Optimal Information Disclosure
true valuation $V_i$

initial noisy signal $v_i$: initial distribution $f_i, F_i; f_i (1 - F_i)$ monotone hazard ratio

conditional distribution of $V_i$

$h_{i|v_i} = h_i (V_i | v_i), H_{i|v_i} = H_i (V_i | v_i)$

additional signal $z_i$

assume $\mathbb{E}[V_i | v_i, z_i]$ is increasing in $z_i$
• first order stochastic dominance: \( \partial H_{iv_i}/\partial v_i < 0 \)
• monotonicity conditions:

\[
(i) \quad \frac{\partial H_{iv_i}(V_i)}{h_{iv}(V_i)} \text{ increasing in } V_i;
\]

\[
(ii) \quad \frac{\partial H_{iv_i}(V_i)}{h_{iv}(V_i)} \text{ increasing in } v_i;
\]

• compare to the conditions in Baron and Besanko (1984), guarantees monotonicity of the allocation
instead of \((z_i, v_i)\), we consider observing a new signal
\((s_i(z_i, v_i), v_i)\)

- \(s_i\) is strictly increasing in \(z_i\), hence preserves information of \(z_i\)
- now, \(s_i\) is a transformation of \(z_i\), but one which makes \(s_i\)
independent of \(v_i\)
- hence we call \(s_i\) "shock", "new information"
Lemma

1. There exists function $u_i$ and $s_i$ such that

$$u_i(v_i, s_i(z_i, v_i)) = V_i,$$

such that $u_i$ is strictly increasing in $z_i$ and $s_i(z_i, v_i)$ is independent of $v_i$.

2. All $s_i$ satisfying (1) are monotone transformation of each other.
Proof of Orthogonalization

- define $s_i (z_i, v_i) \triangleq H_{iv_i} (V_i)$, we identify $s_i$ as the percentile of the distribution of $V_i | v_i$
- observe that for any $y \in [0, 1]$

\[
\Pr (H_{iv_i} (V) \leq y) = \Pr (V \leq H_{iv_i}^{-1} (y)) = H_{iv_i} \left( H_{iv_i}^{-1} (y) \right) = y
\]

- that is $s_i$ is uniform on $[0, 1]$ irrespective of $v_i$ and hence independent of $v_i$
- finally let

\[
u_i (v_i, s_i) = H_{iv_i}^{-1} (s_i)
\]

- and monotoncity and equivalence of $s_i$ are easy to establish
Implications of Utility Function

- earlier assumption \((i)\) is equivalent to
  \[ u_{i12} \leq 0 \]

- earlier assumption \((ii)\) is equivalent to
  \[ u_{i11}/u_{i1} \leq u_{i12}/u_{i2} \]

- open question, does this imply that certain multidimensional conditions for incentive compatibility are satisfied

- probability integral transform (Angus (1994)): In statistics, the probability integral transform relates to the result that data values that are modelled as being random variables from any given continuous distribution can be converted to random variables having a uniform distribution.
Implications of Independent News

- consider the benchmark case when $s_i$ is observed
- optimal allocation follows the virtual utility rule (Proposition 1):
  \[ u_i(v_i, s_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} u_{i1}(v_i, s_i) \]  
  \[ (1) \]
- important implications follow regarding the allocation rule

Lemma

1. $X_i^*$ is continuous in $v_i, s_i$;
2. $X_i^*$ is weakly increasing in $v_i, s_i$;
3. If $v_i > \hat{v}_i$, $s_i < \hat{s}_i$, and $u_i(v_i, s_i) = u_i(\hat{v}_i, \hat{s}_i)$, then $X_i^*(v_i, s_i) \geq X_i^*(\hat{v}_i, \hat{s}_i)$. 
we can now ask whether we want to report consistently (as a weak form of truthtelling) in the continuation game (this is Lemma 4 of ES)

Lemma

In period $t = 1$, buyer $i$ with type $v_i$ who reported $\hat{v}_i$ and has observed $s_i$ will report

$$\hat{s}_i = \sigma_i (v_i, \hat{v}_i, s_i)$$

such that

$$u_i (v_i, s_i) = u_i (v_i, \sigma_i (v_i, \hat{v}_i, s_i))$$

this allows us to describe the continuation payoff function (including optimal deviation strategies in period 1) and leads to theorem 1
Sufficient Conditions

- the conditions (i) and (ii) guarantee that (1) is satisfied pointwise, not only in expectation over \( s_i \), hence they are sufficient conditions, but not necessary, as they are overly strong.
- in the benchmark case (with complete information) a necessary and sufficient condition that \( X_i^* \) can be implemented is for all \( v_i, \hat{v}_i \):

\[
\begin{align*}
\int \int u_{i1} (y, s_i) X_i^* (y, s_i) dydG (s_i) \\
\geq \\
\int \int (u_{i1} (y, s_i) - u_i (\hat{v}_i, s_i)) X_i^* (\hat{v}_i, s_i) dydG (s_i)
\end{align*}
\]

(2)
- in the original model (with incomplete information), a necessary and sufficient condition that \( X_i^* \) can be implemented is for all \( v_i, \hat{v}_i \):

\[
\int \int u_{i1} (y, s_i) X_i^* (y, s_i) dydG (s_i)
\]
notice that we can write

\[ u_i (v_i, s_i) = V_i \]

and

\[ u_{i1} (v_i, s_i) = \frac{\partial H_{iv_i} (V_i) / \partial v_i}{h_{iv} (V_i)} \]

thus the virtual utility is

\[ V_i - \frac{1 - F_i (v_i)}{f_i (v_i)} \cdot \frac{\partial H_{iv_i} (V_i) / \partial v_i}{h_{iv} (V_i)} \]
How does the Example violate the ES conditions

- consider the differentiability conditions