This problem set is due 2/13/13

1. Prove the following lemma that we used in class. Lemma: Suppose f satisfies cycle monotonicity. For any \( r, s, t \in T \) with \( s \neq t \), we have

\[
\text{dist}_{T_f}(r, t) \leq \text{dist}_{T_f}(r, s) + l(s, t).
\] (0.1)

2. Prove the following result that we discussed in class: Theorem: The following are equivalent for the allocation rule f: (1.) The allocation rule f is DSIC; (2.) The type graph \( T_f \) has no cycle of negative length. (3.) The allocation graph \( A_f \) has no cycle of negative length.

3. We called a generalized Groves payment rule

\[
p_i(v) = h_i(v_{-i}) - \frac{1}{\lambda_i} \sum_{j \neq i} \lambda_j v_j(f^n(v)) + \kappa(f^n(v)),
\]

if \( \lambda_i > 0 \), where \( h_i : V_{-i} \to \mathbb{R} \) is an arbitrary function. Prove the sufficiency part of (Roberts 1979) theorem. Theorem: Every affine maximizer satisfying UIA can be implemented by the generalized Groves payment rule. (See (Mishra 2012), p8 for the definition of UIA).

References
