Economics 121b: Intermediate Microeconomics
Problem Set 4:
Pareto Efficiency and Competitive Equilibrium
2/20/12

This problem set is due on Monday, 2/27/12, in class. To receive full credit, provide a complete defense of your answer.

1. Edgeworth Box and Pareto Efficiency. Consider an island economy with Friday and Robinson. They have agreed to share their resources and they have also agreed that the weight that Friday receives in the economy is \( w_f \in (0,1) \) and the weight that Robinson receives is \( w_r = 1 - w_f \). Their preferences are different. Friday likes bananas more than coconuts, and Robinson likes coconuts more than bananas (i.e. you can assume that \( \alpha \in (0,5,1) \)). The utility function are given by:

   \[ u_f(x_f) = \alpha \ln x_{fb} + (1 - \alpha) \ln x_{fc}, \]

and

   \[ u_r(x_r) = (1 - \alpha) \ln x_{rb} + \alpha \ln x_{rc}. \]

The endowment of the island is given by

   \[ e_b = 2, \quad e_c = 2c \geq 2 \]

(a) For every weight \( w_f \in (0,1) \), find the allocation which would maximize the social surplus given the weights; in other words, we are interested in finding the allocation \((x_{fb}, x_{fc}, x_{rb}, x_{rc})\) which maximizes the sum:

   \[ w_f u_f(x_f) + (1 - w_f) u_r(x_r) \]

subject to the resource constraints of the economy.

(b) For every weight \( w_f \in (0,1) \), can you find an initial endowment of bananas and coconuts among Robinson and Friday and a pair of prices that the Pareto efficient allocation actually constitutes an equilibrium of the market. (Here we decentralize the Pareto efficient allocation via a market equilibrium.) It is sufficient to discuss the case of \( c = 1 \).
2. Edgeworth Box and Competitive Equilibrium. Consider again Robinson and Friday. Now, Robinson and Friday have the same endowment of bananas and coconuts:

\[ x^f_b = x^r_b = 1 \]
\[ x^f_c = x^r_c = c > 1 \]

but their preferences are different.

(a) We normalize the price of bananas to be equal to one, or \( p_b = 1 \). Compute the demand function of Robinson and Friday as a function of the price for coconuts \( p_c \).

(b) Find the equilibrium price of this island economy and determine the net trading quantities.

i. Start with the case of \( c = 1 \).

ii. Continue to describe the equilibrium for general \( c > 1 \) and describe the intuition.