Expenditure and Welfare
Elasticities (Price)

- Price elasticities:

\[ \varepsilon_{i,j} = \frac{dx_i^*}{dp_j} \approx \frac{\% \text{ change in demand } x_i^*}{\% \text{ change in price } p_j} \]

- Price elasticities are typically negative (but often we refer to them in absolute values, hence disregard the negative sign)

- Using log functions we get a linear relationship (as if often appears in empirical work)

\[ \varepsilon_{i,j} = \frac{dx_i^*}{dp_j} \approx \frac{\partial \log x_i^*(p, l)}{\partial \log l} \]
income elasticities:

\[ \varepsilon_{i,l} = \frac{\frac{dx_i^*}{dl}}{\frac{x_i^*}{l}} = \frac{\frac{dx_i^*}{l}}{\frac{dl}{l}} \approx \frac{\% \text{ change in demand } x_i^*}{\% \text{ change in income } l} \]
• inferior good:

\[ \varepsilon_{i,l} = \frac{dx_i^*}{dl} \frac{l}{x_i^*} < 0, \]

• normal good:

\[ \varepsilon_{i,l} = \frac{dx_i^*}{dl} \frac{l}{x_i^*} \in (0, 1), \]

• luxury good

\[ \varepsilon_{i,l} = \frac{dx_i^*}{dl} \frac{l}{x_i^*} > 1 \]
the budget share of commodity $i$ is:

$$s_i = \frac{p_i x_i^*}{l}$$

an implication of the luxury good is that its budget share is increasing with income $l$:

$$\frac{ds_i}{dl} = \frac{p_i \frac{dx_i^*}{dl} l - p_i x_i^*}{l^2} > 0$$

$\Leftrightarrow \ \frac{dx_i^*}{dl} l - x_i^* > 0$

$\Leftrightarrow \ \frac{dx_i^*}{dl} > \frac{x_i^*}{l}$

$\Leftrightarrow \ \frac{dx_i^*}{dl} \frac{l}{x_i^*} > 1$. 
Expenditures and Revenues

- own price elasticity

\[ \varepsilon_{i,i} = \frac{dx_i^*}{dp_i} \frac{x_i^*}{p_i} \]

- revenue is given by:

\[ R_i = p_i x_i^* \]

- demand is called inelastic if

\[ R_i' (p_i) > 0 \iff \varepsilon_{i,i} \in (-1, 0) \]

- demand is called elastic if

\[ R_i' (p_i) > 0 \iff \varepsilon_{i,i} \in (-\infty, -1) \]

- demand is called unit-elastic if

\[ R_i' (p_i) = 0 \iff \varepsilon_{i,i} = -1 \]
Elasticity and Revenues

- recall the revenues

\[ R_i' (p_i) = x_i^* + p_i x_i'^* (p_i) > 0 \]

\[ \Leftrightarrow \frac{dx_i^* (p_i)}{dp_i} > - \frac{x_i^*}{p_i} \]

\[ \Leftrightarrow \frac{dx_i^* (p_i)}{dp_i} x_i^* > -1 \]

- recall the elasticity

\[ \varepsilon_{i,i} = \frac{dx_i^*}{dp_i} x_i^* \]

- revenue is given by:

\[ R_i = p_i x_i^* \]

- demand is called inelastic if

\[ R_i' (p_i) > 0 \Leftrightarrow \varepsilon_{i,i} \in (-1, 0) \]
• Estimating Marshallian and Hicksian demand
Data and Giffen Good

- From Ireland to China, from Potatoes to Rice