remaining interval $\alpha \in [\alpha_c, 1]$, the inequality $q_E(\alpha) < Q_E(\alpha)$ leads to $q_I(\alpha) > Q_I(\alpha)$. The best response function (10) together with the market clearing condition (9) implies that $p_I(\alpha) > P_I(\alpha)$, which in turn leads to $p_E(\alpha) > P_E(\alpha)$.

Proof of Proposition 6. (1) By Lemma 2.

(2) It follows directly from Lemma 2 that $p_E(\alpha)$ is convex for $\mu(\alpha) < s$ and concave for $\mu(\alpha) > s$.

(3) By Lemma 2.

(4) By Lemma 2. ∎

Proof of Proposition 7. The case of $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) < v_E(\alpha)$ was argued in the text. Suppose now that $\pi_E(Q_E(\alpha), Q_I(\alpha) \mid \alpha) > v_E(\alpha)$. Suppose first that $q_E < Q_E(\alpha)$, then we want to show that at q_E, $\pi_E(q_E, q_I(q_E) \mid \alpha) > v_E(\alpha)$. The argument is by contradiction. Suppose not; then it would follow from the Bellman equation that $V_E''(\alpha) > 0$, but then $q_E < Q_E(\alpha)$ cannot be an equilibrium as the entrant would have an incentive to deviate and increase the quantity. By a similar argument, we can exclude the possibility of $q_E > Q_E(\alpha)$ where $\pi_E(q_E, q_I(q_E) \mid \alpha) > v_E(\alpha)$ holds simultaneously.

Finally, we present sufficient conditions to rule out possible equilibria in the region where $q_E > Q_E(\alpha)$ and $\pi_E(q_E, q_I(q_E) \mid \alpha) < v_E(\alpha)$. Observe that for all q_E sufficiently close to $Q_E(\alpha)$:

$$q_E \frac{\partial \pi_E(q_E, q_I(q_E) \mid \alpha)}{\partial q_E} < \pi_E(q_E, q_I(q_E) \mid \alpha) - v_E(\alpha). \tag{28}$$

A sufficient condition to rule out equilibria with $q_E > Q_E(\alpha)$ is therefore that the derivative of the lhs is always below the derivative of the rhs for $q_E > Q_E(\alpha)$. Using the fact that

$$\pi_E(q_E, q_I(q_E) \mid \alpha) = p_E(q_E, q_I(q_E))\, q_E$$

we may rewrite the inequality (28) as

$$q_E^2 \frac{\partial p_E(q_E, q_I(q_E))}{\partial q_E} < -v_E(\alpha).$$

The sufficient condition can then be written as

$$2\frac{\partial p_E(q_E, q_I(q_E))}{\partial q_E}+q_E\left(\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_E^2}+\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_I \partial q_E}\,q_I'(q_E)\right)<0. \tag{29}$$

As the first term is strictly negative independent of $F(\theta)$, it is sufficient to show that

$$\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_E^2}+\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_I \partial q_E}\,q_I'(q_E)\leqslant 0.$$

Consider first $\mu(\alpha)\leqslant s$, then the equilibrium price of the entrant can be written as

$$p_E=\mu(\alpha)\,F^{-1}(1-q_E-q_I),$$

and hence

$$\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_E^2}=\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_I \partial q_E}.$$

The sufficient condition (29) can then be written as

$$2\frac{\partial p_E(q_E, q_I(q_E))}{\partial q_E}+q_E\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_E^2}(1+q_I'(q_E))<0. \tag{30}$$

By the assumption of concavity of the profit function of the duopolist,

$$2\frac{\partial p_E(q_E, q_I(q_E))}{\partial q_E}+q_E\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_E^2}<0.$$

Now if

$$\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_E^2}>0,$$

then (30) holds since $q_I'(q_E) < 0$ by the stability of the best response. On the other hand, if

$$\frac{\partial^2 p_E(q_E, q_I(q_E))}{\partial q_E^2} < 0,$$

then (30) holds since

$$\frac{\partial p_E(q_E, q_I(q_E))}{\partial q_E} < 0$$

and $1 + q_I'(q_E) > 0$.

Next suppose that $\mu(\alpha) > s$. Then the price of the entrant is given by

$$p_E = \mu(\alpha)\, F^{-1}(1-q_E) + s[F^{-1}(1-q_E-q_I) - F^{-1}(1-q_E)]$$

Let $H(\cdot)$ be the inverse function of F, or $H = F^{-1}$, and let h be the first derivative of H. The condition (29) can be written as:

$$-2[h(1-q_E)(\mu(\alpha)-s) + h(1-q_E-q_I)\, s]$$
$$+ q_E[h'(1-q_E)(\mu(\alpha)-s) + (1+q_I'(q_E))\, h'(1-q_E-q_I)\, s] < 0. \quad (31)$$

By the concavity of the profit function of the monopolist, we know that

$$-2h(1-q_E-q_I)(\mu(\alpha)-s) + q_E h'(1-q_E-q_I)(\mu(\alpha)-s) < 0 \quad (32)$$

and also that

$$-2h(1-q_E)(\mu(\alpha)-s) + q_E h'(1-q_E)(\mu(\alpha)-s) < 0. \quad (33)$$

But since $0 < 1 + q_I'(q_E) < 1$,

$$(-2h(1-q_E-q_I)(\mu(\alpha)-s) + q_E h'(1-q_E-q_I)(\mu(\alpha)-s))(1+q_I'(q_E)) < 0,$$

and therefore

$$-2h(1-q_E-q_I)\, s + (1+q_I'(q_E))\, q_E h'(1-q_E-q_I)\, s < 0. \quad (34)$$

Finally, adding (33) and (34) yields (31). ∎

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