Game Theory: Dominance and Mixing
• players:

\[ i = 1, 2, ..., I \]

• actions:

\[ a_i \in A_i \]

• action profile, vector

\[ a = (a_1, ..., a_I) = (a_i, a_{-i}) \]

• payoff functions, utility functions:

\[ u_i : \times_{j=1}^I A_j \to \mathbb{R} \]

and we write

\[ u_i (a_1, ..., a_i, ..., a_I) = u_i (a_i, a_{-i}) \]

• a game \( \Gamma \) is triple

\[ \Gamma = \{\{1, .., I\}, \{A_i\}_i, \{u_i\}_i\} \]
Dominant and Dominated Strategies

- A strategy $a_i$ is dominant for $i$ if
  \[ u_i(a_i, a'_i) \geq u_i(a', a'_i), \text{ for all } a'_i \in A_i, a'_{-i} \in A_{-i} \]

- A strategy $a_i$ is (strictly) dominated for $i$ if for all $a_{-i}$ there exists $a'_i$ such that
  \[ u_i(a_i, a_{-i}) < u_i(a'_i, a_{-i}) \]

- Iteratively eliminated strictly dominated strategies:

  \[
  \begin{array}{ccc}
  L & M & R \\
  T & 2,3 & 2,1 & 3,2 \\
  B & 1,2 & 4,2 & 1,3 \\
  \end{array}
  \]

- The Cournot duopoly ...
consider the matching pennies game:

\[
\begin{array}{ccc}
     & H & T \\
H & -1, 1 & 1, -1 \\
T & 1, -1 & -1, 1 \\
\end{array}
\]

a pure strategy equilibrium does not exist, and hence we look for a mixed strategy equilibrium.

a mixed strategy \( \sigma_i \) is a probability distribution such that:

\[
\sigma_i (a_i) \geq 0, \text{ and } \sum_{a_i \in A_i} \sigma_i (a_i) = 1
\]
even when pure strategy equilibria exists, they may not be the only ones

battle of the sexes again:

\[
\begin{array}{cccc}
B & O \\
B & 3,1 & 0,0 \\
O & 0,0 & 1,3 \\
\end{array}
\]

we begin with the best response
a strategy is not a single action, but an entire plan of what to do in the game

the Stackelberg leadership game
• consider the linear demand function of the final consumer facing a retailer $p_r$

$$q_r = 1 - p_r$$

• the retailer has to procure from a wholeseller who in turn charges a price $p_w$

• retailer and wholeseller have zero marginal cost
consider the optimal price of the retailer for a fixed cost $p_w$:

final consumer facing a retailer $p_r$

$$\max (1 - p_r) (p_r - p_w)$$

and hence

$$1 - 2p_r + p_w = 0 \rightarrow p_r = \frac{1}{2} (p_w + 1)$$

wholeseller have zero marginal cost:

$$\max \left( 1 - \frac{1}{2} (p_w + 1) \right) p_w$$

and hence

$$\left( 1 - p_w - \frac{1}{2} \right) = 0 \rightarrow p_w = \frac{1}{2} \rightarrow p_r = \frac{3}{4}$$