1 Lecture 9: Elasticities Continued

Definition 1. A luxury good is defined as one for which the elasticity of income is greater than one. Therefore for a luxury good $i$,

$$\epsilon_{i,I} = \frac{dx^*_i}{x^*_i dI} > 1$$

We can also define luxury good in the following alternative way.

Definition 2. If the budget share of a good is increasing in income then it is a luxury good.

Before we explain the equivalence of the two definitions let us first define the concept of budget share. Budget share of good $i$, denoted by $s_i(I)$, is the fraction of income $I$ that is devoted to the expenditure on that good. Therefore,

$$s_i(I) = \frac{p_i x^*_i(p, I)}{I}$$

Now to see how the two definitions are related we take the derivative of $s_i(I)$ w.r.t. $I$.

$$\frac{ds_i(I)}{dI} = \frac{dx^*_i}{x^*_i} \frac{p_i I - p_i x^*_i}{I^2}$$

Now if good $i$ is luxury then we know that,

$$\frac{ds_i(I)}{dI} > 0$$

$$\iff \frac{dx^*_i}{x^*_i} \frac{p_i I - p_i x^*_i}{I^2} > 0$$

$$\iff \frac{dx^*_i}{dI} p_i I - p_i x^*_i > 0$$

$$\iff \frac{dx^*_i}{dI} p_i I - p_i x^*_i > 0$$
Therefore we see that the two definitions of luxury good are equivalent. Hence a
luxury good is one which a consumer spends more, proportionally, as her income
goes up.

**Definition 3. Revenue** from a good $i$ is defined as the following:

$$R_i(p_i) = p_i x_i^*$$

Differentiating $R_i(p_i)$ w.r.t. $p_i$ we get,

$$R'_i(p_i) = x_i^* + p_i \frac{dx_i^*}{dp_i}$$

$$= x_i^* \left[ 1 + \frac{p_i}{x_i^*} \right] \frac{dx_i^*}{dp_i}$$

$$= x_i^* \left[ 1 + \epsilon_{i,i} \right]$$

We say that:

Demand is inelastic if,

$$R'_i(p_i) > 0 \Rightarrow \epsilon_{i,i} \in (-1, 0)$$

Demand is elastic if,

$$R'_i(p_i) < 0 \Rightarrow \epsilon_{i,i} \in (-\infty, -1)$$

Demand is unit-elastic if,

$$R'_i(p_i) = 0 \Rightarrow \epsilon_{i,i} = -1$$