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Venture capital financing, moral hazard, and learning

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Abstract

We consider the provision of venture capital in a dynamic agency model. The value of the venture project is initially uncertain and more information arrives by developing the project. The allocation of the funds and the learning process are subject to moral hazard. The optimal contract is a time-varying share contract which provides intertemporal risk-sharing between venture capitalist and entrepreneur. The share of the entrepreneur reflects the value of a *real option*. The option itself is based on the control of the funds. The dynamic agency costs may be high and lead to an inefficient early stopping of the project. A positive liquidation value explains the adoption of strip financing or convertible securities. Finally, relationship financing, including monitoring and the occasional replacement of the management improves the efficiency of the financial contracting.

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1. Introduction

1.1. Motivation

Venture capital has become a major vehicle for the funding of start-up firms. In many countries, most notably the United States, venture capital is now the financing mode of choice for projects where "learning" and "innovation" are important. Because of their innovative nature, venture projects carry a substantial risk of failure. Only a minority of start-ups are high-return investments: 20% or less are frequent estimates for the fraction of projects where investors can successfully "cash out", mostly through IPO's. Of the remaining, a majority is sold off privately or merged which can mean anything between a modest success and scantly disguised failure with substantial losses. A minority is liquidated implying a complete write-off of the investment. [1]

One of the most challenging problems in venture financing is to determine when to release funds for continued development and when to abandon a project. Many aspects of the venture capital industry suggest that practitioners are well aware that they face a sequence of starting and stopping problems in the financing of a venture. [2] An essential feature of any venture project is the necessity to fund the project in order to learn more about the uncertain return of the venture. This process often starts with the provision of seed financing to set up a business plan. The simultaneity of the financing decision and the acquisition of information about the investment project is characteristic for ventures and more generally for the financing of innovation. Surprisingly, the dynamic interaction of both aspects has received little attention in the literature.

This paper proposes a simple model to analyze the optimal financing of venture projects when learning and moral hazard interact. A wealth constrained entrepreneur offers an investment opportunity to a venture capitalist. The project can either succeed or fail. The successful completion requires funds for the development of the project. The rate of investment controls the probability of success. A higher investment level accelerates the process of discovery. As the project continues to receive financing without achieving success, the agents change their assessment about the likelihood of future success. Eventually the prospects may become too poor to warrant further investment.

The entrepreneur controls the allocation of the funds and the investment effort is unobservable to the investor. The control over the funds implies that the entrepreneur also controls the flow of information about the project. The

[1] For estimates, see Poterba (1989), Sahlman (1990), Sagari and Guidotti (1991), Gompers (1995) and Amit et al. (1997).

[2] The staging of the funds, documented in Lerner (1994) and Gompers (1995) is perhaps the most prominent aspect of the sequential nature of venture financing.

solution of the agency conflict has to take into account the intertemporal incentives for the entrepreneur. Suppose, in any given period, the entrepreneur would consider to divert the capital flow for her private consumption. In the following period she would then be marginally more optimistic about the future of the project than the venture capitalist. The entrepreneur would know that the project did not receive any capital in the preceding period and hence could not possibly generate a success. But the venture capitalist would continue to believe that the entrepreneur did as instructed. In consequence he interprets the fact that no success has been observed as "bad news" about the project. Following a deviation, the entrepreneur will therefore keep her posterior belief about future success constant while the posterior belief of the investor is necessarily downgraded. The reward for the entrepreneur therefore consists of two components. She needs to be compensated for the foregone private benefits but also for the downgrading of her expectations about the future of the project. The longer the experimentation horizon, the larger is the option value of the diversion. In fact, the compensation could become so large as to surpass the net value of the project. In turn this implies the possibility of financial constraints in the form of an inefficient and premature end of the project.

1.2. Results and empirical implications

The optimal share contract and the financial constraints allow for a number of empirical implications.

First, our paper provides an analysis of the optimal evolution of the shares of entrepreneur and venture capitalist. How the parties should optimally split the prize should depend on the funding horizon and the flow of funds. Our model predicts that the share of the entrepreneur decreases towards the end. Initially, the entrepreneur's share can rise or fall, depending on the discount rate and the degree of initial optimism. The *expected* share of the entrepreneur, however, always decreases over time. Similarly, the *expected return* of the venture capitalist decreases over time and he may even make losses if the project approaches the stopping time. Empirical findings indicate that the entrepreneur is indeed penalized if the project takes too much time as her equity fraction is diluted from financing round to financing round. [3]

Second, we obtain results for the security design by extending the model to positive liquidation values in case of abandonment. The liquidation value is received by selling tangible assets or intermediate results. The venture capitalist should then either receive strip packages *combining common stock and debt* or *convertible securities*. The optimal contract should reward the entrepreneur

[3] As new funds are provided in exchange with stock purchase agreements, the shares of the entrepreneur become diluted, see e.g. Sahlman (1990), Lerner (1995) and Gompers (1995).

only in the case of success. The venture capitalist should therefore keep a "hard" claim in case of failure. However, if the venture capitalist would hold exclusively hard claims, then a premature liquidation is likely. [4]

Third, a pure share or equity contract could be financed at *arm's length*, implying that it could be traded in financial markets. However an arm's length share contract may leave too much surplus to the entrepreneur and the project would be terminated too early. Our model accounts thus naturally for the observation that venture financing is typically *relationship* financing. Costly monitoring and the option to replace the entrepreneur may become desirable. Relationship specific financing permits the extension of the funding horizon. [5]

Fourth, short-term refinancing of the project can never be optimal. We call short-term financing, as distinct from staged financing, a financial policy where the entrepreneur attracts funds on a competitive basis in each period. Towards the end of the efficient investment horizon, the expected return is insufficient to cover the necessary outlays for both partners. The efficient solution can only be achieved by a long-term contract which allows for intertemporal risk-sharing. More precisely, the venture capitalist subsidizes continuation of the project toward the end in exchange for higher expected profits at the beginning. Consistent with this result, Sahlman (1990) observes that venture capitalists are protected against competition by preemptive rights and Anand and Galetovic (1997) report that venture financing is frequently supported by long-term relationships.

We may finally remark that up-front financing as well as staged financing are consistent with our model. Since the optimal contract satisfies the intertemporal incentive constraints, the funds will be allocated by the entrepreneur as intended regardless of how the funds are provided over time. [6] In Section 7 we discuss the extension of the model to multiple signals, where stage financing arises as the unique optimal financial arrangement.

1.3. Related literature and overview

The theoretical research on venture finance has only recently emerged. In Hart and Moore (1994), the option of the entrepreneur to repudiate her financial obligations limits the feasible amount of outsider claims. Neher (1997) extends their approach to stage financing as an instrument to implement the optimal investment path. Admati and Pfleiderer (1994) show that a fixed

[4] The predominance of convertible preferred stock is documented by Sahlman (1990) and Trester (1998).

[5] The frequent usage of monitoring and replacement of management is documented in Gorman and Sahlman (1989), Sahlman (1990) and Lerner (1995).

[6] Sahlman (1990) discusses up-front financing, even in the presence of staging, and Gompers (1995) analyzes the structure of stage financing.

fraction equity contract may give robust optimal incentives if it is efficient to allocate the control rights to the venture capitalist. Berglöf (1994) considers convertible debt in a framework of incomplete contracts to transfer control rights to the value-maximizing party. Chan et al. (1990) explain the optimal transition of control between entrepreneur and venture capitalist in a model with initial uncertainty about the skill of the entrepreneur. Hellmann (1996) explains the willingness of the entrepreneur to relinquish control rights by a trade-off between equity and debt induced incentives. Trester (1998) argues that the problem of an entrepreneur dissipating the firm's assets can be mitigated if the investor has no option to declare default and seize the assets. Cornelli and Yosha (1997) analyze the problem of an entrepreneur manipulating short-term results for purposes of "window-dressing".

The paper is organized as follows. The model is presented in Section 2. The value of the project is characterized in Section 3. The structure and efficiency of short and long-term contracting is examined in Section 4. We extend the model in Section 5 by allowing for a positive liquidation value of the project and discuss issues of security design in this context. In Section 6, we analyze the relationship specific instruments such as monitoring and the occasional replacement of the entrepreneur. Section 7 discusses possible extensions of the model. Section 8 concludes.

2. The model

The venture is presented as an investment project with uncertain returns in Section 2.1. The successful realization of the venture is positively correlated with the volume of financing it receives. As the flow of investment sinks into the project, entrepreneur and investor update their assessment of the prospects of the project. The evolution of the posterior belief of eventual success represents the *learning process* of the agents, which is analyzed in Section 2.2. The moral hazard problem between the entrepreneur and the investor as well as the financing possibilities of the project are finally described in Section 2.3.

2.1. Project with uncertain returns

An entrepreneur owns a project with uncertain return. The project is either "good" with prior probability α_0 or "bad" with prior probability $1 - \alpha_0$. If the project is "good", then in every period t, there is a certain probability that the project is successfully completed, in which case it yields a fixed payoff R. The probability of success in period t, *conditional* on the project being good, is denoted by p_t. The probability p_t is in turn an increasing function of the investment flow in period t. Or inversely, a success probability p_t requires an investment flow of $c(p_t)$ in period t. We assume that $c(p_t)$ is a linear function of p_t:

$$c(p_t) = cp_t, \quad c > 0. \tag{1}$$

The maximal probability of success in each period is denoted by p (without any subscript), where $0 < p < 1$ and any probability $p_t \in [0, p]$ is feasible in each period. In other words, any investment beyond $c(p)$ does not increase the probability of success.

If the project is "bad", then it will never yield a return and the probability of success is zero independent of the capital flow. The project can receive financing over any number of periods and time is discrete and denoted by $t = 0, 1, \ldots, T$. The investment process either stops with a successful completion or the project is eventually abandoned when the likelihood of future success becomes sufficiently small.

The uncertainty of the project is resolved over time by an *experimentation process*, where the likelihood of success is positively correlated with the investment in the project. The investment problem is simple as the investment only influences the conditional probability of success in every period and independent of time. In particular, the investment flow does not influence the value of the successful realization, R, or the scrap value if the project should be abandoned. In Section 7 we shall discuss how these modifications, as well as time dependent probabilities, would enrich the predictions of our model.

2.2. Learning

As the experimentation process develops over time, entrepreneur and investor learn more about the prospects of the project. If success has not yet occurred at period t, then the participants in the project update their beliefs about the type of the project. We next determine the evolution of the posterior beliefs. We denote by α_{t+1} the posterior belief that the project is good, based on no discovery until and including t. The evolution of the posterior belief α_{t+1}, conditional on no success, is given by Bayes' rule as a function of the prior belief α_t and the capital flow cp_t as:

$$\alpha_{t+1} = \frac{\alpha_t(1 - p_t)}{\alpha_t(1 - p_t) + 1 - \alpha_t}. \tag{2}$$

The posterior belief α_{t+1} thus decreases over time when success has not been realized. The decline in the posterior belief is stronger for larger investments, as the participants in the venture become more pessimistic about the likelihood of success. The posterior belief, again conditional on no success yet, can be represented as a function of the initial belief α_0 and the sequence of investments until t, $\{cp_0, cp_1, \ldots, cp_t\}$:

$$\alpha_{t+1} = \frac{\alpha_0 \prod_{s=0}^{t} (1 - p_s)}{\alpha_0 \prod_{s=0}^{t} (1 - p_s) + 1 - \alpha_0}. \tag{3}$$

Under a constant investment policy $p_t = \hat{p}$, the evolution of α_t is a discrete version of a decreasing logistic function:

$$\alpha_{t+1} = \frac{\alpha_0\left(1-\hat{p}\right)^{t+1}}{\alpha_0\left(1-\hat{p}\right)^{t+1}+1-\alpha_0}. \tag{4}$$

The evolution of the posterior belief, conditional on no success, under two different constant allocation policies is displayed in Fig. 1.

The posterior belief changes only slowly if the participants have very precise beliefs about the nature of the project, i.e. if α_t is close to either 0 or 1. Correspondingly the event of no success is most informative if the agents have very diffuse beliefs, i.e. α_t is close to $\frac{1}{2}$. In this case the posterior beliefs change most rapidly. In any case, a higher investment level accelerates the rate at which the posteriors change over time as displayed in Fig. 1.

2.3. Moral hazard and financing

The entrepreneur has no wealth initially and seeks to obtain external funds to realize the project. Financing is available from a competitive market of

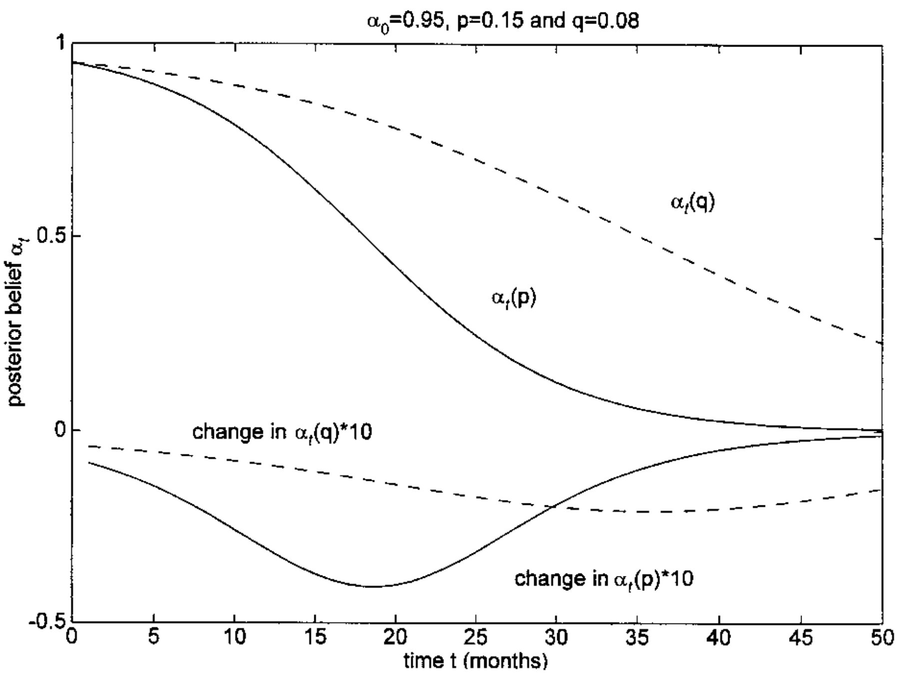

Fig. 1. Volume of financing cp and evolution of posterior belief.

venture capitalists. Entrepreneur and venture capitalists have initially the same assessment about the likelihood of success, which is given by the prior belief α_0. The entrepreneur and venture capitalists are both risk-neutral and have a common discount factor $\delta \in (0, 1)$.

The funds which are supplied by the venture capitalist are to be allocated by the entrepreneur to generate the desired success R. However the (correct) allocation of the funds to the project is unobservable to the venture capitalist, and thus a moral hazard problem arises between financier and entrepreneur. Indeed the entrepreneur can "shirk" and decide to (partially) withhold the investment and divert the capital flow to her private ends. An equivalent, but perhaps more classical formulation of the same moral hazard problem is following one: the efficient application of the investment requires effort, which is costly for the entrepreneur. By reducing the effort, the entrepreneur also reduces the probability of success and hence the efficiency of the employed capital. In both cases, a conflict of interest arises about the use of the funds.

Initially, the entrepreneur can suggest financial contracts to any of the venture capitalists. The selected venture capitalist then decides whether to accept or reject. The contract can be contingent on time, outcome and the capital provided by the investor. However, due to the moral hazard nature of the financing problem, the contract cannot be made contingent on the (correct) application of the funds. The design of the contract has to ensure that incentive compatibility and individual participation constraints are satisfied. The contract may contain a clause prohibiting that the entrepreneur continues the project once the contract has expired, for example by transferring ownership of the idea to the venture capitalist. If such a prohibition is not made, then the entrepreneur can again suggest financial contracts to any of the venture capitalists. For the moment, we abstractly consider contingent contracts, but in the appropriate places, we shall discuss which standard securities will be able to perform the tasks of the contingent contracts. We neglect renegotiation of contracts throughout this paper. [7]

Finally, we wish to emphasize that while there is no initial asymmetry in the information between financier and entrepreneur, the asymmetry may arise over time as the project receives funding. The source of the asymmetry is the unobservability of the allocation of funds. If entrepreneur and investor have different assessments over how the funds have been employed, then in turn they will have different posterior probabilities over the likelihood of success. Before we consider the optimal financial contract between entrepreneur and investor, we first analyze the efficient investment policy in the absence of the moral hazard problem.

[7] See Bergemann and Hege (1998) for a discussion of renegotiation in dynamic agency problems.

3. Value of the venture

The social value of the venture project with prior belief α_0 is maximized by an optimal investment policy and an optimal stopping point. At the stopping point the project is abandoned and no further investment is undertaken. The stopping point itself can either be characterized by the posterior belief α_T at the stopping point or the *real time* T at which the project is abandoned. For any given investment policy $\{cp_0, \dots, cp_T\}$ there is naturally a one-to-one relation between α_T and T through Bayes' rule as developed in Eq. (3).

We denote by $V(\alpha_t)$ the value of the project with posterior belief α_t under optimal policies. The optimal policies can be obtained by standard dynamic programming arguments. Consider first the optimal stopping point α_T. Clearly, the project should receive funds as long as the expected returns from the investment exceed the costs, or

$$\alpha_T p_T R - cp_T \geqslant 0, \tag{5}$$

for some $p_T \in [0,\ p]$. Conversely, if the current expected returns do not exceed the investment cost, or

$$\alpha_T p_T R - cp_T < 0, \tag{6}$$

then it is optimal to abandon the project, as future returns will only decline further. It follows from Eqs. (5) and (6) that the *efficient boundary point* α^* between the investment region and the stopping region is given by

$$\alpha^* p_T R - cp_T = 0 \iff \alpha^* = \frac{c}{R}. \tag{7}$$

The posterior belief α^* at which stopping occurs decreases when either the return R increases or the marginal cost c of generating success decreases.

We notice next that if indeed the last investment occurs at α_T, then it is optimal to choose $p_T = p$ due to the linear structure of the problem. Hence we obtain the value in the terminal period:

$$V(\alpha_T) = \alpha_T pR - cp. \tag{8}$$

The value of the venture is then obtained recursively by the dynamic programming equation:

$$V(\alpha_t) = \max_{p_t}\{\alpha_t p_t R - cp_t + (1 - \alpha_t p_t)\delta V(\alpha_{t+1})\}, \tag{9}$$

where the posterior belief α_{t+1} is determined by the incoming belief α_t and the investment cp_t in period t through Bayes' rule as expressed in Eq. (2). The value function (9) represents the implications of an investment policy cp_t on current and future returns. An increase in cp_t is costly but it increases the probability of a successful completion today and the associated expected returns $\alpha_t p_t R$. At the same time, it becomes less likely that the project will have to be continued tomorrow as $1 - \alpha_t p_t$ decreases. Finally, if success should not occur in period t

even with large investment flow cp_t, then the posterior belief α_{t+1} will decrease and correspondingly the continuation value of the project, $V(\alpha_{t+1})$.

The value function (9) also indicates that the linearity in p_T which the terminal period problem (5) displays, is preserved in the intertemporal investment problem (9) as well. The optimal policy is therefore to invest maximally at the level of cp as long as the posterior belief is above α^* and stop as soon the posterior belief falls for the first time below the boundary point α^*. For transparency, we may translate this policy into a stopping time policy T^* in *real time*. In this case we ask how long can we maximally invest cp and still maintain posterior beliefs above the stopping point α^*. The optimal stopping time T^* is then given by

$$T^* = \max\left\{ T \,\middle|\, \frac{\alpha_0(1-p)^T}{\alpha_0(1-p)^T + 1 - \alpha_0} \geqslant \alpha^* \right\}. \tag{10}$$

Evidently, the optimal stopping time T^* depends on the initial belief α_0 at which the project is started, $T^* \triangleq T^*(\alpha_0)$, but we usually suppress the dependence on α_0 as a matter of convenience. The stopping time T^* then represents the time elapsed between starting at α_0 and arriving for the last time at a posterior belief exceeding α^*. The socially efficient investment policy and the value of the venture can then be obtained from the solution of the recursive problem (9):

Proposition 1 (Optimal investment policy).

(*i*) *The optimal policy is to invest maximally cp until* T^*.

(*ii*) *The social value of the venture is given by*

$$V(\alpha_0) = \alpha_0 p(R-c)\frac{1-\delta^{T^*}(1-p)^{T^*}}{1-\delta(1-p)} - (1-\alpha_0)cp\frac{1-\delta^{T^*}}{1-\delta}. \tag{11}$$

Proof. See Appendix A. □

The value function $V(\alpha_0)$ presents an intuitive decomposition of the value of the project. [8] The first term in Eq. (11) is the expected value of the project conditional on the project being good. Notice that the value of the project is discounted at a rate which compounds the pure discount rate δ and the probability of *no* discovery $(1-p)$ which results in the factor $\delta(1-p)$. The second term captures the case that the project is bad which occurs with probability $(1-\alpha_0)$. In this case, costly experimentation will continue until the stopping time T^* is reached and discounting occurs at the rate δ until the project is stopped at T^*. With the description of the socially efficient investment policy

[8] For future reference, we denote by $V_T(\alpha_0)$ the value of the project if financing occurs at the maximal rate cp, until T, where obviously $V_{T^*}(\alpha_0) = V(\alpha_0)$.

in the background, we next turn to the financial contracting between entrepreneur and venture capitalist.

4. Financial contracting

We begin in Section 4.1 by analyzing the provision of venture capital under short-term contracts. The optimal short-term contracts are simple share contracts between entrepreneur and investor. However as a relationship governed by short-term contracts has almost no scope for intertemporal transfers, short-term contracts are generally inefficient and will lead to a premature end of the venture. Consequently, we investigate in Section 4.2 the structure of long-term contracts and in Section 4.3 their efficiency properties. The model is extended in Section 5 to allow for a positive liquidation value of the project and issues of security design appear in this context.

4.1. Short-term contracts

The venture capitalist offers his funds for the project in exchange against a share of the uncertain returns of the venture. Evidently the expected returns for the venture capitalist must be large enough to justify his investment. At the same time, the entrepreneur must have sufficient incentives to truthfully invest the funds in the project. As the project can only be successfully completed and yield R if the entrepreneur applies the funds correctly, it follows that the incentives provided to the entrepreneur should maximally discriminate with respect to the signal R. With the wealth constraint of the entrepreneur, the most high-powered incentive contract is obviously the following: she receives a positive share of R if the project was a success and nothing otherwise. Due to the binary nature of the project, success or failure, these contracts, which we call *share contracts*, form indeed the class of optimal contracts in this environment.

Define by S_t the share of the entrepreneur if R is realized in period t. The corresponding share of the venture capitalist is $(1 - S_t)$. In a short-term contract, the venture capitalist only promises to provide funds for a single period, and then reconsiders financing in the subsequent periods. The expected return from the current investment must at least exceed the cost of the investment and since there is competition among the venture capitalists, in equilibrium the venture capitalist will just break even. Hence the short-term contract must satisfy for any level of funding cp_t the following *participation constraint*:

$$\alpha_t p_t (1 - S_t) R = cp_t. \tag{12}$$

On the other hand, the remaining share S_t for the entrepreneur has to be large enough for her to invest the funds in the project and not divert them to her private ends. This forms the *incentive compatibility constraint* for the entrepreneur, which is formally stated by

$$\alpha_t p_t S_t R \geqslant c p_t. \tag{13}$$

By combining Eqs. (12) and (13) we infer that any financing under short-term contracts can only be continued as long as the inequality

$$\alpha_t p_t R \geqslant 2 c p_t \tag{14}$$

is satisfied. The critical value of the posterior belief, denoted by α^s, where short-term contracting will cease is thus given by

$$\alpha^s = \frac{2c}{R}.$$

By comparing α^s with the socially efficient stopping point α^* obtained in the previous section, we immediately obtain the following Proposition.

Proposition 2 (Short-term financing). *The venture project is stopped prematurely under short-term financing as* $\alpha^s > \alpha^*$.

The premature stopping indicated by $\alpha^s > \alpha^*$ is naturally equivalent to a funding horizon T^s which is shorter than the efficient horizon T^*, where T^s is determined as T^* in Eq. (10). This is a simple, but important benchmark result. It indicates that efficient financing requires some form of *intertemporal risk sharing* which can only be sustained by commitments made through long-term contracts. The necessity of intertemporal risk sharing is easy to grasp. As the posterior belief α_t deteriorates over time, the expected value which the parties expect to split decreases as well. Eventually, the competing claims emanating from the investment problem of the financier Eq. (12) and the agency problem of the entrepreneur Eq. (13) lead to a conflict. In this respect, financing a venture resembles a team problem where both investor and entrepreneur contribute. The venture capitalist must earn the equivalent of cp_t to justify his investment, while the entrepreneur must earn the equivalent of cp_t if she were to employ the funds towards the proper end. As α_t decreases these compensations eventually cannot be covered anymore from the *expected proceeds* in period t and it follows that short-term contracts will necessarily terminate too early.

4.2. Long-term contracts

In a regime of short-term financing the investor has to break even in every period as there was no commitment on either side to continue the relationship. Long-term contracts can improve the intertemporal risk-sharing by replacing the sequence of participations constraints (12) for every period by a single *intertemporal participation constraint*, which covers the entire funding horizon.

By offering the investor a larger share of the return in the early stages of the financing, his shares in the later stage can be lowered, and hence the project can be continued beyond α^s. But a long-term contract with the associated funding commitments offers the entrepreneur a rich set of alternative actions, many of them not desirable from the investor's point of view. For example, if the entrepreneur is promised additional funding in the next period, then she may consider to divert the funds today and bet instead on a positive realization of the project tomorrow. By implication, the incentives for the entrepreneur today have to be sufficiently strong, in particular relative to the incentives offered tomorrow. These dynamic considerations then generate a rich set of predictions about the sharing rules over time. In a *first step* we ask what the *minimal share* of the entrepreneur has to be for her to truthfully apply *any given sequence* of funds to the project. The *second step* identifies the *incentive compatible* funding policy which maximizes the value for the entrepreneur and hence is adopted in equilibrium.

The solution to the minimization problem is, again, obtained explicitly by dynamic programming methods. Consider first the final period of the contract, denoted by T. The share S_T has to be high enough for her to invest the funds in the project rather than to divert them, or formally S_T has to satisfy:

$$\min_{S_T} \alpha_T p_T S_T R \geqslant c p_T. \tag{15}$$

The minimal share S_T in the ultimate period T is then given by

$$S_T = \frac{c}{\alpha_T R} \tag{16}$$

and the expected value of this arrangement to the entrepreneur is denoted by $E_T(\alpha_T)$. Solving the problem recursively we obtain a sequence of value functions, denoted by $E_T(\alpha_t)$, where α_t is the current posterior belief and T is the length of the entire contract. Consequently $T - t$ is the number of remaining periods in the contract. The incentive problem in period t is given by

$$E_T(\alpha_t) \triangleq \min_{S_t}\{\alpha_t p_t S_t R + \delta(1 - \alpha_t p_t) E_T(\alpha_{t+1})\} \tag{17}$$

subject to

$$E_T(\alpha_t) \geqslant \alpha_t p' S_t R + c(p_t - p') + \delta(1 - \alpha_t p') E_T(\alpha') \quad \forall p' \in [0, p_t). \tag{18}$$

We notice the intertemporal structure of the problem. If the entrepreneur correctly employs the funds, then with probability $\alpha_t p_t$ success occurs in period t. On the other hand, no success occurs with probability $1 - \alpha_t p_t$ in which case the project continues, but the prospects of future success will appear dimmer as α_{t+1} is given by

$$\alpha_{t+1} = \frac{\alpha_t(1 - p_t)}{1 - \alpha_t p_t} < \alpha_t.$$

The inequality Eq. (18) requires that for a given allocation of funds cp_t, the share S_t has to be large enough to prevent any diversion of funds. The diversion of funds, represented by $p' < p_t$, affects the payoffs for the entrepreneur in two ways. Consider first the contemporaneous effect. The likelihood of success today will be smaller as $p' < p_t$, but the entrepreneur enjoys the utility from the diverted funds $c(p_t - p')$. The second and dynamic effect is that a continuation of the contract in the next period becomes more likely. And as less funds are applied to the project today, there is less reason to change the posterior belief and clearly:

$$\alpha' = \frac{\alpha_t(1-p')}{1-\alpha_t p'} > \frac{\alpha_t(1-p_t)}{1-\alpha_t p_t} = \alpha_{t+1} \quad \text{for } p' \in [0, p_t).$$

After all, the event of no success should not surprise and lead to a smaller change in the posterior belief as less resources have been devoted to the project. In consequence, the entrepreneur will be more optimistic about *future* success as she has seen (and provided) less evidence against it, and naturally the continuation value will be higher with more optimistic beliefs:

$$\alpha' > \alpha_{t+1} \Longleftrightarrow E_T(\alpha') > E_T(\alpha_{t+1}).$$

If the entrepreneur were to withhold the funds, the *private* belief α' would diverge from the *public* belief α_{t+1}, and to fend off the informational asymmetry, the entrepreneur is granted an *informational rent*. The source of the rent is the control over the conditional probability p_t and through it over the learning process, and we shall refer to it as the *learning rent.*

It is now apparent that there are two forces which help to realign the interest of the entrepreneur with the ones of the investor. First, a larger share S_t if success occurs today relative to the share S_{t+1} if success occurs tomorrow. Second, the discounting of future returns at the rate δ depresses the incentives of the entrepreneur to postpone the successful realization of the project. In consequence, the more myopic the entrepreneur is, the less binding are the intertemporal incentive constraints.

The solution S_t of the minimization problem represented by (17) and (18) delivers the expected value $E_T(\alpha_t)$ the entrepreneur receives for a given funding policy, where $E_T(\alpha_t)$ satisfies the recursive equation:

$$E_T(\alpha_t) = \alpha_t p_t S_t R + \delta(1 - \alpha_t p_t) E_T(\alpha_{t+1}). \tag{19}$$

In the first step, we were concerned with the minimal share the entrepreneur has to receive for any given level of funding. In the next step we ask how and when the funds should be released so as to maximize the value of the venture. As the market for venture capital is competitive, in equilibrium the net value of the project will belong entirely to the entrepreneur. Formally, the problem is then given by

$$\max_{\{cp_0, cp_1, \ldots, cp_T\}} V_T(\alpha_0) \tag{20}$$

subject to

$$V_T(\alpha_0) \geqslant E_T(\alpha_0). \tag{21}$$

The constraint (21) incorporates the sequence of incentive constraints through $E_T(\alpha_0)$ and the participation of the investor by requiring non-negative profits in form of the inequality. The preceding analysis provides the important hints on the solution of Eqs. (20) and (21) as well. Consider any aggregate investment the investor would like to contribute to the project and the remaining issue is only how the funds should be distributed over time. If the funds are invested only slowly into the project, then the funding commitment necessarily extends over a longer horizon and the investor faces more intertemporal incentive constraints. These constraints increase the share of the entrepreneur and hence decrease the investor's share. Thus the optimal solution is to invest in each period up to the efficient level cp. The complete solution of S_t and $E_T(\alpha_t)$ based on the programs (20) and (21) is summarized as follows.

Proposition 3 (Share contract and entrepreneurial value).

(*i*) *The value function of the entrepreneur is given by*

$$E_T(\alpha_t) = cp\alpha_t \frac{1-\delta^{T-t}}{1-\delta} + cp(1-\alpha_t)\frac{1-\left(\frac{\delta}{1-p}\right)^{T-t}}{1-\frac{\delta}{1-p}}. \tag{22}$$

(*ii*) *The share function of the entrepreneur is given by*

$$S_t = \frac{c(1-p)}{\alpha_t R} + \frac{cp}{R}\frac{1-\delta^{(T-t)}}{1-\delta} + \frac{cp(1-\alpha_t)}{\alpha_t R}\frac{1-\left(\frac{\delta}{1-p}\right)^{T-t}}{1-\frac{\delta}{1-p}}. \tag{23}$$

Proof. See Appendix A. □

The intertemporal contract S_t ensures that the entrepreneur employs the capital in every period towards the discovery process. The three elements in the share contract as displayed in Eq. (23) may seem rather inaccessible at first, but can be decomposed and traced to the different aspects of the agency problem, namely (*i*) static agency costs, (*ii*) intertemporal agency costs, and (*iii*) informational agency costs.

(*i*) If the project would be financed only for single period and hence $T = t = 0$, then the minimal share for the entrepreneur to act properly would be

$$\frac{c}{\alpha_t R}. \tag{24}$$

(*ii*) If the investor would like to fund the venture until time $T > 0$, but could *observe* the evolution of the posterior α_t, so that the entrepreneur would not have access to the learning rent, then the minimal share offered at time t would have to be increased by

$$\frac{cp}{R}\frac{1-\delta^{(T-t)}}{1-\delta}. \tag{25}$$

The new aspect in the intertemporal agency problem is the option to withhold financing for a single period, but continue afterwards as instructed until T. To prevent the delay in any period, the investor has to provide stronger incentives. The higher additional compensation is necessary as the deadline T, at which funding is stopped is relatively remote. As the deadline comes closer, or $T-t$ is decreasing, the need for additional incentives becomes weaker. Notice also that the Eq. (25) only depends on the "time to go" and is independent of the posterior belief α_t.

(*iii*) Finally the informational agency costs are represented by

$$\frac{cp(1-\alpha_t)}{\alpha_t R}\frac{1-\left(\frac{\delta}{1-p}\right)^{T-t}}{1-\frac{\delta}{1-p}}, \tag{26}$$

which forms the basis for the learning rent of the entrepreneur. It depends on the value of the current beliefs and the rate p at which updating of the posterior beliefs occurs relative to the rate of discounting δ. The rate of updating p is the quantity of information the entrepreneur controls at each instant of time. The informational rent is hence increasing in the quantity of information under influence of the entrepreneur, but dampened by discounting, as the value of information today is larger than tomorrow. The three elements Eqs. (24)–(26) together determine the share of the entrepreneur. An illustration of the decomposition of the sharing rule is given in Fig. 2.

The behavior of the shares S_t over time is thus determined by an underlying option problem. The reward implied by the shares S_t has to be equal to the value of the option of diverting funds for a single period. The value of this particular option is determined as any regular option by the volatility of the underlying state variable and the time over which the option can be exercised. Here, the volatility is the conditional probability p at which updating occurs and the time of the option right is the remaining length of the funding, $T-t$.

The optimal (arm's length) contract is hence a time-varying *share* or *equity* contract. The time-varying share contract presents the solution to a *real option* problem, where the real option is the control of information in every period. Some implications for the intertemporal sharing of the returns are recorded next:

Proposition 4 (Evolution of shares over time).

(*i*) *S_t has at most one interior extremum in t and then it is a maximum.*

(*ii*) *For δ sufficiently close to 1, S_t is monotonically decreasing.*

Proof. See Appendix A. □

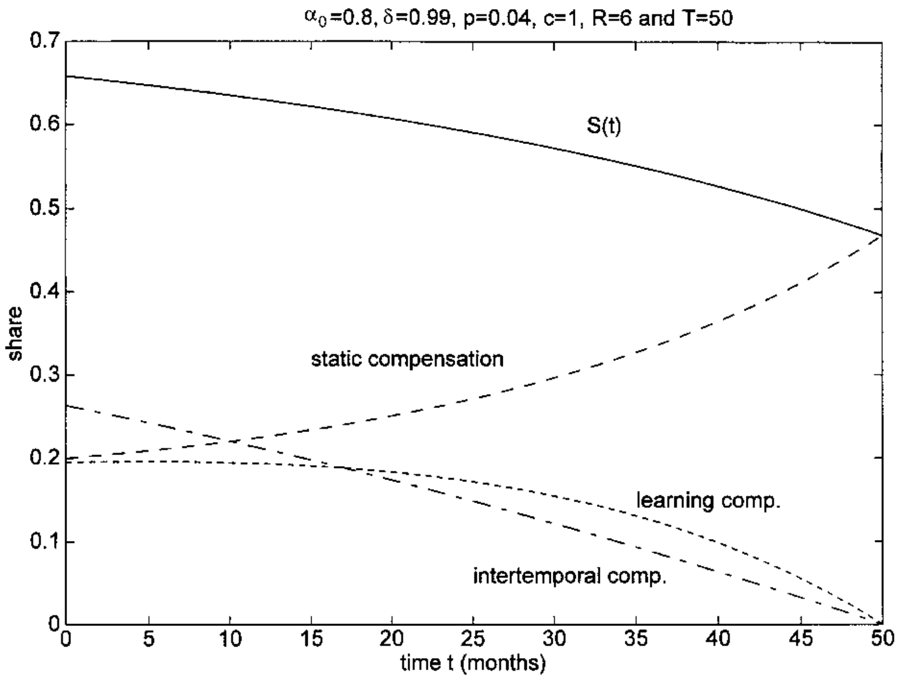

Fig. 2. Evolution of the share $S(t)$ over time and decomposition.

Earlier in this section we identified two elements which provide incentives to allocate the funds truthfully in period t: (a) decreasing shares over time and (b) sufficiently strong discounting. Proposition 4 identifies the interplay between these two forces. If discounting does not work because the remaining time horizon is too short (i) or discounting is too weak (ii), then the shares have to fall over time. On the other hand, the shares of the entrepreneur can only increase initially (i), when discounting (due to the length of the funding horizon $\hat{T}$) is sufficiently strong to insure incentive compatibility.

4.3. Equilibrium and inefficiency

The efficient funding policy and the associated value $E_T(\alpha_0)$ to the entrepreneur for any given funding horizon T is identified in Proposition 3. The final question is to what horizon the venture capitalist is willing to extend his commitment and whether the efficient horizon T^* can be attained. The entrepreneur participates in the project until T if the expected net value of the project exceeds the expected value the entrepreneur receives, as represented by the inequality Eq. (21): $V_T(\alpha_0) \geqslant E_T(\alpha_0)$. The efficient horizon T^* can be achieved if at T^* we have

$$V_{T^*}(\alpha_0) \geqslant E_{T^*}(\alpha_0). \tag{27}$$

The equilibrium contract and the implied funding horizon is determined with the assistance of Proposition 3 and the participation constraint (21) of the investor. We have to distinguish two cases. If the project is not sufficiently rich to be continued until T^*, or in other words if

$$V_{T^*}(\alpha_0) < E_{T^*}(\alpha_0),$$

then the funding horizon is determined by $\hat{T}$, where $\hat{T}$ is the largest time horizon for which the value of the project exceeds the value of the compensation for the entrepreneur

$$\hat{T} = \max\{T + 1 | V_T(\alpha_0) > E_T(\alpha_0)\}. \tag{28}$$

In the second case, the project is sufficiently rich to allow for an efficient financing, or formally:

$$V_{T^*}(\alpha_0) \geqslant E_{T^*}(\alpha_0), \tag{29}$$

in which case funding is extended until T^*. In equilibrium, the net value of the project has to be allocated completely to the entrepreneur, or

$$V_{\hat{T}}(\alpha_0) = E_{\hat{T}}(\alpha_0),$$

due to competition among the venture capitalists. Hence if the inequality holds strictly in either Eqs. (28) and (29), then the remaining surplus has to be given to the entrepreneur in a way compatible with the incentive constraints. In the case of Eq. (28) this can be achieved by a one period continuation as a "winding-down" phase, where $S_{\hat{T}}$ is determined as in Proposition 3 but with a smaller capital flow $cp_{\hat{T}} < cp$, as cp itself would violate the participation constraint of the investor by definition of $\hat{T}$. In the case of Eq. (29), the project is rich enough to guarantee the entrepreneur a larger share than the one determined by S_t in Proposition 2. But as these shares also have to satisfy the sequence of incentive constraints, their intertemporal behavior will be similar to S_t. The efficiency properties of the long-term contracting are summarized in the following proposition.

Proposition 5 (Inefficiency).

(i) Long-term contracts allow for an extended funding horizon $\hat{T}$ (relative to short-term financing), but never exceed T^*.

(ii) The funding horizon $\hat{T}$ of the optimal long-term contracts may not attain T^*.

(iii) The funding horizon $\hat{T}$ increases in R and α_0 and decreases in c.

Proof. See Appendix A. □

As the funding horizon $\hat{T}$ increases with improved conditions for the project, one may wonder whether the inefficiency indicated in Proposition 5 (*ii*) will eventually disappear entirely. The answer here is rather subtle. As, to take one example, the return R increases, $\hat{T}$ increases but so does T^*. Eventually T^* may become so large that the informational rent of the entrepreneur becomes too large and the project will have to be stopped at some $\hat{T} < T^*$. The example in Fig. 3 illustrates that better projects (with larger R) will indeed receive longer funding commitments as Proposition 5 predicts, but the equilibrium allocation may never attain social efficiency.

5. Liquidation value and security design

In this section, we introduce a liquidation value of $L_t > 0$, which is collected whenever the project ends without having succeeded. The value L_t represents intermediate outcomes of the venture and captures the proceeds from selling the remaining tangible assets if the project is liquidated. The value L_t is a deterministic function of time and does not depend on the behavior of the entrepreneur. We assume that $L_t \geqslant \delta L_{t+1}$ for all t. The liquidation value hence contains no information about the actions of the entrepreneur. It is meant to

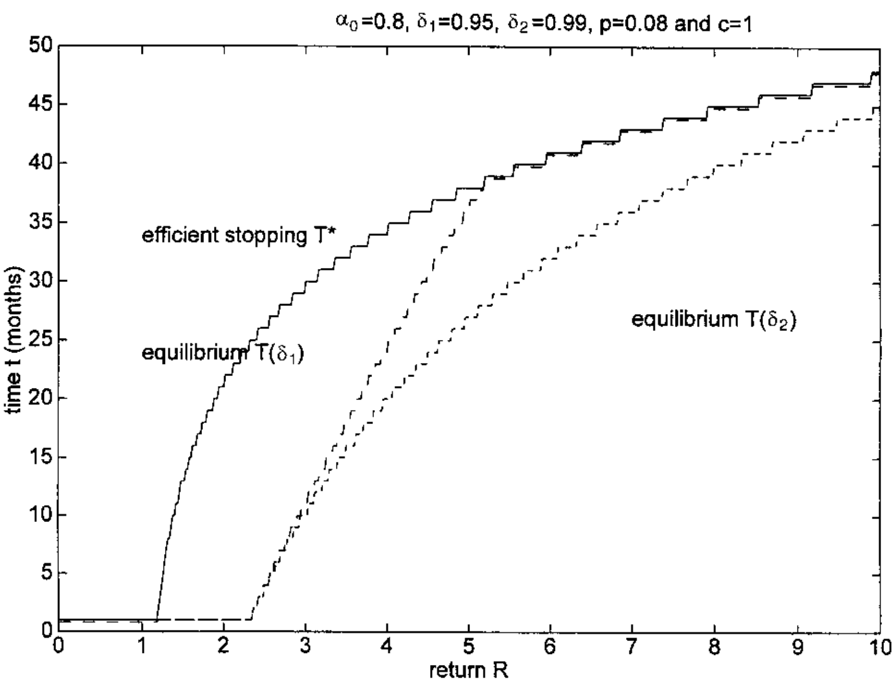

Fig. 3. Return R and duration of financing T.

represent the value created by verifiable actions. The condition on the growth of L_t merely ensures that the project is not continued without the entrepreneur. [9]

The liquidation value enhances the social value of the project. If the project is supposed to be liquidated in T, then the expected net present value is $(1-\alpha_0+\alpha_0(1-p)^T)\delta^T L_T$, which is the probability to reach T multiplied by the discounted value in T. The optimal contract should award the liquidation payoff so as to relax the financial constraint on the funding horizon $\hat{T}$. The idea of the optimal share contract under limited liability is to reward the entrepreneur if and only if she was successful. But a pure equity contract would give the entrepreneur a part of the liquidation value. This necessarily weakens the incentive structure, as by diverting funds, she would increase the likelihood of reaching the final period T, and with it an additional payoff $S_T L_T$. The optimal contract therefore needs to split the claims in the case of success from the ones in the case of liquidation. A pure equity contract cannot achieve this and a mixture between debt and common equity, or a convertible security become necessary.

Define a *debt contract* as a profile of debt claims $D_t \geqslant 0$, which are puttable at any time t and bear no coupon. Similarly, a time-varying *common stock contract* is denoted by a profile of fractions $A_t \geqslant 0$ of total equity that the entrepreneur receives if the project is terminated in t (successfully or unsuccessfully). The optimal financial contract can then be achieved by a mixture of debt and common equity, or a convertible security.

Proposition 6 (Security design).

(i) An optimal contract is provided by a mixture of debt and common equity with

$$D_t = \delta D_{t+1} \quad \forall t < T, \qquad D_T = L_T, \qquad A_t = S_t \frac{R}{R-D_t}. \tag{30}$$

(ii) A convertible preferred stock held by the investor with a nominal value of D_t and converted into a share $1-S_t$ of common stock if exercised in period t also represents an optimal contract.

Proof. See Appendix A. □

In "strip financing" the venture capitalist retains equity *and* debt, and the debt claim increases at the rate r, where $1/(1+r)=\delta$, until D_T reaches L_T. The incentive compatible equity share A_t of the entrepreneur, which would be S_t if indeed R would be distributed, has to be increased to account for the seniority claim of D_t on R in the case of success. An equivalent distribution

[9] The evolution of L_t could also be stochastic as long as $L_t \geqslant \delta \mathbb{E} L_{t+1}$.

of the pay-offs can be achieved by a convertible preferred stock, as indicated by Proposition 6(*ii*). The time-varying *conversion price* is given by $D_t/(1 - S_t)$. [10]

Interestingly, there is some empirical evidence that debt becomes more important as a financing tool towards later stages of venture projects. [11] In our model debt has a function only at the termination date and this might provide an element to understand this pattern in the dynamic capital structure in venture financing. The preceding discussion already indicated that in many situations a pure sharing contract cannot be optimal. The corollary presents precise conditions.

Corollary 1. *A common stock contract A_t does not constitute an optimal contract if the financing ends inefficiently early: $\hat{T} < T^*$.*

A combination of debt and equity could relax the financial constraints faced by a pure equity solution. Common equity is inefficient because *equity gives too little of the liquidation value* to the venture capitalist in case the project fails. We note finally that a pure debt contract may be inefficient for exactly the opposite reason. *Debt may convey too much of the liquidation value* to the venture capitalist and encourage premature liquidation.

6. Monitoring and job rotation

In this section aspects of relationship financing are considered which may reduce the inefficiency indicated in Proposition 5: (*i*) monitoring and (*ii*) changing the management. Both modifications imply the transfer of substantial control rights to the venture capitalist and hence point to the *relationship aspect* of venture capital financing. The focus is on the optimal *timing* of these control instruments.

Consider first the possibility of monitoring the research effort of the venture capitalist. Monitoring is costly and the venture capitalist has to spend $mp_t > 0$ to monitor the entrepreneur in period t. [12] In return, the venture capitalist receives an accurate signal about the application of the funds. The signal is verifiable and hence punishment in case of a deviation (a signal which differs from the contractually agreed effort) can be implemented. Hence, the moral hazard problem is eliminated in the periods where monitoring takes place.

[10] We recall that S_t is the *minimal* incentive compatible share if the entire R is distributed.

[11] See Amit et al. (1997).

[12] The cost of monitoring is thus linear in the flow of funds cp_t. It can be conceived as an accounting or control system, the cost of which are increasing in the size of the operation to be monitored.

The optimal timing is determined by the costs and benefits of monitoring in the intertemporal agency problem. The *expected present cost* of monitoring in period t is simply mp_t adjusted by discounting and the probability of reaching period t, as the project may be successfully completed before t. Hence the expected present cost of monitoring are decreasing with time. The benefits of monitoring come from the *static* as well as the *intertemporal* component in the share S_t. If monitoring occurs in period t then the share of entrepreneur can be set to $S_t = 0$, as there is no need to provide any incentives in period t. But monitoring in period t also affects the incentives in all preceding periods. We recall that the value of diverting funds in $t' < t$ was partially due to the possibility to have a share in the success later on. Monitoring in period t excludes this option at least for period t and hence the share of the entrepreneur can be reduced marginally in all periods *preceding* t. The direct benefits from monitoring ($S_t = 0$) are clearly decreasing in t due to discounting, but the indirect benefits increase as the number of periods which enjoy the marginal reduction increase. Efficient monitoring occurs where the reduction in the informational rent for the entrepreneur *per* unit cost of monitoring is maximized.

Proposition 7 (Monitoring policy). *The optimal policy is to monitor towards the end of the project.*

Proof. See Appendix A. □

Monitoring then occurs towards the end of the venture as the discounting of the costs and the indirect benefits dominate the direct savings due to monitoring uniformly. Notice also that in the monitoring phase all the residual gains are allocated to the investor.

The timing of the monitoring in our model is partially due to the uniform rate at which information is generated. More generally, the benefits of monitoring in period t are increasing in the amount of information which is generated in period t. Some other implications of a more general information structure will be discussed in the next section.

Consider next the possibility of replacing the current manager, entrepreneur or not, by a new manager. The financing horizon $\hat{T}$ of the project may then be subdivided into several managerial job spells. We investigate the following extension of the model. In any period t, the entrepreneur or the incumbent manager can be replaced by a new manager. There are no differences between the original entrepreneur and successive managers, in particular concerning productivity and moral hazard. As before, the entrepreneur initially owns the project and tries to capture as much of the surplus as possible.

Managerial job rotation reduces the informational rent of each manager by restricting the duration of each individual manager. However, for the (initial) entrepreneur *and* the investor outside managers are costly, as the founding partners have to concede shares to the future managers. The objective for the initial pair is therefore to minimize the pay-outs to succeeding managers. For this reason, the replacement of the entrepreneur should be scheduled as late as possible.

Each manager requires a compensation which is determined exactly as the shares S_t of the entrepreneur. As there is no cost in replacing the manager, the pay-out is minimized when the manager changes in every period. [13] The cost of developing the venture with a sequence of manager is then equal to $cp_t + cp_t$, where one term accounts for the cost of investment and the other for the incentive costs of the manager. Proposition 2 indicates that the efficient termination then occurs at T^s.

Proposition 8 (Replacement).

(i) It is optimal to replace the entrepreneur if and only if the long-term contract would otherwise stop at $\hat{T} < T^s$.

(ii) With optimal replacement the project will be stopped at T^s. The entrepreneur remains in place until the value of the project net of compensations to the managers is disbursed to her. Thereafter, a new manager is brought in every period.

Proof. See Appendix A. □

The replacement of the founding entrepreneur constitutes an empirical regularity in the venture capital industry, see Gorman and Sahlman (1989). We may also add that earn outs are inefficient in this framework. [14] The rationale is essentially the same as the one exposed in Section 5. A quit payment works against the idea of making all benefits contingent on success. Thus, one insight of the present model is that earn outs, by providing insurance in the case of failure, are a costly practice in an environment with an uncertain completion date.

7. Discussion and extensions

In this paper, the venture is characterized by a simple binary structure. In each period success is possible and the likelihood of success depends on the belief α_t and the intensity p_t at which the project is developed. The investment flow influences only the probability by which success is generated, and the prize

[13] We note that the general analysis suggested here carries over to more general settings with switching costs or decreasing efficiency of subsequent managers.

[14] See Sahlman (1990) for the observation that the founding entrepreneur often receives little protection against the threat of being put aside.

R is constant over time. In this section we discuss how several extensions would modify our results. We consider in particular (*i*) time-varying returns R_t, (*ii*) multiple signals and staging and (*iii*) a time-varying information flow.

Time varying returns R_t. The value of the successful realization, R, was assumed to be constant throughout this paper. But the investment process may also have a cumulative effect on the value of a successful realization and hence lead to an increasing sequence of R_t over time. An increasing value R_t would tend to reduce the inefficiency problem documented in Proposition 5. With an increasing R_t, the share S_t of the entrepreneur could be reduced further towards the end as the incentive constraints are based on the composite S_tR_t. Conversely, a decreasing sequence R_t would make funding even more precarious. The positive effect of an increasing R_t points to the importance of value creation and production of tangible assets during the investment process. [15]

Multiple signals and staging. This paper portrays a simple venture which ends after a single positive signal is received. Clearly the arrival of good and bad news may be a more complex process. Frequently, projects are divided into various stages which are defined by the completion of certain intermediate results. In these circumstances continued financing may be conditional on the successful completion of earlier stages.

The basic model we analyzed describes the evolution of the incentives in any component of such staged projects. The entire project would simply be a sequence of such stages, each giving rise to an optimal stopping time. In each individual problem R_t would constitute the continuation value after having received an intermediate result. Each stopping problem determines how long to wait for the arrival of an intermediate signal before the project is stopped. Otherwise the previous analysis carries over and the share contract for each stage concerns the sharing of the incremental value produced in that stage.

The sequential arrival of information then supports *stage financing* as the optimal arrangement. To see this, suppose the contracting parties had the choice to contract either on a strong final signal (i.e. completion of the marketable product) or on a finer sequence of intermediate signals. The sequencing, implied by the staging, splits the horizon over which the intertemporal incentive constraints have to be compounded. In other words, the entrepreneur realizes that she must produce the intermediate result first in order to receive continued financing. This reduces her incentives to procrastinate in the intermediate periods.

Time varying information flow. The investment flow cp_t controls the conditional probability of success p_t, and via p_t, the evolution of the posterior from α_t to α_{t+1}. The marginal cost of generating success in terms of probability was

[15] Gompers (1995) presents evidence that the value of tangible assets and the length of the financing rounds increase with the duration of the venture.

assumed to be constant in the level of p_t (up to the maximum p) as well as constant over time.

The optimal investment problem with a general increasing and convex cost function $c(p_t)$ shares the same structural features as the model here, but, naturally, would not lead to an explicit solution as presented in Proposition 1 and 3. If the costs of generating success vary over time, then the information flow, associated with p_t, would be timed so as to coincide with periods where the costs are relatively low. Periods with a higher p_t would then constitute periods with a "learning boost". We saw in the previous section that the benefits of monitoring are positively related to the volatility of information. Monitoring would then tend to occur in periods where much information is produced. In the presence of sequential arrival of information, we would expect monitoring to be most prevalent in periods before the next financing stage. [16]

8. Conclusion

This paper investigated the provision of venture capital when the investment flow controls the speed at which the project is developed. As the binary outcome of the project is uncertain, the speed of development influences the (random) time at which the project yields success *and* the information which is acquired by the investment flow. The role of the entrepreneur is to control the application of the funds which are provided by the venture capitalist.

The paper provides a rationale for long-term contracting as these contracts achieve best the goal of distributing the entrepreneur's return over time in a way which maximizes the research horizon. It is further shown that the compensation of the entrepreneur is similar to an option contract, and as such depends on the length of the contract and the volatility of the information induced through her actions. The option expresses the value of the intertemporal incentive constraint. As the value of the option may become exceedingly large, relationship financing may become necessary. In consequence, the optimal timing of monitoring and replacement of the entrepreneur are analyzed.

The paper focuses on the financing of venture projects. But the interaction between investment and learning process and the incentives necessary to implement both processes is central for the financing of R&D in general. The present work may therefore be considered as a step in developing further insights into the optimal financing of innovation.

[16] See Lerner (1995) for empirical evidence on such a correlation.

Acknowledgements

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Appendix A

This appendix collects the proofs to all propositions in the paper.

Proof of Proposition 1. The value of an *arbitrary* investment policy starting at t and ending after T, denoted by $\{cp_t, cp_{t+1}, \ldots, cp_T\}$, can be expressed by telescoping the returns over time as

$$\sum_{s=t}^{T} \delta^{s-t} \left((\alpha_s p_s R - cp_s) \prod_{r=t}^{s-1} (1 - \alpha_r p_r) \right), \tag{A.1}$$

where, by convention, $\prod_{r=t}^{t-1} (1 - \alpha_r p_r) = 1$. From Bayes' rule in Eq. (2) we obtain

$$\prod_{r=t}^{s-1} (1 - \alpha_r p_r) = \frac{\alpha_t}{\alpha_s} \prod_{r=t}^{s-1} (1 - p_r) \tag{A.2}$$

and

$$\frac{\alpha_t}{\alpha_s} = \alpha_t + (1 - \alpha_t) \frac{1}{\prod_{r=t}^{s-1} (1 - p_r)}. \tag{A.3}$$

By substituting Eqs. (A.2) and (A.3) into Eq. (A.1), we obtain after collecting terms:

$$\sum_{s=t}^{T} \delta^{s-t} \left((\alpha_t p_s R - \alpha_t cp_s) \prod_{r=t}^{s-1} (1 - p_r) - cp_s (1 - \alpha_t) \right). \tag{A.4}$$

Consider now the optimal investment policy cp_t in period t. By rewriting (A.4) as

$$p_t(\alpha_t R - c) + \sum_{s=t+1}^{T} \delta^{s-t}\left(\alpha_t p_s (R-c)(1-p_t)\prod_{r=t+1}^{s-1}(1-p_r) - cp_s(1-\alpha_t)\right), \tag{A.5}$$

it appears immediately that Eq. (A.5) is linear in p_t, and thus it is either optimal to allocate the capital maximally at cp or not to allocate any capital at all. Hence until the project is stopped, in each period capital is allocated at the maximal rate cp. In consequence the stopping time is given by T^* as defined in Eq. (10). By setting $p_s = p$ for all $s = t, t+1, \ldots, T^*$, we obtain:

$$V(\alpha_t) = \sum_{s-t}^{T^*} \delta^{s-t}\left(\alpha_t p(R-c)(1-p)^{s-t} - (1-\alpha_t)cp\right),$$

and in particular for α_0,

$$V(\alpha_0) = \alpha_0 p(R-c)\frac{1-\delta^{T^*}(1-p)^{T^*}}{1-\delta(1-p)} - (1-\alpha_0)cp\frac{1-\delta^{T^*}}{1-\delta}. \qquad \square$$

The proof to Proposition 3 relies on Lemma A.1, which describes the optimal policy for the entrepreneur.

Lemma A.1 (Optimal policy of the entrepreneur).
(*i*) *The optimal policy for the entrepreneur is always either* $p' = 0$ *or* $p' = p_t$.
(*ii*) *If the optimal policy is* $p' = p_t$ *for some* α_t, *then* p_t *remains the optimal policy for all* $\alpha > \alpha_t$.

Proof. (*i*) Consider first the expected value of the entrepreneur for an arbitrary assignment of shares, under the assumption that she truthfully applies the funds to the project:

$$\sum_{s=t}^{T} \delta^{s-t}\left(\alpha_s p_s S_s R \prod_{r=t}^{s-1}(1-\alpha_r p_r)\right),$$

which can be expressed, after using Eqs. (A.2) and (A.3), as

$$\sum_{s=t}^{T} \delta^{s-t}\left(\alpha_t p_s S_s R \prod_{r=t}^{s-1}(1-p_r)\right),$$

or, after separating the behavior in t, as

$$\alpha_t p_t S_t R + (1-p_t)\sum_{s=t+1}^{T} \delta^{s-t}\left(\alpha_t p_s S_s R \prod_{r=t+1}^{s-1}(1-p_r)\right).$$

If the entrepreneur would consider diverting funds in period t, then she wishes to maximize the value of the deviation p', which is given by

$$\alpha_t p' S_t R + (1 - p') \sum_{s=t+1}^{T} \delta^{s-t} \left(\alpha_t p_s S_s R \prod_{r=t+1}^{s-1} (1 - p_r) \right) + c(p_t - p'). \tag{A.6}$$

The expression Eq. (A.6) is linear in p' and the first part of the lemma follows directly.

(*ii*) By the linearity of Eq. (A.6), if it is optimal to choose $p' = p_t > 0$ under α_t, then we have

$$\alpha_t p_t S_t R - \alpha_t p_t \sum_{s=t+1}^{T} \delta^{s-t} \left(p_s S_s R \prod_{r=t+1}^{s-1} (1 - p_r) \right) \geqslant c p_t > 0, \tag{A.7}$$

where the first inequality is due to the optimality and the second is due to $p_t > 0$. By monotonicity, the first inequality is preserved by any $\alpha > \alpha_t$. □

Proof of Proposition 3. (*i*) Consider the value function $E_T(\alpha_0)$ of the entrepreneur in the initial period $t = 0$, obtained solving Eqs. (17) and (18) recursively. By Lemma A.1, $E_T(\alpha_0)$ has to be equal to the value generated by a maximal deviation today which is followed by compliance in all future periods:

$$E_T(\alpha_0) = c p_0 + \sum_{t=1}^{T} \delta^t \left(\alpha_{t-1} p_t S_t R \prod_{s=1}^{t-1} (1 - \alpha_s p_s) \right), \tag{A.8}$$

which in turn is equivalent to $E_T(\alpha_0) = c p_0 + (\alpha_0/\alpha_1)\delta E_T(\alpha_1)$. The general recursion of the value function then yields:

$$E_T(\alpha_t) = c p_t + \frac{\alpha_t}{\alpha_{t+1}} \delta E_T(\alpha_{t+1}), \tag{A.9}$$

from which we obtain after recursive substitution

$$E_T(\alpha_0) = \sum_{t=0}^{T} \delta^t \frac{\alpha_0}{\alpha_t} c p_t, \tag{A.10}$$

and rearranging by using Eq. (A.3) we get:

$$E_T(\alpha_0) = \alpha_0 c \sum_{t=0}^{T} \delta^t p_t + (1 - \alpha_0) c \sum_{t=0}^{T} \delta^t \frac{p_t}{\prod_{s=0}^{t-1} (1 - p_s)}, \tag{A.11}$$

which in turn is equivalent to Eq. (22) if $p_t = p$ for all t.

(*ii*) The share contract S_t is obtained by equating Eqs. (A.9) and (19) which yields: $\alpha_t p_t S_t R = c p_t + (\alpha_t/\alpha_{t+1})\delta p_t E_T(\alpha_{t+1})$, and after using Eq. (A.9) again, we get

$$\alpha_t p_t S_t R = (1 - p_t) c p_t + p_t E_T(\alpha_t), \tag{A.12}$$

which yields immediately Eq. (23) after replacing $E_T(\alpha_t)$ with the explicit expression obtained in Eq. (A.11) if $p_t = p$ for all t. Finally, $p_t = p$ follows immediately from Proposition 1 and the fact that the incentive compatible contract S_t only depends on the continuation value as just described. □

Proof of Proposition 4. (*i*) Consider the share function S_t as derived in Eq. (23) and substitute α_t by Bayes' rule Eq. (4) to obtain

$$S_t = \left(1 + \frac{1-\alpha_0}{\alpha_0(1-p)^t}\right)\frac{c(1-p)}{R} + \frac{cp}{R}\frac{1-\delta^{(T-t)}}{1-\delta} + \frac{1-\alpha_0}{\alpha_0(1-p)^t}\frac{cp}{R}\frac{1-\left(\frac{\delta}{1-p}\right)^{T-t}}{1-\frac{\delta}{1-p}}. \tag{A.13}$$

It is then sufficient to prove that the share function $S_t \equiv S(t)$ as a *continuous function* of t has at most one interior extremum, and that it has to be a maximum. Based on Eq. (A.13), one obtains after some rearranging: $S'(t) \gtrless 0 \iff$

$$\left(\frac{1-p}{\delta}\right)^t\left(\frac{\delta^T}{1-\delta} + \frac{1-\alpha_0}{\alpha_0\left(1-\frac{\delta}{1-p}\right)}\left(\frac{\delta}{1-p}\right)^T\right) \lessgtr \frac{1-\alpha_0}{\alpha_0\left(1-\frac{\delta}{1-p}\right)}\frac{p\ln(1-p)}{(1-\delta)\ln\delta}, \tag{A.14}$$

from which it follows after inspection that $S'(t)$ can cross zero at most once and only from above.

(*ii*) By inspecting again condition Eq. (A.14), we find that as $\delta \to 1$, the right-hand side of the equivalent condition eventually becomes positive and the left hand side negative, which proves the claim. □

Proof of Proposition 5. (*i*) Consider any project with $\alpha_t > \alpha^s > \alpha_{t+1} > \alpha^*$, when financing occurs at the rate of cp. Then, by Proposition 2, there will be no financing in period $t+1$ with short-term contracts, although it would be efficient, since $\alpha_{t+1} > \alpha^*$. Under a long-term contract, the investor can be given $\alpha_t(1-S_t)pR > pc$ in period t, and $\alpha_{t+1}(1-S_{t+1})pR < pc$, and have his intertemporal participation constraint balanced over the periods. This then allows financing to proceed strictly longer than under a short-term contract and enhance the efficiency of the contractual arrangement.

(*ii*) The example associated with Fig. 3 verifies the claim.

(*iii*) Consider the difference $V_T(\alpha_0) - E_T(\alpha_0)$ for a given T, as given by Proposition 1 and 3. The difference is increasing in R and α_0 and decreasing with c. The equilibrium horizon $\hat{T}$ can be increased with increases in the difference $V_T(\alpha_0) - E_T(\alpha_0)$. □

Proof of Proposition 6. The value of the project with liquidation value, denoted by $V_T^L(\alpha_0)$, is given by

$$V_T^L(\alpha_0) = V_T(\alpha_0) + \delta^T\left(\alpha_0(1-p)^T + (1-\alpha_0)\right)L_T,$$

where $V_T(\alpha_0)$ is as defined earlier in Proposition 1. The duration $\hat{T}$ of the contract is determined by

$$\hat{T} = \max\left\{T+1\middle|V_T^L(\alpha_0) > E_T(\alpha_0)\right\}, \tag{A.15}$$

where $E_T(\alpha_0)$ is as defined in Proposition 3 and the last period $\hat{T}$ is again a "winding-down" period which insures that in equilibrium:

$$V_{\hat{T}}^L(\alpha_0) = E_{\hat{T}}(\alpha_0). \tag{A.16}$$

The entrepreneur receives the loan necessary to pursue the project until $\hat{T}$ in advance.

(i) Since the debt claim grows at the rate r, the investor is indifferent over the particular period t at which he claims payment of the debt as long as he is assured to receive D_t in the particular period t. Moreover, in any period in which he does not claim the debt there is some probability that a success occurs. Hence he will claim the debt only if no success has been observed in period $\hat{T}$. By condition Eq. (A.16) he is willing to participate. Finally, for the entrepreneur, her share will be higher, namely A_t as specified in Eq. (30), to compensate her for the debt claim which has seniority. But $A_t(R - D_t)$ is just equal to the minimal compensation S_tR, necessary to satisfy the intertemporal incentive constraints as proven in Proposition 3. Hence the contract induces her to truthfully direct the funds to the project as well. Thus the specified mixture of debt and equity implements the optimal outcome.

(ii) The argument is almost identical to the one provided under (i). □

Proof of Corollary 1. Suppose the entrepreneur receives a share $A_T > 0$ of the liquidation proceeds if liquidation occurs in T. The modified incentive constraint in period T then becomes

$$\alpha_T p A_T R + (1 - \alpha_T p) A_T L_T \geqslant cp + A_T L_T,$$

which implies that surplus the entrepreneur can guarantee himself in T increases from cp to $cp + A_T L_T$. From Eq. (A.12) in the proof of Proposition 3, we can then infer that this in fact increases A_t in all periods. Thus the value of the entrepreneur, denoted by $E_T^L(\alpha_0)$ when she receives a share of the liquidation value, is higher than when she does not. If $\hat{T}$, defined by

$$\hat{T} = \max\left\{T+1\middle|V_T^L(\alpha_0) > E_T^L(\alpha_0)\right\},$$

is indeed smaller than T^*, then we clearly have $\bar{T} > \hat{T}$, with $\bar{T}$ being defined by

$$\bar{T} = \max\left\{T+1\middle|V_T^L(\alpha_0) > E_T(\alpha_0)\right\},$$

since $E_T(\alpha_0) < E_T^L(\alpha_0)$, for all T. But this implies in particular that $V_{\bar{T}}^L(\alpha_0) > V_{\hat{T}}^L(\alpha_0)$ as $\bar{T} > \hat{T}$. Since in equilibrium we have $V_{\hat{T}}^L(\alpha_0) = E_{\hat{T}}^L(\alpha_0)$ and $V_{\bar{T}}^L(\alpha_0) = E_{\bar{T}}(\alpha_0)$, it follows that a pure sharing contract is not optimal and will not be chosen in equilibrium by the entrepreneur. □

Proof of Proposition 7. Monitoring is costly and reduces the social surplus which can be distributed between entrepreneur and venture capitalist. The benefit of monitoring is the reduction in $E(\alpha_0)$ which can then be used to extend the length of the project financing. Monitoring should therefore occur in those periods where the reduction in $E(\alpha_0)$ is maximized *per unit cost of monitoring*. The expected cost of monitoring in period t is given by $C(t) \triangleq \delta^t(\alpha_0(1-p)^t + 1 - \alpha_0)mp$. The benefit of monitoring in period t only is $B(t) \triangleq E(\alpha_0) - E^t(\alpha_0)$, where the superscript t denotes the period in which monitoring occurs. We next compute the benefit of monitoring explicitly. Suppose we would monitor in period t, then the value to the entrepreneur in t is

$$E^t(\alpha_t) = \delta(1-\alpha_t p)E(\alpha_{t+1}) = \delta(1-\alpha_t p)\sum_{s=t+1}^{T} \delta^{s-(t+1)} \frac{\alpha_{t+1}}{\alpha_s} cp,$$

after using Eqs. (A.9) and (A.10). Rewritten in terms of α_t, we obtain

$$E^t(\alpha_t) = \delta(1-p)\sum_{s=t+1}^{T} \delta^{s-(t+1)} \frac{\alpha_t}{\alpha_s} cp.$$

The value function in $t-1$, with monitoring in period t, can be obtained by backwards induction:

$$E^t(\alpha_{t-1}) = cp + \delta^2(1-\alpha_{t-1}p)\frac{\alpha_t}{\alpha_{t+1}} \sum_{s=t+1}^{T} \delta^{s-(t+1)} \frac{\alpha_{t+1}}{\alpha_s} cp. \tag{A.17}$$

For the general recursion we would like to write all numerators in terms of α_{t-1}, and obtain with $\alpha_t(1-\alpha_{t-1}p) = \alpha_{t-1}(1-p)$, the following expression for (A.17):

$$E^t(\alpha_{t-1}) = \frac{\alpha_{t-1}}{\alpha_{t-1}} cp + \delta^2(1-p)\sum_{s=t+1}^{T} \delta^{s-(t+1)} \frac{\alpha_{t-1}}{\alpha_s} cp.$$

The general recursive value function is then obtained by

$$E^t(\alpha_0) = \sum_{s=0}^{t-1} \delta^s \frac{\alpha_0}{\alpha_s} cp + (1-p)\sum_{s=t+1}^{T} \delta^s \frac{\alpha_0}{\alpha_s} cp,$$

and hence the gains from monitoring are

$$B(t) = \delta^t \frac{\alpha_0}{\alpha_t} cp + p\sum_{s=t+1}^{T} \delta^s \frac{\alpha_0}{\alpha_s} cp,$$

or equivalently, by Eqs. (A.10) and (A.12), $B(t) = \delta^t \alpha_0((1-p)cp + pE(\alpha_t))$. We then need to maximize

$$\max_t \frac{B(t)}{C(t)} \iff \max_t \frac{(1-p)cp + pE(\alpha_t)}{(1-p)^t}. \tag{A.18}$$

We shall show that the maximum is always achieved at $t = T$, and as t is discrete, it is sufficient to show that

$$\frac{(1-p)cp+pE(\alpha_{t+1})}{(1-p)^{t+1}}-\frac{(1-p)cp+pE(\alpha_t)}{(1-p)^t}>0, \quad \forall t\leqslant T,$$

which is verified by substituting $E(\alpha_t)$ by $E(\alpha_{t+1})$, using Eq. (A.9). □

Proof of Proposition 8. (*i*) The social cost of continuation with a sequence of one-period managers is $cp + cp$, where one term reflects the cost of financing and the other term the incentive costs to the manager. The efficient stopping point for this arrangement is therefore α^s. The entrepreneur is only replaced by the manager when it leads to the creation of surplus, which by implication can only occur if exclusive financing with the entrepreneur would only lead to $\hat{T} < T^s$.

(*ii*) The compensation to the managers is minimized by replacing them in every period as they receive no intertemporal rents in this case. From the viewpoint of the entrepreneur and the venture capitalist, the compensation of new managers is a cost as similar to the monitoring cost and hence the sequential employment of entrepreneur and manager is as in the case of monitoring. □

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