Economics 121b: Intermediate Microeconomics  
Problem Set 5 – Suggested Solutions  
2/29/12

1. (a) Expected utility of the lottery:

\[ E[U(x_1, x_2)] = \frac{3}{8} \times 0 + \frac{5}{8} \times \sqrt{100} = 6.25 \]

Utility of the expected value of the lottery:

\[ u(EV) = \sqrt{\frac{3}{8} \times 0 + \frac{5}{8} \times 100} = \sqrt{62.5} \approx 7.91 \]

The utility of the expected value of the lottery is larger than the expected utility of the lottery, hence this decision maker is risk averse.

(b) Certainty equivalent \( c \) of the lottery:

\[ u(c) = 6.25 \quad \Leftrightarrow \quad c = (6.25)^2 = 39.0625 \]

The certainty equivalent is smaller than the expected value of the lottery, which equals 62.5

(c) Consider the lottery \((x_1, \ldots, x_K; \pi_1, \ldots, \pi_K)\). If a decision maker is risk averse, he has a concave utility function. Let \( u(\cdot) \) be a concave (Bernoulli) utility function. By the definition of concave functions,

\[ u \left( \sum_{k=1}^{K} \pi_k x_k \right) \geq \sum_{k=1}^{K} \pi_k u(x_k) \quad (1) \]

with the inequality being strict if \( u(\cdot) \) is strictly concave. The certainty equivalent is defined as

\[ u(c) = \sum_{k=1}^{K} \pi_k u(x_k) \quad (2) \]

Equations (1) and (2) imply

\[ u \left( \sum_{k=1}^{K} \pi_k x_k \right) \geq u(c) \quad \Leftrightarrow \]

\[ \sum_{k=1}^{K} \pi_k x_k \geq c \]

Hence, the certainty equivalent is smaller than the expected value.
(d) The indifference curve is obtained as before in the two-good case:

\[ \pi_1 \sqrt{x_1} + \pi_2 \sqrt{x_2} = \bar{U} \iff x_2(x_1) = \left( \frac{\bar{U} - \pi_1 \sqrt{x_1}}{\pi_2} \right)^2 \]  

(3)

(e) The slope of the indifference curve is obtained by taking the derivative of equation (3):

\[ \frac{dx_2}{dx_1} = \frac{\bar{U} - \pi_1 \sqrt{x_1}}{\pi_2} \left( -\frac{1}{2} \frac{\pi_1}{\pi_2} \right) 
= \left( \frac{\pi_1}{\pi_2} \right)^2 - \frac{\pi_1 \bar{U}}{\pi_2^2 \sqrt{x_1}} \]  

(4)

Setting \( x_1 = x_2 \) in the utility function yields \( x_1 = \bar{U}^2 \). Plugging this into equation (4) yields

\[ \left. \frac{dx_2}{dx_1} \right|_{x_1=x_2} = \left( \frac{\pi_1}{\pi_2} \right)^2 - \frac{\pi_1 \bar{U}}{\pi_2^2 \sqrt{x_1}} 
= \frac{\pi_1^2 - \pi_1}{(1 - \pi_1)^2} 
= -\frac{\pi_1}{\pi_2} \]

where the last two equalities use the fact \( \pi_2 = 1 - \pi_1 \). At the point where the payoffs in both states are the same, the agent is willing to trade off payoffs in both states at the same rate as the ratio of the two probabilities. Note that this is similar to the solution of the utility maximization problem: At the optimal point (since the agent is risk averse it is optimal to equalize payoffs in both states) the slope the indifference curve is equal to the “price ratio” (the ratio of the probabilities).

(f) Take the second derivative of the indifference curve:

\[ \frac{d^2 x_2}{dx_1^2} = \frac{\pi_1 \bar{U}}{\pi_2^2 \sqrt{x_1}^3} > 0 \]

Hence, the indifference curve is convex and we have convex preferences.

2. The Arrow-Pratt measure of absolute risk aversion for the function \( u \) is defined by

\[ r^u(x) = -\frac{u''(x)}{u'(x)} \]
Let \( g(x) = f(u(x)) \), hence

\[
\begin{align*}
rg(x) &= -g''(x)g'(x) \\
&= -f''(u(x))u'(x) + f'(u(x))u''(x) \\
&= -\frac{f''(u(x))u'(x)}{f'(u(x))} - \frac{f'(u(x))u''(x)}{f'(u(x))} \\
&= -\frac{f''(u(x))}{f'(u(x))} \frac{u''(x)}{u(x)} \\
&= rf(u(x)) + ru(x)
\end{align*}
\]

Since \( rf(u(x)) \) and \( ru(x) \) are positive, \( rg(x) \) is larger than \( ru(x) \). However, if \( f \) is linear, \( f'' = 0 \) and \( rg(x) \) and \( ru(x) \) are the same.

3. (a) Expected utility of the lottery:

\[
E[U(x_1, x_2)] = \frac{1}{2} \sqrt{36} + \frac{1}{2} \sqrt{64} = 7
\]

(b) Expected utility of each agent under the arrangement:

\[
E[U(x_1, x_2)] = \frac{1}{4} \sqrt{36} + \frac{1}{4} \sqrt{64} + \frac{1}{4} \sqrt{36} + 14 + \frac{1}{4} \sqrt{64} - 14 \approx 7.04
\]

(c) No arrangement can yield higher expected utility than in (a) since no agent would be willing to transfer money to the other.

(d) Insurance only works in situations where risks are uncorrelated, e.g., it is unlikely that all the houses in the same city burn down at the same time or everyone’s car is stolen. Other natural disasters such as floods and earthquakes affect many people at the same time, which makes transfers to insure against these risks problematic.