1 Lecture 14: Theory of Production

We can use tools similar to those we used in the consumer theory section of the class to study firm behaviour. In that section we assumed that individuals maximize utility subject to some budget constraint. In this section we assume that firms will attempt to maximize their profits given a demand schedule and production technology.

Firms use inputs or commodities $x_1, \ldots, x_I$ to produce an output $y$. The amount of output produced is related to the inputs by the production function $y = f(x_1, \ldots, x_I)$, which is formally defined as follows:

**Definition 1.** A production function is a mapping $f : \mathbb{R}_+^I \rightarrow \mathbb{R}_+$.

The prices of the inputs/commodities are $p_1, \ldots, p_I$ and the output price is $p_y$. The firm takes prices as given and independent of its decisions.

Firms maximize their profits by choosing the optimal amount and combination of inputs.

$$\max_{x_1, \ldots, x_I} p_y f(x_1, \ldots, x_I) - \sum_{i=1}^I p_i x_i. \quad (1)$$

Another way to describe firms’ decision making is by minimizing the cost necessary to produce an output quantity $\bar{y}$.

$$\min_{x_1, \ldots, x_I} \sum_{i=1}^I p_i x_i \text{ s.t. } f(x_1, \ldots, x_I) \geq \bar{y}.$$  

The minimized cost of production, $C(\bar{y})$, is called the cost function.

We make the following assumption for the production function: positive marginal product

$$\frac{\partial f}{\partial x_i} \geq 0$$
and declining marginal product
\[
\frac{\partial^2 f}{\partial x_i^2} \leq 0.
\]

The optimality conditions for the profit maximization problem (1) and the FOCs for all \( i \)
\[
p_y \frac{\partial f}{\partial x_i} - p_i = 0.
\]
In other words, optimal production requires equality between marginal benefits and marginal cost of production. The solution to the profit maximization problem then is
\[
x_i^*(p_1, \ldots, p_I, p_y), \quad i = 1, \ldots, I
\]
\[
y^*(p_1, \ldots, p_I, p_y),
\]
i.e., optimal demand for inputs and optimal output/supply.

The solution of the cost minimization problem (4), on the other hand is
\[
x_i^*(p_1, \ldots, p_I, \bar{y}), \quad i = 1, \ldots, I,
\]
where \( \bar{y} \) is the firm’s production target.

**Example 1.** One commonly used production function is the Cobb-Douglas production function where
\[
f(K, L) = K^\alpha L^{1-\alpha}
\]
The interpretation is the same as before with \( \alpha \) reflecting the relative importance of capital in production. The marginal product of capital is \( \frac{\partial f}{\partial K} \) and the marginal product of labor is \( \frac{\partial f}{\partial L} \).

In general, we can change the scale of a firm by multiplying both inputs by a common factor: \( f(tK, tL) \) and compare the new output to \( tf(K, L) \). The firm is said to have **constant returns to scale** if
\[
 Tf(K, L) = f(tK, tL),
\]
it has **decreasing returns to scale** if
\[
 Tf(K, L) > f(tK, tL),
\]
and **increasing returns to scale** if
\[
 Tf(K, L) < f(tK, tL).
\]
Example 2. The Cobb-Douglas function in our example has constant returns to scale since
\[
f(tK, tL) = (tK)^\alpha (tL)^{1-\alpha} = tK^\alpha L^{1-\alpha} = tf(K, L).
\]

Returns to scale have an impact on market structure. With decreasing returns to scale we expect to find many small firms. With increasing returns to scale, on the other hand, there will be few (or only a single) large firms. No clear prediction can be made in the case of constant returns to scale. Since increasing returns to scale limit the number of firms in the market, the assumption that firms are price takers only makes sense with decreasing or constant returns to scale.

2 Lecture 15: Imperfect Competition

2.1 Pricing Power

So far, we have considered market environments where single agent cannot control prices. Instead, each agent was infinitesimally small and firms acted as price takers. This was the case in competitive equilibrium. There are many markets with few (oligopoly) or a single firms, (monopoly) however. In that case firms can control prices to some extent. Moreover, when there are a few firms in a market, firms make interactive decisions. In other words, they take their competitors’ actions into account. In Section ??, we will use game theory to analyse this type of market structure. First, we cover monopolies, i.e., markets with a single producer.

2.2 Monopoly

If a firm produces a non-negligible amount of the overall market then the price at which the good sells will depend on the quantity sold. Examples for firms that control the overall market include the East India Trading Company, Microsoft (software in general because of network externalities and increasing returns to scale), telecommunications and utilities (natural monopolies), Standard Oil, and De Beers.

For any given price there will be some quantity demanded by consumers, and this is known as the demand curve \( x : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) or simply \( x(p) \). We assume that consumers demand less as the price increases: the demand function is downward sloping or \( x'(p) < 0 \). We can invert this relationship to get the inverse demand function \( p(x) \) which reveals the price that will prevail in the market if the output is \( x \).

If the firm is a monopolist that takes the demand data \( p(x) \) as given then its goal is to maximize
\[
\pi(x) = p(x)x - c(x)
\]
by choosing the optimal production level. For the cost function we assume $c'(x) > 0$ and $c''(x) \geq 0$, i.e., we have positive and weakly increasing marginal costs. For example, $c(x) = cx$ satisfies these assumptions (a Cobb-Douglas production function provides this for example). The monopolist maximizes its profit function (2) over $x$, which leads to the following FOC:

$$p(x) + xp'(x) - c'(x) = 0.$$  (3)

Here, in addition to the familiar $p(x)$, which is the marginal return from the marginal consumer, the monopolist also has to take the $xp'(x)$ into account, because a change in quantity also affects the inframarginal consumers. For example, when it increases the quantity supplied, the monopolist gets positive revenue from the marginal consumer, but the inframarginal consumers pay less due to the downward sloping demand function. At the optimum, the monopolist equates marginal revenue and marginal cost.

**Example 3.** A simple example used frequently is $p(q) = a - bq$, and we will also assume that $a > c$ since otherwise the cost of producing is higher than any consumer’s valuation so it will never be profitable for the firm to produce and the market will cease to exist. Then the firm want to maximize the objective

$$\pi(x) = (a - bx - c)x.$$  

The efficient quantity is produced when $p(x) = a - bx = c$ because then a consumer buys an object if and only if they value it more than the cost of producing, resulting in the highest possible total surplus. So the efficient quantity is

$$x^* = \frac{a - c}{b}.$$

The monopolist’s maximization problem, however, has FOC

$$a - 2bx - c = 0$$

where $a - 2bx$ is the marginal revenue and $c$ is the marginal cost. So the quantity set by the monopolist is

$$x^M = \frac{a - c}{2b} < x^*.$$

The price with a monopoly can easily be found since

$$p^M = a - bx^M = a - \frac{a - c}{2} = \frac{a + c}{2} > c.$$

Figure 1 illustrates this.
A monopoly has different welfare implications than perfect competition. In Figure 1, consumers in a monopoly lose the areas A and B compared to perfect competition. The monopolist loses area C and wins area A. Hence, there are distributional implications (consumers lose and the producer gains) as well as efficiency implications (overall welfare decreases).

We can write the monopolist’s FOC (3) in terms of the demand elasticity introduced in Section ?? as follows:

\[
p(x^*) + x^*p'(x^*) = c'(x^*) \iff \\
p(x^*) \left[1 + \frac{x^*p'(x^*)}{p(x^*)}\right] = c'(x^*) \iff \\
p(x^*) = \frac{c'(x^*)}{1 + \epsilon_p^{-1}}.
\]

Since \(\epsilon_p < 0\), we have that \(p(x^*) > c'(x^*)\), in other words, the monopolist charges more than the marginal cost. This also means that if demand is very elastic, \(\epsilon_p \to \infty\), then \(p(x^*) \approx c'(x^*)\). On the other hand, if demand is very inelastic, \(\epsilon_p \approx -1\), then \(p(x^*) \gg c'(x^*)\).