1. **Third Degree Price Discrimination.** Consider a monopolist (say a local movie theatre in Fort Lauderdale) which has two distinct client groups, adults and seniors. The inverse demand for the adults is given by

\[ p(q_A) = a - bq_A, \]

and the inverse demand of retirees is given by

\[ p(q_B) = \frac{a}{3} - \frac{b}{3}q_R. \]

(a) Describe the demand function in the two markets graphically and then compute the demand elasticity in each market.

(b) Compute the demand function \( q(p) \) under the assumption that the movie theater can only offer a single price to both segments of the market. (Hint: at a given price add the demand of the adults and senior market. You need to go from the inverse demand function to the demand function.) Illustrate the aggregate demand function in contrast to the demand functions in each segment. Now compute the optimal price of the movie theatre when it can only offer a single and common price to the market segments. Who goes to the movies and who doesn’t?

(c) Next we allow the movie theatre to offer different prices in each segment and customers cannot misrepresent their identity. What is the optimal price in each one of the markets?

(d) Compare the welfare of the consumers and the revenue of the consumer after the introduction of different prices across different segments. Explain.

2. Consider the oligopoly model we discussed in class with \( I \) competitors and linear demand and cost functions:

\[ p(q) = a - bq, \quad c(q) = c \cdot q \]
In class, we discussed informally, the outcome under a collusive agreement under which a representative firm $i$ would choose (for all other firms) a quantity that would maximize the industry profit. Formally describe the resulting optimization problem, and derive the optimal collusive agreement. Discuss how the optimality conditions differ in the collusive agreement from the Cournot oligopoly agreement.

3. Consider the oligopoly model we discussed in class with $I = 2$ competitors and linear demand and cost functions:

$$p(q) = a - bq, \quad c(q) = c \cdot q.$$  

(a) Formally derive the best response function for every firm $i = 1, 2$:

$$BR_i(q_j)$$

(b) Consider any arbitrary starting point $q^0 = (q_1^0, q_2^0) \in [0, (a - c) / b]$ and repeatedly apply the best response function, so that

$$(q_1^1, q_2^1) = (BR_1(q_2^0), BR_1(q_1^0))$$  

and more generally

$$(q_1^k, q_2^k) = (BR_1(q_2^{k-1}), BR_1(q_1^{k-1})).$$ (1)

i. Graphically describe the behavior of (1).

ii. Show formally that the limit $\lim_{k \to \infty} (q_1^k, q_2^k) = ((a - c) / 3b, (a - c) / 3b)$, independently, i.e. for every $q^0$. Briefly, explain your findings verbally.

4. The duopoly model we have been working with views firms as choosing their quantities of output, with the market then setting a common price for the two firms. Here is an alternative model. Firms 1 and 2 set prices $p_1$ and $p_2$. If $p_1 < p_2$, firm 1 sells quantity $A - Bp_1$ and firm 2 sells nothing. Similarly, if $p_1 > p_2$, firm 2 sells quantity $A - Bp_2$ and firm 1 sells nothing. If $p_1 = p_2$, each firm sells half of the quantity $A - Bp_1 = A - Bp_2$. Suppose that each firm has constant marginal cost $c$, so that the cost of producing output $x_i$ for firm $i$ is $C(x_i) = cx_i$.

(a) Find the equilibrium prices $p_1$ and $p_2$ in this market. This is not a job for calculus, because the firms’ payoffs are not differentiable functions of $p_1$ and $p_2$. Instead, try a few combinations of prices, and for each one, ask yourself whether each firm is doing the best it can given the other firm’s price, or whether either firm has an incentive to change its price. You have an equilibrium when each firm is setting a price that maximizes its profits, given the price of the other firm.

(b) How does the outcome you’ve found in [4a] compare to the equilibrium of the quantity-setting model, or to the competitive outcome in this market? Now suppose you are involved in a case before the
Justice Department, in which a merger is to be evaluated that will leave a market with only two firms. The concern is that this merger will lead to higher consumer prices. If you represented one of the firms that wanted to merge, which model of the resulting duopoly market would you be inclined to use as the basis for your analysis? Which would you use if you worked for the Justice Department? As an outsider, which do you think is more appropriate?

**Reading Assignment:** NS Chapter 14, 15