Oligopoly:
Cournot, Bertrand, Stackelberg
- market power
- many firms: \( i = 1, 2, \ldots, I \) with cost function
  \[
  c_i(q_i), \quad c'_i(q_i) \geq 0, \quad c''_i(q_i) \geq 0
  \]
- each firm \( i \) produces quantity \( q_i \)
- aggregate quantity is
  \[
  q = \sum_{i=1}^{I} q_i; \quad q_{-i} = \sum_{j \neq i} q_i
  \]
- inverse demand associates price to quantity offered:
  \[
  p(\cdot) = q^{-1}(\cdot), \quad p'(q) < 0
  \]
Antoine Augustin Cournot (28/8/1801 – 31/3/1877): the supply of spring water

- Teacher of August Walras, father of Leon Walras, first to draw supply and demand curves, 30 years before Marshall

- The problem of the oligopolist is to choose quantity \( q_i \): 

\[
\max_{q_i} \left\{ p(q_i, q_{-i}) q - c(q_i) \right\}
\]

- Solving the oligopolist’s problem

\[
p'(q_i, q_{-i}) q_i + p(q_i, q_{-i}) - c'_i(q_i) = 0
\]

- Oligopolist is more aggressive than monopolist
Linear Demand and Constant Marginal Cost

- consider inverse demand
  \[ p(q) = a - bq, \quad a, b > 0 \]
- consider cost function (constant marginal cost)
  \[ c_i(q_i) = c \cdot q_i, \quad c > 0 \]
- the problem for the oligopolist is
  \[
  \max_q \left\{ \left( a - b \sum_j q_j \right) q_i - cq_i \right\}
  \]
- the first order condition lead to the best response:
  \[
  a - b \sum_{j \neq i} q_j - 2bq_i - c = 0 \iff q_i = \frac{a - b \sum_{j \neq i} q_j - c}{2b}
  \]
• we have the best response:

\[
\left( a - b \sum_{j \neq i} q_j - 2bq_i \right) - c = 0 \Leftrightarrow q_i = \frac{a - b \sum_{j \neq i} q_j - c}{2b}
\]

or

\[ q_i = BR_i (q_{-i}) \]

• the action are strategic substitutes

\[
\frac{dq_i}{dq_j} = \frac{dBR (q_j)}{dq_j} = -\frac{1}{2} < 0
\]

• the intersection of the best responses form an equilibrium, a Nash equilibrium
Nash Equilibrium

- an equilibrium (Nash) equilibrium is achieved if for all $i$:

$$q_i = \frac{a - b \sum_{j \neq i} q_j - c}{2b}$$

- by symmetry, we have $q_i^* = q_j^*$:

$$q_i^* = \frac{a - b (l - 1) q - c}{2b} \iff q_i^* = \frac{a - c}{b(l + 1)}$$

- aggregate supply is

$$lq_i^* = \frac{l}{l + 1} \frac{a - c}{b},$$

where

$$\frac{l}{l + 1} > \frac{1}{2}, \text{ for all } l > 1.$$
• firm $a$ is the first mover, firm $b$ is the second mover
• leadership advantage
• value of commitment
- firms compete in prices rather than quantities
- firm with lowest price receives the entire demand
- demand function is

\[ q \left( \min \{p_1, \ldots, p_l\} \right) \]

- all firms have same marginal cost:

\[ c(q_i) = cq_i \]
Differentiated Products

- consider

\[ q_i = a_i - p_i + bp_j \]

with \( b \in (0, 1) \)

- a higher price of a competitor leads to an increase of price \( j \)
- strategic complements
Hotelling Pricing Game

- linear city
- horizontally differentiated products
- marginal cost equal to zero