(Commodity) Bundling

- Adams and Yellen, Commodity Bundling and the Burden of Monopoly, QJE 1976
- in the movie "Five Easy Pieces" Jack Nicholson enters a diner and order toast and coffee. The waitress informs him that toast cannot be order separately. He is forced to order a chicken salad sandwoch without chickenn, salad, mayonnaise.
- three strategies
  - pure component pricing
  - pure bundle pricing
  - mixed bundle pricing
- valuation of the consumer is

\[ u = u_1 + u_2 \]

where \( u_1 \) and \( u_2 \) are (independently) distributed
graphic representation of pricing strategies in utility space

- pure component pricing leads to four rectangles
- pure bundling leads to a line with slope $-1$
- mixed bundling leads to four areas, rectangles and slope $-1$ due to the coexistence of pure bundling and pure component pricing
- often referred to as tying, Whinston (1987, unpublished) shows that a firm with monopoly power in one market can use it to establish monopoly power in a second markets, even if the products are completely unrelated
Sufficient Condition for Optimality of Bundling

- McAfee, McMillan, Whinston: "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values", QJE 1989
- explicit condition for the optimality of mixed bundling to component pricing
- pure bundling can be shown to be weakly dominated in all cases to mixed bundling
- Corollary 1: With independent values, mixed bundling strictly dominates pure component pricing.
- graphic illustration: start with optimal prices in the absence of bundling
- now introduce a bundle equal to the sum of the optimal prices...
- now increases the price of the second good by $\varepsilon$, by optimality the direct gains are netting to zero, but there are indirect gains in the presence of the bundled price
Multi-Dimensional Screening

- binary types, binary allocations
- utility of the agent is given by:
  \[ u_i^A (x^A) + u_j^B (x^B) \]
- utility of the principal is given by:
  \[ v_i^A (x^A) + v_j^B (x^B) \]
- probability
  \[ \alpha_{ij} = \Pr (i \in \{L, H\}, j \in \{L, H\}) \]
• consider the problem where only the five downward incentive constraints are (possibly) binding
• graphic illustration
The Nature of the Local Constraints

1. Strong Positive Correlation (all downwards constraints are binding)
2. Weak Positive Correlation (only the box constraints are binding)
3. Negative Correlation (towards activity $A$) $hh \rightarrow hl$ is not binding
4. Negative Correlation (towards activity $B$) $hh \rightarrow lh$ is not binding
Is the Relaxed Problem the Complete Problem?

- only in the case of positive correlation
- otherwise orientation of the constraints can change
- with negative correlation, it may go from $hl \rightarrow hh$ and $hl \rightarrow lh$ in case c
- with negative correlation, it may go from $lh \rightarrow hh$ and $lh \rightarrow hl$ in case d
An Example: Revenue Decrease

Maximal Revenue with Multiple Goods, Hart and Reny, 2012, unpublished

- valuation is given by
  \[ x_1, x_2 \]

- consider the following mechanism: buyer is offered good 1 at price 1, good 2 at price 2, or get both goods for price 4

- we can then map the optimal choice by the bidder as a function of his valuation

  \[ b(x_1, x_2) = \max \{ 0, x_1 - 1, x_2 - 2, x_1 + x_2 - 4 \} \]

- now here comes the surprising aspect:
  - suppose the valuation increases from (1.3, 2.4) to (1.7, 2.6), then it is optimal to switch to buy good 1 rather than good 2, and so the revenue increases.
An Example: Revenue Decrease with Optimal Mechanism

- let the valuations be given by

\[
\begin{align*}
(1, 1) & \quad p = \frac{1}{4} \\
(1, 2) & \quad p = \frac{1}{4} - \alpha \\
(2, 2) & \quad p = \alpha \\
(2, 3) & \quad p = \frac{1}{2}
\end{align*}
\]

- for \(0 \leq \alpha \leq 1/4\), the distribution is first order stochastic dominant
- nevertheless maximal revenue decreases with \(0 \leq \alpha \leq 1/12\)
• Result 1: maximal revenues equals $11/4 - \alpha$ for $0 \leq \alpha \leq 1/12$
• given the mechanism, the revenue is given by

$$\frac{1}{4} \cdot 1 + \left(\frac{1}{4} - \alpha\right) \cdot 2 + \alpha \cdot 1 + \frac{1}{2} \cdot 4 = 11/4 - \alpha$$

• we leave the optimality of the mechanism outside
• the same example can be done with independent valuations
• in single object auction, there always exists a deterministic mechanism

• with many objects that is not the case, consider the following example with correlated values

\[
\begin{align*}
(1, 0) & \quad p = \frac{1}{3} \\
(0, 2) & \quad p = \frac{1}{3} \\
(3, 3) & \quad p = \frac{1}{3}
\end{align*}
\]

and the following mechanism

<table>
<thead>
<tr>
<th>value</th>
<th>quantity</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 0)</td>
<td>(\frac{1}{2}, 0)</td>
<td>\frac{1}{2}</td>
</tr>
<tr>
<td>(0, 2)</td>
<td>(0, 1)</td>
<td>2</td>
</tr>
<tr>
<td>(3, 3)</td>
<td>(1, 1)</td>
<td>5</td>
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</tbody>
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• this induces the best response (bid)

\[ b(x_1, x_2) = \max \left\{ \begin{align*}
0, & \frac{1}{2}x_1 - \frac{1}{2}, \\
& x_2 - 2, \\
& x_1 + x_2 - 5
\end{align*} \right\} \]

• and yields a revenue of 2.5

• by contrast the best deterministic mechanism is given by (in fact component pricing) which yields a revenue of \( \frac{7}{3} \):

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• what happens if we replace the fractional constraint (i.e. the expectation) by its component \((0, 0)\) and \((1, 1)\)

• in either case, the revenue would actually decrease (by losing revenue directly for \((0, 0)\) and forcing lower revenues on the bundle \((3, 3)\) to 4.

• in the single unit case, replacing the lottery by its constituting elements leads exactly to the average

• multiple incentive constraints are at force at every single allocation
Returning to the Single Unit Environment

- to complete the comparison let us consider the single unit environment
- replacing the lottery outcome when there is a single good:

<table>
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<th>price</th>
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<th>$p'$</th>
<th>$q''$</th>
<th>$p''$</th>
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